宜蘭大學

土木工程學系碩士班

93~97 學年度
工程數學考古題

一、 $\frac{1}{x}\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(4x\frac{\partial U}{\partial x}\right)$ ，

B.C.: $U(1,t)=0,\quad U(e^2,t)=0$

I.C.: $U(x,0)=\pi$

a、試求其特徵值與特徵函數。

b、解出 $U(x,t)$ ，並寫出其展開式至前五項。

共（34%）。

二、Evaluate the following integrals 計算下列各積分之值（33%）。

(A) $\int_1^2 (2x-6x^4+5)dx$

(B) $\int_1^2 (x-1)(x+2)dx$

(C) $\int_1^2 \frac{dx}{x^2}$

三、求解 dy/dx + y = x , y(0) = 9 。（33%）

一、對於一個定義於正實軸($t \geq 0$)的實數值函數 $g(t)$，可以定義此函數之富利葉轉換(Fourier Transform) $\hat{g}(f)$ 爲

$$\hat{g}(f) \equiv \int_0^\infty g(t) \mathrm{e}^{-2\pi \mathrm{i} f t} dt$$

$\hat{g}(f)$ 爲定義於整個實軸(即 f 爲任意實數)的複數值函數。

請依據此定義回答以下問題：

a　證明：對於任意實數 f ，請證明 $\hat{g}(f)+\hat{g}(-f)$ 的虛數部份爲零。(10%)

b　求出 $p(t)=e^{-2(t-3)}$ 的富利葉轉換 $\hat{p}(f)$ 。(10%)

c　求出 $r(t)=e^{-t^2}$ 的富利葉轉換 $\hat{r}(f)$ 。(13%)

二、PDE ： $\dfrac{\partial U}{\partial t}=\dfrac{\partial^2 U}{\partial x^2}$

B.C. $U(0,t)=0, U(1,t)=1$

I.C. $U(x,0)=\begin{cases} 2x, 0<x<\frac{1}{2} \\ 1, \frac{1}{2}<x<1 \end{cases}$

a　解出 $U(x,t)$ ，並寫出其展開式至前三項。(30%)

b　試問當 t 趨近於無限大時，U 應爲何？(4%)

三、 4^{th} order ODE ： $\dfrac{d^4 y}{dx^4}+y=0$ ，

a　解出其 general solution。（23%）

b　試舉出一實務例解釋其物理意義。（10%）

九十七學年度研究所碩士班考試入學
土木工程學系碩士班甲組
工程數學考科

1. 考慮 A=$\begin{pmatrix} 3 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 4 \end{pmatrix}$，求解 A 的特徵值(eigenvalues)與特徵向量(eigenvectors)。 (25 分)

2. 某地區爆發紅火蟻災情，自四月一日當天上午 10:00 發現第一個蟻窩以來，至四月三日上午 10:00 蟻窩數量已達 5 個，已知蟻窩增加率(蟻窩數量對時間的變化率)與目前蟻窩數量成正比，試問若無有效方法控制災情的情況下，預估四月十一日上午 10:00 蟻窩數量將達多少個？ (25 分)

3. 一橫樑(如圖所示)左端嵌入牆中，右端無支撐，橫樑上方之荷重如下

$$w(x) = \begin{cases} w_0 \dfrac{2}{L} x, & 0 < x < L/2 \\ 0, & L/2 < x < L \end{cases}$$

已知橫樑變形量 $y(x)$ 之控制方程式為

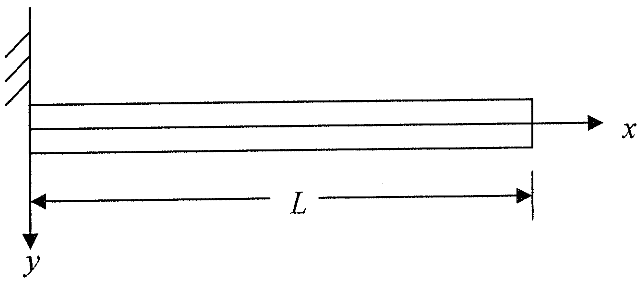
$$EI \frac{d^4 y}{dx^4} = w(x)$$

式中 EI 為常數。橫樑左右邊界條件如下

左邊界： $y(0) = 0, \quad y'(0) = 0$

右邊界： $y''(L) = 0, \quad y'''(L) = 0$

試求解 $y(x)$。 (25 分)



4. 求解下列方程式之 $u(x, y)$。 (25 分)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, \quad 0 < y < b$$

$$u(0, y) = 0, \quad \left.\frac{\partial u}{\partial x}\right|_{x=a} = -hu(a, y), \quad h > 0, \quad 0 < y < b$$

$$u(x, 0) = 0, \quad u(x, b) = 4\frac{x}{a}\left[1 - \left(\frac{x}{a}\right)\right], \quad 0 < x < a$$

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電子工程學系碩士班

93~97 學年度
工程數學考古題

九十五學年度研究所碩士班考試入學
電機工程學系碩士班
工程數學考科

1.Solve each of the differential equations in following.

