第一類習題：級數解法

1. Find two power series solutions of the differential equation $\left(x^2-1\right)y''+xy'-y=0$ about the ordinary point $x=0$.【94 中興機械 20%，93 成大電機 20%】

2. Using power series method, $y=\sum_{n-0}^{\infty}c_n x^n$, to solve $y''+3x^2y'-6y=x$, $y(0)=0$, $y'(0)=2$.【93 交大電子 8%】

3. Find the series solution of $xy'-y-x-1=0$ in power of $(x-1)$.【94 輔仁電子 16%】

4. $y''+(2x-2)y'+\left(x^2-2x+2\right)y=0$, $y(1)=3$, $y'(1)=-1$, find series solution about $x=1$ at least four nonzero terms.【92 元智電機 30%】

5. Find a power series solution in powers of x of the following differential equation.
$y''-4xy'+\left(4x^2-2\right)y=0$ 【94 淡江化工 25%】

6. 單選題，每題恰有一解，答對一小題給 3 分，答錯或不答，不給分也不扣分。
For the IVP $\begin{cases}\left(x^2-2x+3\right)y^{(2)}-3y^{(1)}+\left(x-2\right)y=0\\ y\left(2\right)=-20,\ y^{(1)}\left(2\right)=-2\end{cases}$, the power-series solution about the initial point is $y\left(x\right)=\sum_{n=0}^{\infty}a_n\left(x-2\right)^n$. Then
(1) $a_0=$ (A)1 (B)-1 (C)2 (D)-2 (E)none (3%)
(2) $a_1=$ (A)1 (B)-1 (C)2 (D)-2 (E)none (3%)
(3) $a_2=$ (A)1 (B)-1 (C)2 (D)-2 (E)none (3%)
(4) $a_3=$ (A)1 (B)-1 (C)2 (D)-2 (E)none (3%) 【94 交大電機】

7. For the following equation,
$$\frac{d^2y}{dx^2}-e^{2x}y=0$$
Please find the solution based on the power series method and write out first 5 non-zero terms in the solution.【93 清大電機 10%】

8. The Legendre equation is given as $\left(1-x^2\right)y''-2xy'+n\left(n+1\right)y=0$ when n is a given number. Use power series method $y=\sum_{m=0}^{\infty}a_m x^m$ to solve the ODE.

(1) Derive the recurrence equation. (5%)

(2) Express a_2, a_4 in terms of a_0. (3%)

(3) Express a_3, a_5 in terms of a_1. (3%)

(4) Find the general solution $y=a_0y_1+a_1y_2$. (3%)

(5) Let $n=0$, find y_1 and y_2. (3%)

(6) Prove that y_2 in (5) can be written as $y_2=\frac{1}{2}\ln\frac{1+x}{1-x}$. (3%)

(7) Solve $y\left(1-x^2\right)y''-2xy'=0$ by $z=y'$, compare the solution in (6). (10%)

【93清大電機10%】

9. Use the Maclaurin series to solve the general solution.
$\left(x^2+1\right)y''+2xy'=0$, $y\left(0\right)=0$, $y'\left(0\right)=1$.【93 台大電機 7%】

10. Given $\left(x-1\right)y''+y'+2\left(x-1\right)y=0$,

(1) Find two linearly independent power series solutions with center $x=4$, with at least four nonzero terms for each series solution. Justify the solutions are linearly independent.

(2) Solve for $y\left(4\right)=1$, $y'\left(4\right)=1$.【90 元智電機控制組 30%】

11. $\left(1-x^2\right)y''-xy'+y=x$, find series solution.【91 彰師光電 10%】

12. 以下的答案中，那些是$y''-\left(1+x\right)y=0$的解。(複選)

(A) $y(x)=1+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{18}x^4+\frac{1}{36}x^5+\cdots$

(B) $y(x)=x+\frac{1}{6}x^3+\frac{1}{12}x^4+\frac{1}{120}x^5+\cdots$

(C) $y=\sum_{n-0}^{\infty}c_nx^n$, $c_1=0$, $c_{k+2}=\frac{c_k+c_{k-1}}{\left(k+1\right)\left(k+2\right)}$, $k=1,2,3,\cdots$

