

勤益科技大學

精密機械與製造科技研究所

93~97 學年度

工程數學考古題

國立勤益技術學院九十三年學年度研究所招生初試試題卷 (共二頁)

所別：精密機械與製造科技研究所 組別：甲、乙 身份別：一般生，在職生
科目：工程數學 准考證號碼：□□□□□□□□ (考生自填) 第一頁

考生注意事項：

- 一、考試時間 100 分鐘
- 二、不得帶字典、翻譯機、計算機等
- 三、請先核對試題、答案卷(試卷)與准考證上之所組別與考試科目是否相符。
- 四、請依照題目順序在答案卷作答。

1. Formulate a boundary value problem model of heat conduction in a slender bar of length L if the left end is insulated and the right end is kept at temperature T_0 . The initial temperature in the cross section at x is $f(x)$. (5%)
2. An elastic prismatic rod is vibrated in longitudinal direction by the wave equation as the following. If at time zero, the rod is stretched by a differential deformation and released from the static state.

$$c^2 u_{xx} - u_t = 0, \quad \text{for } t > 0, 0 < x < L$$

$$u_x(0, t) = 0, \quad u_x(L, t) = 0, \quad \text{for } t \geq 0$$

$$u(x, 0) = (1 + \alpha)x, \quad u_t(x, 0) = 0, \quad \text{for } 0 \leq x \leq L$$

- (a) Give a word statement of the constant c . (3%)
 - (b) Find a series solution. (15%)
3. The state of stress in a plate lying in the x - y plane is given $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$. Using Hooke's law, the strain stress relation is $\{\epsilon\} = [S]\{\sigma\}$, where $[S]$ is the compliance matrix.

(a) Calculate the inverse matrix of $[S]$. (7%)

(b) When the strain are $\epsilon_{xx} = 1, \epsilon_{yy} = 2, \epsilon_{xy} = 3$, and the Poisson ratio is ν ,

find the plane stresses. (5%)

$$[S] = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1+\nu}{E} \end{bmatrix}$$

4. The stress state at a point a machine element with respect to a Cartesian coordinate system is $\sigma_{xy} = 1, \sigma_{xx} = 1, \sigma_{yy} = -2, \sigma_{zz} = 3, \sigma_{yz} = 0, \sigma_{zx} = 3$ (MPa).

(a) Write the stress tensor in matrix form. (5%)

背面還有試題

國立勤益技術學院九十三年學年度研究所招生初試試題卷 (共二頁)

所別：精密機械與製造科技研究所 組別：甲, 乙 身份別：一般生, 在職生

科目：工程數學 准考證號碼：□□□□□□□□ (考生自填) 第二頁

(b) Compute the principle stresses and corresponding directions. (6%)

5. Find the general solution of the equation: $y' = \frac{y}{x} + \frac{x}{2y}$. (8%)

6. Analyze the quadratic form $f(x_1, x_2) = 3x_1^2 + 3x_2^2 + 2x_1x_2$.

(a) Find the standard form of $f(x_1, x_2)$. (5%)

(b) If $f(x_1, x_2) = 4$, draft it in the principle axes. (7%)

7. Find the standard form of the quadratic form: $-2x_1x_2 + 2x_3^2$. (6%)

8. Find the general solution of $x^2y'' + xy' + 4y = f(x)$,

(a) if $f(x) = 0$; (4%) (b) if $f(x) = \sin(2 \ln x)$. (8%)

9. Solve the differential equation (8%)

$$y'''' + y' = 0, \text{ with } y(0) = 0, y'(\pi) = 0, y''\left(\frac{\pi}{2}\right) = -1.$$

10. Solve the differential equation

$$y' + y = \begin{cases} 0, & 0 \leq t \leq \pi \\ 3 \cos t, & \pi \leq t \end{cases} \text{ with } y(0) = 0. (8\%)$$

國立勤益技術學院九十四學年度研究所碩士班招生筆試試題卷

所別:精密機械與製造科技研究所

組別:甲、乙

身分別:一般生

科目:工程數學

准考證號碼:

(考生自填)

考生注意事項:

- 一、考試時間100分鐘
- 二、應考人可攜帶不具備存程式功能之計算機
- 三、無論是否使用計算機，作答時均須詳列解答過程。
- 四、請依照題目順序在答案卷作答。
- 五、共兩頁

1. [15%] Consider the driven mechanical oscillator governed by the differential equation

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

where $m = 0.4\text{kg}$, $c = 5\text{N} - \text{s}/\text{m}$, $k = 1400\text{N}/\text{m}$, and let $F(t)$ is a periodic excitation force

$$F(t) = \begin{cases} 2 & 0 < t < \pi \\ -2 & \pi < t < 2\pi \end{cases} \quad F(t+2\pi) = F(t)$$

Find the steady-state oscillation?