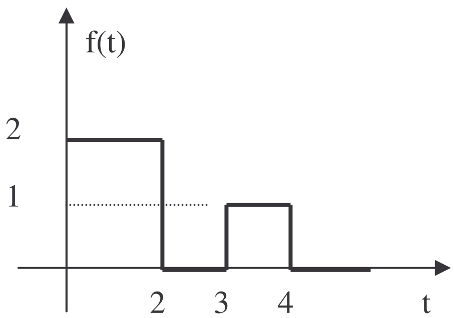
(a) $xy' = x^{-1}y^2 + y$ (10%)

(b) $y'' - 8y' + 16y = 8\sin 2x$ (10%)

2.Find the orthogonal trajectories of the curves (10%)

$$y = \frac{1}{2}x^2 + 3$$

3.Find the Laplace transforms⋯{f (t)} for the given $f(t)$. (10%)



4.Solve the initial-value problem in following. (10%)

$y'' + y' - 2y = 5t + e^{2t}$, $y(0) = y'(0) = 1$

5.Evaluate $\oint_c e^{1/z}\,dz$, for c any closed path not passing through the origin. (10%)

6.Let A be a square matrix such that $A^{-1} = A^t$. Prove that $|A| = \pm 1$. (10%)

7.Given $f(x) = xe^{-|x|}$,

(a) Find the Fourier integral representation of $f(x)$. (8 %)

(b) Evaluate $\int_0^\infty \frac{\omega \cdot \sin(\omega)}{(1+\omega^2)^2}\,d\omega$, using the results of (1). (7 %)

8.Use residue theorem to evaluate the inverse Laplace transform of $\frac{1}{\sqrt{s+1}}$.

(Hint： $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$ & $\int_0^\infty e^{-ax^2}dx = \frac{\sqrt{\pi}}{2\sqrt{a}}$) (15 %)

1.Solve the differential equation in following.

$$y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$$ (10%)

2. Solve the differential equation in following. (10%)

$$y' + y = (xy)^2$$

3.Find the inverse Laplace transforms of following equation. (10%)

$$F(s) = \frac{s+1}{(s^2+4s+13)(s^2+4s+3)}$$

4.Solve the following equation using Laplace transforms. (10%)

$$y'' + y' = g(t) \quad , \quad g(t) = \begin{cases} 0 & , \quad 0 \le t \le 2 \\ 2 & , \quad t > 2 \end{cases} \quad , \quad y(0) = y'(0) = 0$$

5.(1). A sinusoidal voltage $2\sin \omega t$, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave ($fig.1$). Find the Fourier series of the resulting periodic function $f(t) = \begin{cases} 0 & if \quad -\frac{\pi}{\omega} < t < 0, \\ 2\sin \omega t & if \quad 0 < t < \frac{\pi}{\omega} \end{cases}$ (10%)

(2).Using (1) to evaluate $\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \cdots = ?$ (5%)

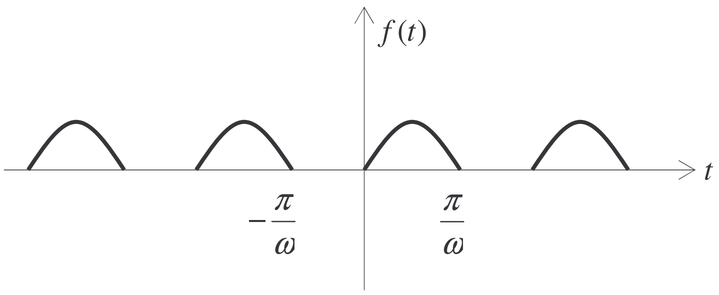


fig.1

6.Prove Cauchy Integral Formula. Let $f(z)$ be differentiable on an open set G. Let C be a closed path in G enclosing only points of G. Then, for any z_0 enclosed by G,

$$f(z_0) = \frac{1}{2\pi i}\oint_c \frac{f(z)}{z - z_0}dz \quad .$$ (15%)

7.(1). Evaluate the following integral by Residue theorem.

$$\int_{-\infty}^{\infty}\frac{1}{(s-2)^2(s^2+9)}ds = ?$$ (10%)

(2). Use residue theorem to evaluate the inverse Laplace transform of $\frac{1}{(s-2)^2(s^2+9)}$.

