雲林科技大學

電機系

91~97 學年度
工程數學考古題

國立雲林科技大學
九十一學年度研究所碩士班入學考試試題

系所：電機系
科目：工程數學

1. Find the general solution for the following differential equations. (20 %)
   (a) $y^2 - 6xy + (3xy - 6x^2)y' = 0$. (10 %)
   (b) $y'' + 9y = x\cos(3x)$. (10 %)
2. There are two solutions that are solved for the equation, $y'' + xy = 0$, in the power series,

$$y_1(x) = 1 - \frac{1}{6}x^3 + \frac{1}{180}x^6 - \frac{1}{12960}x^9 + \dots$$

$$y_2(x) = x - \frac{1}{12}x^4 + \frac{1}{504}x^7 - \frac{1}{45360}x^{10} + \dots$$

   Can you verify the solutions are linearly independent ? (10 %)
3. Find the Laplace transform for the following function (10 %)
   $f(t) = e^t[1 - \cosh(2t)]$.
4. Find the inverse Laplace transform for the following function. (10 %)

$$F(s) = \frac{se^{-s}}{(s^2+4)^2}$$

5. Find the steady-state solution $y(t)$ of $y'' + 0.02y' + 25y = r(t)$, where

$$r(t) = \begin{cases} t + \dfrac{\pi}{2} & \text{if } \ -\pi < t < 0, \\ -t + \dfrac{\pi}{2} & \text{if } \ 0 < t < \pi, \end{cases} \quad \text{and } \ r(t + 2\pi) = r(t). \qquad (10\%)$$

6. True or false. Give a reason or a counterexample.
   (a) If the columns of a matrix are linearly dependent, so are the rows. (3%)
   (b) A symmetric matrix times a symmetric matrix is symmetric. (3%)
   (c) The inverse of a symmetric matrix, if exists, is symmetric. (3%)
   (d) Let A, B be $n \times n$ matrices. If $AB = B$, then $A = I$ (I is the $n \times n$ identity matrix). (3%)
7. The complete solution to $A\mathbf{x} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$ is $\mathbf{x} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 2 \end{bmatrix}, c \in \mathbf{R}$. Find A. (10%)
8.
   (a) Find a basis for the subspace W_1 of vectors $[a,b,c,d]^T$ with $a + c + d = 0$ (6%)
   (b) Find a basis for the subspace W_2 of vectors $[a,b,c,d]^T$ with $a + b = 0$ and $c = 2d$. (6%)
   (c) What is the dimension of the intersection $W_1 \cap W_2$. (6%)

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九十二學年度碩士班入學招生考試試題

系所：電機系

科目：工程數學 (乙)

1. Find the general solution for each of the following differential equations.

   (a) $\frac{dy}{dx} = \frac{2y+y\cos x}{2x+\sin x}$ (10%)

   (b) $(3x-4)^2 \frac{d^2y}{dx^2} + 3(3x-4)\frac{dy}{dx} + 36y = 0$ (15%)

2. Find the inverse Laplace transform of the function $F(s) = \tan^{-1}\left(\frac{2}{s}\right)$. (10%)

3. Use the Laplace transformation to solve the following differential equation

$$y'(t) + 2y(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}, \quad y(0) = 0$$ (15%)

4. For what value(s) of α does the following system of equations have (i) no solution? (ii) a unique solution? (iii) infinitely many solutions?

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 1 \\ 2x_1 + \alpha x_2 + 6x_3 &= 6 \\ -x_1 + 3x_2 + (\alpha - 3)x_3 &= 0. \end{aligned}$$

   In cases (ii) and (iii), describe the general solution. (15%)

5. When $a + b = c + d$, show that $[1, 1]^T$ is an eigenvector and find both eigenvalues of

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$ (15%)

6. Let

$$A = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 4 \\ 2 & 1 & 2 \end{bmatrix}.$$

   This matrix is singular with *rank* one. Find three eigenvalues and three linearly independent eigenvectors. (10%)

7. Suppose that f is a solution of $y'' - t^2 y = y$. Show that $F\{f(t)\}$ is also a solution, where $F\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$ is the Fourier transform of f. (10%)

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系所：電機系
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1. Find the general solution for each of the following differential equation.