2.[15%] (a) Draw a labeled sketch of the graph of the function $f(t)$, where $H(t)$ is a Heaviside function (unit step function).

(b) Find the Laplace transform of $f(t)$? $f(t) = H(t - \pi)\cos(t)$

3. [10%] Solve $y(t)$ in the equation

$$y = t - 9 \int_0^t y(\tau)(t - \tau)d\tau$$

4.[15%] The one dimensional heat equation subject to the given conditions

$$u(0, t) = 0 \quad u(L, t) = 0$$

$$u(x, t) = \begin{cases} \pi/2 & 0 < x < L/2 \\ 0 & L/2 < x < L \end{cases}$$

(a) Write out the differential equation (b) Solve the temperature function $u(x, t)$?

< 共兩頁，第一頁 >

5.[10%] Solve the differential equation

$$\frac{d^2u}{dx^2} + \pi^2u - 1 = 0 \quad 0 < x < 1$$

with boundary conditions

$$u(0) = 0 \quad u(1) = 0$$

6.[10%] Determine a formula for the $k - th$ power of the matrix

$$\mathbf{A} = \begin{bmatrix} 7 & -1 \\ 6 & 2 \end{bmatrix}$$

7.[15%] Find the general solution to the differential equation

$$\frac{dy}{dx} = \frac{2x + y}{2y - x}$$

8.[10%] Find the directional derivative of $W = x^2y + 2xy^2 - z^3$ at $(1, 2, 3)$ in the direction $\mathbf{d} = 1\hat{i} - 2\hat{j} + 2\hat{k}$

國立勤益技術學院九十五學年度研究所碩士班招生筆試試題卷

所別:精密機械與製造科技研究所

組別:甲、乙、丙

身分別:一般生

科目:工程數學

准考證號碼:

(考生自填)

考生注意事項:

- 一、考試時間100分鐘
- 二、應考人可攜帶不具儲存程式功能之計算機
- 三、無論是否使用計算機，作答時均須詳列解答過程。
- 四、請依照題目順序在答案卷作答。
- 五、共三頁

1. [10%] The austenite state of steel with $750^{\circ}C$ is quenched with $200^{\circ}C$ media by the following Newton's cooling law. During the first 30 minutes, the metal is cooled down to $300^{\circ}C$

$$\frac{dT}{dt} = k(T - 200)$$

where T is temperature of the metal, t is the quench time, and k is the proportional constant.

Determine (1) the constant k (1/min)

(2) the temperature after the first 60 minutes.

2. [15%] Solve the given initial-value problem

$$\begin{aligned}\frac{dx}{dt} &= 2x - 3y \\ \frac{dy}{dt} &= x - 1\end{aligned}$$

$$x(0) = 0, y(0) = 0$$

3. [10%] Matrix

$$A = \begin{bmatrix} 6 & 3 \\ -10 & -5 \end{bmatrix}$$

Determine (1) the eigenvalues and eigenvectors

(2) A^{10}

< 共三頁，第一頁，精機所工程數學 >

4. [10%] Solve the given initial-value problem

$$y'' + 4y = \delta(t - \pi) \quad y(0) = 0, \quad y'(0) = 2$$

where $\delta(t - \pi)$ is an impulse function.

5. [10%] A linear system $Ax = B$

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 3 \\ 1 & 3 & -2 \\ 2 & -5 & 7 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 5 \\ -7 \\ 19 \end{bmatrix}$$

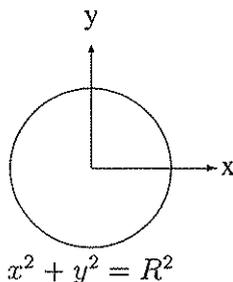
Determine (1) the rank of matrix A

(2) solve the unknowns

(3) sketch the solution in Cartesian coordinates

6. [10%] The homogeneous circular disk of mass m , is located in the $x-y$ plane and described by $x^2 + y^2 = R^2$.

Find the mass moment of inertia (1) I_x (2) I_z



7. [10%] The three components of velocity in a flow field are given by

$$u = x^2 + y^2 + z^2, \quad v = xy + yz + z^2, \quad \text{and} \quad w = -3xz - (z^2/2) + 4.$$

Determine (1) the volumetric dilatation

(2) the rotation velocity.

8. [10%] A plane contains the given points $(1, 3, -1)$, $(0, 1, 0)$, $(0, 1, 1)$,

Determine (1) the normal vector

(2) the equation of the plane

< 共三頁，第二頁，精機所工程數學 >

9.[15%] A torque is applied to the free end of a circular shaft and suddenly removed. The shaft is elastic rotating deformation with fixed end at $x = 0$, and free at $x = 1$. Via nondimensionalizing, the governing equation is given by

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 \theta}{\partial t^2}$$

where θ is the vibrating rotation angle, t is the time variable.