(10%)

8.Given $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (10%)

(1). Find $e^A = ?$

(2). Find $\cos A = ?$

九十七學年度研究所碩士班考試入學
電機工程學系碩士班
工程數學考科

1. (13%) Find the complete solution y for the following linear differential equation

$$(x-2)^2\frac{d^2y}{dx^2}+5(x-2)\frac{dy}{dx}-5y=0.$$

2. (12%) Find the general solution of the following equation

$$(x^2+y^2+2x)dy=2ydx.$$

3. (12%) Let the coorinate vector of $\mathbf{x}$ with respect to the basis B be $[\mathbf{x}]_B$. If $B=\{(1,1,0),(1,0,1),(1,1,1)\}$, $B_2=\{(1,0,0),(1,0,1),(1,1,1)\}$, and $[\mathbf{x}]_B=(1,2,3)$,

Find $[\mathbf{x}]_{B_2}$.

4. (7%) (a) Find the eigenvalues and eigenvectors for the matrix $\mathbf{A}$

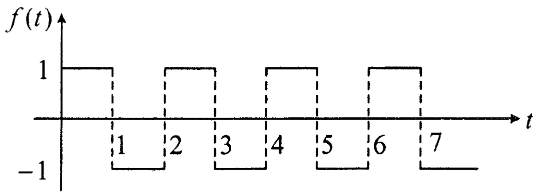
$$\mathbf{A}=\begin{bmatrix}1 & -4\\ 2 & -5\end{bmatrix}.$$

(6%) (b) Find $e^{\mathbf{A}^2}$.

5. (14%) Use power series method to solve the following differential equation. Find the first three nonzero terms of two linearly independent Frobenius solutions.

$$\frac{d^2y}{dx^2}-\left(\frac{1}{2x}\right)\frac{dy}{dx}+\left(\frac{1}{x}\right)y=0$$

6. (12%) Find the Laplace transform $L\{f(t)\}$ in the form of hyperbolic tangent function $\tanh(\cdot)$, where $f(t)$ is the periodical square wave shown in the following figure.



7. (12%) Given a smooth curve $R(t)=3\sin(t)\vec{i}+3\cos(t)\vec{j}+4t\vec{k}$, find the value of term $w=25(\kappa+\tau)$ where κ is the curvature and τ is the torsion of the curve.

8. (12%) Evaluate $\int_C |z|^2 dz$, where C is the straight line segment from 1 to i.

宜蘭大學

電機工程學系碩士班

95~97 學年度
工程數學考古題

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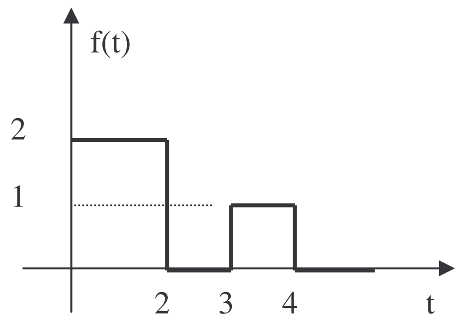

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6.Let A be a square matrix such that $A^{-1} = A^t$. Prove that $|A| = \pm 1$. (10%)

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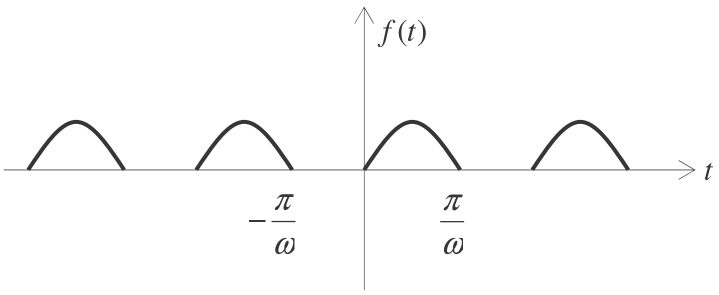

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(1). Find $e^A = ?$

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1. (13%) Find the complete solution y for the following linear differential equation

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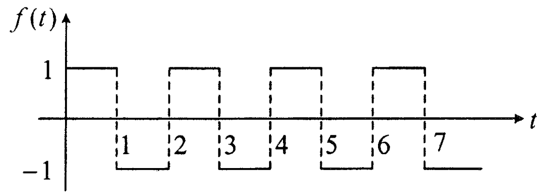


7. (12%) Given a smooth curve $R(t)=3\sin(t)\vec{i}+3\cos(t)\vec{j}+4t\vec{k}$, find the value of term $w=25(\kappa+\tau)$ where κ is the curvature and τ is the torsion of the curve.