(a) $y'' + 10y' + 24y = 1, \quad y(1) = 10, \quad y'(1) = 10$ (10%)

(b) $y''' - 4y'' + 13y' + 50y = -4\cos(2x)$ (10%)

2. Find the Laplace transformation of the following function.

$$f(t) = \begin{cases} 0, & \text{if } 0 \le t < 4 \\ e^{-3t}, & \text{if } 4 \le t < 6 \\ 1+t, & \text{if } t \ge 6 \end{cases}$$ (15%)

3. Find the inverse Laplace transformation of the following function.

$$F(s) = \frac{1}{(s^2+4)(s+12)}$$ (10%)

4. (a) Find the Fourier series for the periodic function $f(x+2\pi) = f(x)$

$$\& \quad f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x^2, & 0 \le x \le \pi \end{cases}$$ (12%)

(b) From (a), Find the value of $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = ?$ (8%)

5. Use the Fourier transform to solve $y''(t) + 6y'(t) + 5y(t) = \delta(t-3)$ (10%)

6. Determine the relationship of a, b, c and the solution(s) of (i) (ii) such that the following system of linear equations has

(i) exactly one solution, (6%)

(ii) an infinite number of solutions, (5%)

(iii) no solution. (4%)

$$\begin{aligned} x + 5y + z &= 0 \\ x + 6y - z &= 0 \\ 2x + ay + bz &= c \end{aligned}$$

7. Find all values of t for which the set S is linear independent.

$$S = \left\{ \begin{bmatrix} t \\ t \\ t \end{bmatrix}, \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} t \\ 0 \\ 1 \end{bmatrix} \right\}$$ (10%)

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1. Given that the Laplace transform $\mathcal{L}\{\frac{2}{t}[1-\cos(t)]\} = \ln(\frac{s^2+1}{s^2})$,

   please find the value of $\mathcal{L}\{\frac{1}{t}[1-\cos(2t)]\}$ (10%)

2. Find the inverse Laplace transform for the following function. (10%)

   $$\frac{se^{-s}}{(s+1)^2(s^2+2s+2)}$$

3. Find the general solution for the following differential equations. (30%)

   (a) $(D^4+5D^2-36)y(x) = 10e^{-2x} + 3\cos(3x)$. (10%)

   (b) $(x^3D^3+3x^2D^2+xD-1)y(x) = 0$. (10%)

   (c) $\frac{dy}{dx} = \frac{6xy-y^2}{3xy-6x^2}$. (10%)

4. Find the Fourier half cosine and Fourier half sine expansions of $f(x)$ for

   $$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 2-x, & 1 \le x < 2 \end{cases}$$ (15%)

5. Solve the following integral equation for the function $f(x)$

   $$\int_0^\infty f(x)\sin(\omega x)dx = \begin{cases} 1, & 0 < \omega < 1 \\ 0, & \omega > 1 \end{cases}$$ (10%)

6. Determine the polynomial $y = a_0 + a_1x + a_2x^2$ whose graph passes through the points (x, y) of (1, 9), (2, 18), and (3, 31). (10%)

7. Consider the following linear equations **Ax=b**,

   $$\begin{cases} x_1 \quad\quad -2x_3 + x_4 = 4 \\ 3x_1 + x_2 - 5x_3 \quad\quad = 8 \\ x_1 + 2x_2 \quad\quad -5x_4 = -4 \end{cases}$$

   Write the solution in the form **x**$= x_h + x_p$, where x_h is the solution of **Ax=0** and x_p is a particular solution of **Ax=b**. (8%)

8. Let A be an 4×4 invertible matrix and $adj\,(A)$ be the adjoint of A. Find the value x of the determinant $|adj(A)| = |A|^x$. (7%)

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1. Apply Laplace transform to solve the equation,
$y''(t)+2ty'(t)-6y=t;\ \ y(0)=0, y'(0)=0$ (10%)

2. Find the inverse Laplace transform for the following function. (10%)
$$\frac{s}{(s+1)^2(s^2+2s+5)}$$

3. Apply Laplace transform to find the solution for the following equations. (10%)
$$x(t)+3\int_0^t[x(\tau)-y(\tau)]d\tau=1$$
$$y(t)+2\int_0^t[2y(\tau)-x(\tau)]d\tau=0$$