The initial and boundary conditions are

$$\theta(0, t) = 0 \quad \frac{\partial \theta}{\partial x}(1, t) = 0$$

$$\theta(x, 0) = \theta_0 x \quad \frac{\partial \theta}{\partial t}(x, 0) = 0$$

Determine (1) the vibrating mode shape function for the shaft.

(2) the $\theta(x, t)$ at the free end ($x = 1$)

< 共三頁，第三頁，精機所工程數學 >

國立勤益技術學院九十六學年度研究所碩士班招生筆試試題卷 (共 1 頁)

所別：精密機械與製造科技研究所 組別：甲、乙、丙 身份別：一般生，在職生

科目：工程數學 准考證號碼：□□□□□□□□□□(考生自填)

考生注意事項：

一、考試時間 100 分鐘。

二、每一題 10 分，滿分 100 分。

三、不得帶字典、翻譯機。應考人攜帶之電子計算器，以具有 +、-、×、÷、%、√、M、三角函數、對數、指數等功能（不具儲存程式功能 Non-programmable）者為限。

四、請先核對試題，答案卷（試卷）與准考證上之所組別與考試科目是否相符。

五、請依照題目順序在答案卷作答，並在最後答案處畫兩條橫線，否則不予計分。

1. 解微分方程式 $y'' + 2y' + 4y = 0$
2. 解 $y'' + 4y' + 3y = 0$; $y(0) = 3$, $y'(0) = 1$
3. 已知邊界值問題 $y'' + \lambda y = 0$ ，邊界條件為 $y(0) = y(L) = 0$ ，其中 λ 為特徵值，解出 $y(x) = ?$
4. $f(z) = \frac{1}{(z-1)}$ 且 $1 < |z|$ ，試做對 $z = 0$ 的 Laurent Series 展開。
5. 解微分方程式 $y' = e^{3x+2y}$ 。
6. 一曲面方程式為 $z^2 - \sqrt{x^2 + y^2} = 0$ 。求其在點 $(3, 4, \sqrt{5})$ 的單位法向量與通過此點的切平面。
7. 求封閉曲面之面積分 $\iint_S \vec{v} \cdot d\vec{A} = ?$ 其中，向量 $\vec{v} = (2x, 3y, 4z)$ ，封閉曲面 $S: x^2 + y^2 - z = 0$ 且 $0 \leq z \leq 1$ 。提示：角錐體積 = $1/3$ (底面積)*高。
8. 求 $e^{at} \cos(bt)$ 之 Laplace Transform，其中 a, b 為常數。
9. 已知 $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ ，求 A 之特徵值與其所對應的特徵向量。
10. 已知 $A = xz^2 \mathbf{i} - 2x^2yz \mathbf{j} + 2yz^4 \mathbf{k}$ ，求在 $P(1, -1, 1)$ 點之旋度 $\nabla \times A$ 。

國立勤益科技大學九十七學年度研究所碩士班招生筆試試題卷

所別：精密機械與製造科技研究所

組別：

科目：工程數學

身份別：一般生, 在職生

准考證號碼：□□□□□□□□ (考生自填) 第一頁

考生注意事項：

一、考試時間 100 分鐘

二、不得帶字典、翻譯機、計算機等

三、請先核對試題，答案卷（試卷）與准考證上之所組別與考試科目是否相符。

四、請依照題目順序在答案卷作答，並在最後答案處畫兩條橫線，否則不予計分。

試題

1. 求解微分方程式 $y'' + 5y' + 6y = e^{-2t}$ 之通解(general solution)。(15%)
2. 求解微分方程式 $y'' + 3y' + 2y = \delta(t-1)$, $y(0) = 0$, $y'(0) = 0$,
其中 $\delta(t-1)$ 是單位脈衝函數。(15%)
3. 試求矩陣 $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ 的特徵值 (eigenvalues) 與特徵向量(eigenvectors)。(15%)
4. 試以矩陣法，求解下列微分方程組。(20%)
$$\dot{x}_1 = -4x_1 + x_2 + x_3$$
$$\dot{x}_2 = x_1 + 5x_2 - x_3$$
$$\dot{x}_3 = x_2 - 3x_3$$
5. 試求力場 $\vec{F} = x^2\vec{i} - z\vec{j} + 2xy\vec{k}$ 沿著曲線： $y=2x$, $z=3y$ ，自 $y=1$ 到 $y=2$ 所作的功。(15%)
6. 解波動方程式 $\frac{\partial u^2}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ 之 D'Alembert solution。(20%)
邊界條件： $-\infty < x < \infty$, $t > 0$
初始條件： $u(x,0) = f(x)$, $\frac{\partial u}{\partial t}(x,0) = g(x)$ 。