8. (12%) Evaluate $\int_C |z|^2 dz$, where C is the straight line segment from 1 to i.

宜蘭大學

機械與機電工程學系碩士班

96~97 學年度

工程數學考古題

1. Evaluating line integral $\int_{c} yzdx + zxdy + xydz$ where the integral path,

   C: $\frac{x-1}{2} = \frac{y-3}{6} = \frac{z-2}{4}$,indicates from (0,0,0) $\rightarrow$ (1,3,2)

2. Consider a system in state variable form: $\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ -k & -3 & -2 \end{bmatrix} \mathrm{X} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} u$,

   Y=$\begin{bmatrix} 1 & 2 & 0 \end{bmatrix} X$

   Find the range of k where the system is stable.

3. Evaluate $\iint_{s} \vec{F} \cdot d\sigma$ where $\vec{F} = xy\vec{i} + xz\vec{j} + (1-z-yz)\vec{k}$; S is the lateral surface of the paraboloid $z=1-x^2-y^2$ for which $z \geqq 0$

4. Solve the equation $\frac{d^2 y}{dt^2} - 2\frac{dy}{dt} + 10y = 0$, with the initial conditions $y(0) = 4,\ \frac{dy}{dt}(0) = 1$.

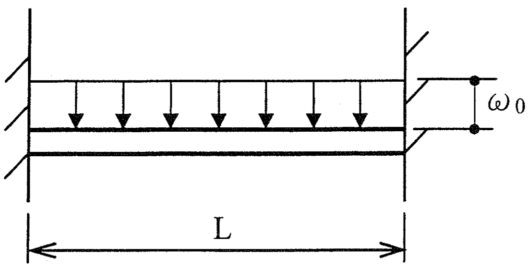
5. Write the following function using unit step functions and find its Laplace transform.

$$f(t) = \begin{cases} 2 & If\, 0 < t < 1 \\ \frac{t^2}{2} & If\, 1 < t < \frac{\pi}{2} \\ \cos t & if\, t > \frac{\pi}{2} \end{cases}$$

1. A beam of length L is clamped at both ends, and a uniform distributed load ω_0 is applied along its length. That is, $\omega(x)=\omega_0$, $0<x<L$. If the deflection $y(x)$ satisfies the following equation:

$$EI\frac{d^4y}{dx^4}=\omega(x)$$

Please find the deflection of the beam.



2. The pressure of material is p, specific volume is v and temperature is T, the relationship between three parameters is pv/T=constant, if one scale s, its deferential form is δS or

$$dS=\frac{dT}{T}-\frac{vdp}{T}\delta S$$, please answer the following equations:

(1)The differential equation is an exact differential or non-exact differential equation? (Please approve it)

(2) Determine s is a state function or route function? (Approve it)

(3)What is the relation between state function and exact differential? (Please describe it)

3. Consider the system represented in state variable form

$$\dot{x}=Ax+Bu$$

$$y=Cx+Du$$

where

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -k & -k & -k\end{bmatrix},\quad B=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

背面尚有試題

$C = [1 \quad 0 \quad 0], \quad D = [0].$

(a) What is the system transfer function?

(b) For what values of k is the system stable?

4. Try to estimate $\oint_C \vec{F} \bullet \overline{dR}$ by using Green's theorem, where $\vec{F} = y\hat{i} - x\hat{j}$ and c: circle $x^2 + y^2 = a^2$.

5. The model of the vibrating membrane for obtaining the displacement u(x,y,t) of a point (x,y) of the membrane from rest (u=0) at time t is $\frac{\partial^2 u}{\partial t^2} = c^2(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})$,
And u=0 on the boundary, u(x,y,0)=f(x,y), $u_t(x,y,0) = g(x,y)$, this is the two-dimensional wave equation with $c^2 = \frac{T}{\rho}$. If a rectangular membrane (x length =a, y length =b), solve this PDE and give the final form of u(x,y,t).