4. Find the Fourier transform for the following function. (10%)
$$\frac{3e^{it}}{t^2-2t+5}$$

5. Find the inverse Fourier transform for the following function. (10%)
$$\frac{1}{(1+\omega^2)(4+\omega^2)}$$

6. Find the general solution for the following differential equations.

(i) $y^2+y-x\dfrac{dy}{dx}=0$ (10%)

(ii) $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}-8y=16+(12x-4)e^{2x}$ (15%)

7. Determine the relationship of a, b, c such that the following system of linear equations has (i) an infinite number of solutions, (5%)
(ii) exactly one solution, (5%)
(iii) no solution. (5%)
$$2x-y+z=a$$
$$x+y+2z=b$$
$$3y+3z=c$$

8. Let $w=\begin{bmatrix}1\\2\\3\\-12\end{bmatrix}$, $v_1=\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$, $v_2=\begin{bmatrix}-1\\-2\\-3\\4\end{bmatrix}$ and $v_3=\begin{bmatrix}0\\0\\1\\1\end{bmatrix}$. Write the vector w as a linear combination of vectors v_1, v_2 and v_3. (10%)

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共 9 題，合計 100 分，請依序作答，否則不計分

1. Find the general solution of the differential equation (10 分)

$$\frac{dy}{dx}=-\frac{3xy+y^2}{x^2+xy}$$ *[Hint: Exact Form- using integrating factor $\phi(x)=x$]*

2. Find the general solution of the differential equation: $y''-2y'-8y=6e^{-2x}$ (15 分)

3. The nonhomogeneous system of linear equations $AX=B$, in which $A=\begin{bmatrix}-1 & 1 & 3\\ 0 & 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-2\\ 4\end{bmatrix}$. Find (1) the reduced row echelon form of augmented matrix $[A\,|\,B]$, (2) the dependent unknowns and independent unknowns, and (3) the general solution of $AX=B$ • (15 分)

4. Let $A=\begin{bmatrix}0 & -2\\ 1 & 3\end{bmatrix}$, find (1) the eigenvalues and eigenvectors of A, and (2) the matrix A^{10} • (10 分)

5. Apply Laplace transform to solve the equation,

$$y''(t)+4y'(t)+4y=3H(t-2);y(0)=0,y'(0)=0,$$

where H(t) is Heaviside function. (10%)

6. Find the inverse Laplace transform for the following function. (10%)

$$\frac{3e^{-2s}}{(s+1)^2(s^2+2s+10)}$$

7. Apply Laplace transform to find the solution for the following equations. (10%)

$$x''(t)-2x'(t)+3y'(t)+2y(t)=3.$$
$$\text{............}2y'(t)-x'(t)+3y(t)=0,$$
$$\text{..........}x(0)=x'(0)=y(0)=0.$$

8. Find the Fourier transform for the following function (10%)

$$f(t)=t[H(t+2)-H(t-2)],$$

where H(t) is Heaviside function.

9.. Find the inverse Fourier transform for the following function (10%)

$$\frac{5e^{i4\omega}\cos(2\omega)}{(9+\omega^2)(4+\omega^2)}$$

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1. Find the general solution of the differential equation (10 分)

$$\left[D^3 - 2D^2 + D\right]y = 2x; \qquad \text{[Note: } D^n y = y^{(n)} = \frac{d^n}{dx^n} y\text{]}$$

2. Find the general solution, $y(x) = c_1 y_1(x) + c_2 y_2(x)$, of the differential equation $x^2 y'' + xy' - y = 0, \quad x > 0$. To explain if y_1, y_2 are linear independent by Wronskain test. (15 分)

3. Let $A = \begin{bmatrix} -1 & 0 \\ 1 & -5 \end{bmatrix}$, find: (1) P, and diagonal matrix $D = P^{-1}AP$,
(2) $(A^2 + 6A + 4I)^5 = ?$ (10 分)

4. To solve the initial value problem , $\begin{array}{l} x_1' = 2x_1 - 10x_2 \\ x_2' = -x_1 - x_2 \end{array}$, $X(0) = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$, by matrix methods。(15 分)