(D) $y(x)=\frac{1}{3}x^3+\frac{1}{15}x^4+\frac{1}{60}x^5+\cdots$

(E) $y=\sum^{\infty}c_n x^n$, $c_0=0$, $c_{k+2}=\frac{c_k+c_{k-1}}{(k+1)(k+2)}$, $k=1,2,3,\cdots$

(F) 以上皆非【91 台大電機 5%】

13. Solve $y''+\sin y=0$, $y(0)=\frac{\pi}{6}$, $y'(0)=0$ by power series method.【94 台大應力 10%】

14. $2y'+xy'+y=0$, solve by power series method.【94 中興材料 10%】

15. $y''-xy=2x$, $y(0)=3$, $y(1)=0$, solve by series method. 【92 淡江化工 15%】

16. Solve $y''+(1+x+x^2+2x^3)y'+3y=3x+5x^2$.【94 交大土木 20%】

17. $y''+y'=0$, solve by series method.【92 海洋光電 16%】

18. $y'''+y''+x^3y=0$, solve by series method.【93 台大生物環境 15%】

19. 已知 $y''-x^2y'+(x+2)y=x$，$y(0)=2$，$y'(0)=1$，試解之並至少寫出前 5 項。【93 台大生機 10%】

20. $y''-2y'+x^3y=0$, find the first nonzero terms of series solution about $x=0$.【93 台科電機 10%】

21. $y''+\sin x\cdot y=0$ solve by series method.【93 中正電機 4%】

22. By using series expansion, find a solution for equation:

(1) $\frac{d^2y}{dx^2}=xy$ with $-\infty<x<\infty$. (7%)

(2) $\frac{d^2y}{dx^2}-2x\frac{dy}{dy}+\lambda y=0$, $-\infty<x<\infty$, and λ being a constant. (8%)【93 海洋光電 15%】

23. $y''+ty'-y=1+t^2$, find series solution about $t=0$.【93 元智光電 20%】

24. Solve $y''+xy=4$, $y(1)=2$, $y'(1)=0$.【92 嘉義生機 10%】

25. $(x^2+1)y''-y'+y=0$, find series solution about $x=0$ at least up to x^4.【92 交大電信 10%】

26. Find the power series solution of the following initial value problem about x = 1.

$xy''-y'+y=0$, $y(1)=2$.【88 雲科機械 15%】

27. Given $(x-1)y''+y'+2(x-1)y=0$.

(1) Find two linearly independent power series solutions with center $x=4$, with at least four nonzero terms for each series solution. Justify the solutions are linearly independent.

(2) Solve for $y(4)=1$, $y'(4)=1$.【90 元智電機控制組 30%】

28. 試以冪級數求解 $\ddot{y}-2x\dot{y}+2y=x$ 。【90 北科土木 15%】

29. 若 $y''+(\cos x)y=0$ ； $y(2)=2$, $y'(2)=1$, 試求其冪級數（Power series）之前四項。【89 北科土木 15%】

30. 以級數展開法，解 $(1-x^2)y''-xy'+y=x$ 。【91 彰師光電 10%】

31. 求下式之級數解： $x^2y''+y'+y=0$ 。【91 交大土木 15%】

32. Consider an ordinary differential equation

$$y''+a(x)y'+b(x)y=0$$

(1) Under what conditions will $x=0$ be an ordinary point? Write a power series form for the solution $y(x)$.

(2) Under what conditions will $x=0$ be a regular singular point? Write a possible power series form for the solution $y(x)$.

(3) Under what conditions will $x=0$ be an irregular singular point? Write a possible power series form for the solution $y(x)$.【86 清大動機 15%】

33. Using power series method about $x=0$ to solve $(1-x^2)y''-2xy'+12y=0$.【88 成大土木 15%】

34. Find the general solution about $x=0$ expressed as $y=c_1y_1+c_2y_2$ for the differential equation $y''-2xy=0$. Show that y_1 and y_2 are linearly independent. Find the interval of convergence for this solution.【87 交大電子 6%】

35. Use power series method to solve $y''-xy'+y=0$.【87 交大機械 15%】

36. Determine the first 5 nonzero terms of the power series solution about $x=0$ for the initial value problem shown below:

$y''-e^x y'+2y=1$; $y(0)=-3,\ y'(0)=1$.【87 台科電機 10%】

37. Apply power series method to solve $y''-3y'+2y=0$.【86 中山資訊 8%】

38. 求 $y''+xy'-y=1+x^2$ 在 $x=0$ 附近之解。【86 台科化工 20%】

39. Solve by power series of $(1-x^2)y''-2xy'=0$.【86 交大機械 10%】

40. There are two solutions that are solved for the equation, $y''+xy=0$, in the power series

$$y_1(x)=1-\frac{1}{6}x^3+\frac{1}{180}x^6-\frac{1}{12960}x^9+....$$

$$y_2(x)x-\frac{1}{12}x^4+\frac{1}{504}x^7-\frac{1}{45360}x^{10}+....$$

Can you verify the solutions are linearly independent?【91 雲科電機 10%】

41. Find a general solution of the Legendre's equation: $\left(1-x^2\right)y''-2xy'+2y=0$ on the interval $-1<x<1$ using the power series method.【91 逢甲電機・電子 20%】

42. 請利用級數(即 $y\left(x\right)=a_0+\sum_{i=1}^{n}a_i x^i$)展開方式解 $y''+y=0$。【91 高科環安 10%】

43. Solve the following second–order differential equation for y as a power series in powers of $\left(x-x_0\right)$ where $x_0=0$: $y''-4xy'+\left(4x^2-2\right)y=0$.【91 清大工程科學 15%】

44. Use power series method to solve the following problem, find at least five terms of a general solution: $y''+2xy'-y=0$.【90 清大工程科學 10%】

45. $y''+2xy'+2y=0$, solve by series method.【90 北科高分子 10%】

46. Find general power series solution of $y''+x^2y=0$.【89 交大環工 15%】

47. $\left(x-1\right)y''+y'+2\left(x-1\right)y=0$, $y\left(4\right)=5$, $y'\left(4\right)=0$, $4\le x\le\infty$。求級數解，只需求最前面 5 項即可。【91 高科機械 20%】

48. 試以冪級數求解 $y''-2xy'+2y=0$。【90 北科大土研所】

49. 試以級數解求解 $xy''-y=0$，並求該解之收斂半徑。【92 交大土研所甲組】

50. Show that the equation $\sin\theta\dfrac{d^2y}{d\theta^2}+\cos\theta\dfrac{dy}{d\theta}+n\left(n+1\right)\left(\sin\theta\right)y=0$ can be transformed in Legendre's equation by means of the substitution $x=\cos\theta$.【86 成大土研所乙組】

51. Solve the following differential equation $\left(1-x^2\right)y''-2xy'+12y=0$.【88 成大土研所丁組】

52. 試求解二階微分方程式 $y''+\left(1+x+x^2+2x^3\right)y'+3y=3x+5x^2$ 的通解，where y

is a function of x.【20%】

53. (1) What is Bessel's equation of order n? Write down the solutions for $n =$ integer and $n \neq$ integer.

(2) What is Legendre's equation? Describe what you know about Legendre polynomials.【20%】

54. $(1-x^2)y'' - xy' + y = x$, find series solution.【91 彰師光電 10%】

55. Please discuss the existence of $y(x)$ by series solution near the $x = 0$ according to the regularity of $f(x)$, $y' - f(x)y = 0$.

(1) If $f(x)$ has ordinary point at $x = 0$

(2) If $f(x)$ has regular singular point at $x = 0$

(3) If $f(x)$ has irregular singular point at $x = 0$

(4) If the above ordinary equation change to $y'' - f(x)y = 0$. What is the different result with (2)?【87 北科電機 20%】

56. Use the Maclaurin series to solve the general solution including the recurrence relation: $y' - x^3 y = 4$【88 雲科電機 15%】

57. (1) Using power series to solve $y' + ky = 0$, in which k is constant.

(2) Is the series from (1) equal to e^{-kx}? Why?【90 交大電機 18%】

58. 試以 Power Series Method 解 $y' = 2xy$。【90 交大土木 20%】

59. Solve $(1-x^2)y' = 2xy$ by power series for $x = 0$.【87 雲科電機 15%】

60. Using the power series method, solve $y' = 2y$ as a power series in powers of $x = 1$.【86 中正電機 10%】

61. The interval of convergence of the power series $\sum_{n-1}^{\infty} \frac{(x-3)^n}{2^n n}$ is

(1) (0,6) (2) [0,6] (3) [1,5] (4) (1,5) 【87 台大電機 5%】