提要 4：如何建立數學模式？(三)

再舉一混合溶液之濃度變化問題說明如何建立數學模式(請參考綠色框線內之說明)。

Physical System
(實際系統)

已知 100 加侖的水槽中溶有 20 磅的鹽，若每分鐘有另一種濃度(每一加侖含 2 磅鹽)的鹽水 5 加侖流入水槽中，充分混合後，每分鐘水槽亦排出 5 加侖的鹽水，試求任意時刻 t 水槽中之鹽重。

Modeling
(模式化)

由物理試驗知，質量守恆定律爲：

後來的鹽重 = 原來的鹽重
+ 流入的鹽重
− 流出的鹽重 (1)

設 t 分鐘時，水槽中之鹽重爲 y 磅，則式(1)可表爲

$$y(t+\Delta t)\text{磅} = y(t)\text{磅} + (10\text{磅/分})(\Delta t\text{分}) - \left(\frac{5\text{加侖/分}}{100\text{加侖}}\right)[y(t)\text{磅}](\Delta t\text{分})$$

故

$$\frac{y(t+\Delta t)-y(t)}{\Delta t} = 10-0.05y(t)$$

當 $\Delta t \to 0$ 時，上式又可改寫爲

$$\frac{dy}{dt} = 10-0.05y$$

另外，剛開始時，水槽中之鹽重爲 20 磅，故初始條件可表爲 $y(0)=20$ 磅。

Mathematical Model
(數學模式)

1. 控制方程式：$\frac{dy}{dt} = 10-0.05y$
2. 初始條件：$y(0)=20$ 磅

Solution Methods
(解題方法)

可採用變數可分離之微分方程式的解法推求問題之解。

Solution(解)

$y = 200-180e^{-0.05t}$

(請參考提要 9 例題 3 之說明)

Notice:

1. 控制方程式 $\frac{dy}{dt} = 10-0.05y$ 實際上乃是質量守恆定律的化身。

習題

1. A tank contains 3 m^3 of pure water initially. Solution with the concentration of 1 mole/m^3 flows into the tank in the flow rate of 1 m^3/min and mixes uniformly with the contents. An equal volume of which is forced out of the tank. Calculate the time required when the concentration in the tank is 0.5 mole/m^3.【94 中原化工所 10%】

2. 一水槽中裝有含 160 公克鹽量之水溶液共 1,000 立方米，假設每單位時間有 40 立方米的海水流入槽中，並均勻混合。海水中每立方米含鹽量爲(1+cos*t*)公克，而槽中水溶液的流出率爲每單位時間 40 立方米，試問槽中水溶液在任意 t 時間的含鹽量 $y(t)$ 爲何？【93 成大機械所 20%】

3. The mixing of the two salt solutions of differing concentrations gives rise to a first-order differential equation for the amount of salt contained in the mixture. Let us suppose that a large mixing tank holds 300 gallons of water in which salt has been dissolved. Another brine solutions is pumped into the large tank at a rate of 3 ga/min, and then when the solution is well stirred it is pumped out at the same rate.
   (a) If the concentration of the solution entering is 2 lb/ga, determine a model for the amount of the salt in the tank at any time.
   (b) If 60 pounds of salts was dissolved in the initial 300 gallons, how much salt would be in the tank after a long time?【89 台大電機所 13%】

4. 一水槽中裝有 200 公升含鹽之水溶液，鹽之濃度爲 1.0 克/公升，現以每分鐘 2 公升之速率加入清水，同時水槽亦以同樣之速率排水，假設在加水及排水的過程中水溶液皆均勻攪拌，請問使水溶液鹽之濃度成爲原來的 1%所需時間？【87 中央土木所 10%】

5. A 400 gallons tank initially is half full of a fluid in which there is 50 gm of salt are dissolved. A mixture consisting of 2 gm of salt per gallon is flowing into the tank at a rate of 10 gallons per minute. Meanwhile, the brine is being drawn off simultaneously at the rate of 4 gallons per minute. Will the amount of the salt in the tank reach 600 gm before the tank is filled?【88 雲科電機所 20%】

6. A 500-gallon tank initially contains 100 gallons of brine in which 5 pounds of salt have been dissolved. Brine containing 2 pounds per gallon is added at the rate of 5 gallons per minute, and the mixture is poured out of the tank at the rate of 3 gallons per minute. Determine how much salt is in the tank at the moment it overflows.【88 台大化工所 10%】