5. Find the Laplace transform for the following functions (10%)

$$[\sin(t-1) + (t^2 - 2)]H(t-1)$$

6. Find the inverse Laplace transform for the following functions.

(a) $\ln[(s+2)/(s-1)]$, (10%)

(b) $\dfrac{se^{-2s}}{(s+2)^2(s^2+4s+8)}$. (10%)

7. Find the sum of the series $\sum_{n=1}^{\infty} (-1)^n / (4n^2 - 1)$. (hint :expand sin(x) in a Fourier cosine series on [0 π] and choose an appropriate value of x. (10%)

8. Find the inverse Fourier transform for function: $\dfrac{2e^{(\omega-2)i}}{[2 + (\omega - 2)i]}$. (10%)

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機械系

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科目：工程數學

1. a) Solve $\frac{d^2y}{dt^2}+\omega^2 y=\cos(\gamma t)$, in which ω and γ are constants, $\gamma \neq \omega$ and $y(0)=y'(0)=0$. (10%)

   b) Evaluate $\lim_{\gamma\to\omega} y(t)$, where $y(t)$ is defined in (a). (5%)

2. Suppose that $y_1(x)$ is a solution of $y''+p(x)y'+q(x)y=0$. Let

$$y_2(x)=y_1(x)\int\frac{e^{-\int p(x)dx}}{y_1^2(x)}dx.$$

   a) Is $y_2(x)$ also a solution of $y''+p(x)y'+q(x)y=0$? Why or why not? (5%)

   b) Are $y_1(x)$ and $y_2(x)$ linearly dependent on any interval on which $y_1(x)$ is not zero? Why or why not? (*Hint: Check the Wronskian* $W(y_1,y_2)$.) (10%)

3. Define $f(x)=\begin{cases}0, & -\pi\le x<0\\ \pi-x, & 0\le x\le\pi.\end{cases}$ Let $g(x)=k_0+k_1\sin(x)+k_2\sin(2x)$, in which k_0, k_1 and k_2 are constants. Find the values of k_0, k_1 and k_2 so that $\int_{-\pi}^{+\pi}(f(x)-g(x))^2\,dx$ is minimized. (10%)

4. Solve $\frac{d^2y}{dt^2}+16y=f(t)$, $y(0)=0$, $y'(0)=1$, where

$$f(t)=\begin{cases}\cos(4t), & 0\le t<\pi\\ 0, & t\ge\pi.\end{cases}$$ (10%)

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Prob. 5 (10%)

Evaluate the given determinant.

$$\begin{vmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & 2 & 3 & 6 \end{vmatrix}$$

Prob. 6 (15%)

Find the inverse of

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Prob. 7 (25%)

Let the electric potential (i.e. the voltage) be given by $V(x,y,z) = 3x^2y - xz$. If a positive charge is placed at P = (1,1,-1), in what direction will the charge begin to move?

(Note: It is known, from electric field theory, that such a charge will begin to move in the direction of maximum rate of voltage drop.)

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1. Consider the following initial value problem (IVP)

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=0,\quad x(0)=1,\quad x'(0)=c$$

where c is a parameter. Find the range of c within which all solutions of the given IVP are non-negative, that is, determine all possible values of c which yield $x(t)\geq 0$ for $t\geq 0$. (10%)

2. Solve $f(t)=t-\int_0^t f(\tau)\exp(t-\tau)d\tau$ for $f(t)$, where $t\geq 0$ and $\exp(\cdot)$ denotes the exponential function. (10%)
(***Hint:*** *Use the Laplace transform and the convolution theorem.*)

3. Is the set $\{1, x, 3x^2-1\}$ orthogonal on the interval $[-1,1]$? Why or why not? (10%)

4. Find the Fourier series of the following periodic function. (10%)

$$f(t)=\begin{cases}1, & 0\leq t<\pi\\ 0, & -\pi\leq t<0\end{cases};\quad f(t+2\pi)=f(t)$$

5. Solve $\frac{d^2x}{dt^2}+2x=f(t)u(t)$, $x(0)=0$, $x'(0)=0$, where $u(\cdot)$ denotes the unit step function, and $f(t)$ is defined in Problem 4. (10%)

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6 . Find the the flux $\int_S \vec{F}\cdot\hat{n}dA$, of $\vec{F}=x\vec{i}+y\vec{j}-z\vec{k}$ across the part of the plane $x+2y+z=8$ lying in the first octant (卦限). (25%)

7 . Solve the following partial differential equation (25%)

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2} \quad \text{for } 0\le x\le L,\ t>0$$

$$u(0,t)=T_1,\ \ u(L,t)=T_2 \quad \text{for } t>0$$

$$u(x,0)=f(x) \quad \text{for } 0\le x\le L$$

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Prob. 1 (25%)

(a) By definition, $u(t-a)$ is 0 for $t < a$, has a jump of size 1 at $t = a$, and is 1 for $t > a$.

Please find the Laplace transform for the function shown below

$$f(t) = e^{(-2t)}u(t-1)$$

(b) Solve the system of equations given as below:

$$y_1' + 2y_1 - y_2 = e^{(-2t)}u(t-1),$$

$$y_2' + y_1 = 0,$$

with $y_1(0) = 0$, $y_2(0) = 0$.

Prob. 2 (25%)

An equation is given as below:

$$xy'' + 2y' + xy = 0, \quad \text{for } x > 0$$

Let its homogeneous solution be $y_h(x) = C_1y_1(x) + C_2y_2(x)$, where C_1, C_2 are arbitrary constants, and one of the basis functions is known as $y_1(x) = \sin(x)/x$.

Please find the other basis function, $y_2(x)$, by **the method of reduction of order**.

[Hint: let $y_2(x) = u(x)\, y_1(x)$ and $d(\cot(x))/dx = -\csc^2(x)$]

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Prob. 3 (25%)

Let S be the part of the cylinder $z = 1 - x^2$ for $0 \le x \le 1$, $-2 \le y \le 2$.

Verify Stokes' theorem if $F = xy\bar{i} + yz\bar{j} + xz\bar{k}$

Hint: Stokes' theorem: Let S be a piecewise smooth orientable surface bounded by a piecewise simple closed curve C. F is a vector field.

$\oint F \cdot dr = \iint_s \nabla \times F \cdot ndS$, where n is an unit normal vector to S.

Prob. 4 (25%)

Solve the following P.D.E.

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < L, t > 0,$$

$$\frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(L, t) = 0 \quad \text{for } t > 0,$$

$$u(x, 0) = f(x) \quad \text{for } 0 \le x \le L.$$

(a) in terms of $f(x)$

(b) if $f(x) = \begin{cases} A & for \quad 0 \le x \le \frac{L}{2} \\ 0 & for \quad \frac{L}{2} \le x \le L \end{cases}$

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請依題號作答並將答案寫在答案卷上，違者不予計分。

1. Consider the following problems.

   a) (5%) Solve $\frac{dy}{dt}+y=2e^{-t}$ subject to $y(0)=1$.

   b) (5%) Solve $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=0$ subject to $x(0)=c_1$, $x'(0)=c_2$.

   c) (5%) Find the constants c_1 and c_2 so that $x(t)=y(t)$ for all $t>0$.

   d) (5%) Find the Laplace transform of $y(t)$.

2. Consider the set $\left\{-1,\ t,\ \frac{3t^2-1}{2}\right\}$ on the interval $[-1,\ 1]$.

   a) (5%) Is the set of functions linearly independent? Why or why not?

   b) (10%) Is the set orthogonal? Why or why not?

   c) (15%) Define $f(t)=\begin{cases} t, & 0\le t\le 1 \\ 0, & -1\le t<0 \end{cases}$ and $g(t)=-\alpha_1+\alpha_2 t$, where α_1 and α_2 are constants. Determine the values of α_1 and α_2 so that $\int_{-1}^{1}(f(t)-g(t))^2\,dt$ is minimized.

   (*Definition*: The inner product of two functions f_1 and f_2 on an interval $[a,\ b]$ is the number $\int_a^b f_1(t)f_2(t)dt$.)

3. Evaluate $\int_C \left(3x^2dx+2yzdy+y^2dz\right)$ along the path C from (0, 3, 5) to (2, 4, 6) (25%)

4. Find the second order PDE u(x,t) whose general solution is expressed in terms of arbitrary function f(x,t) and g(x,t): u(x,t)= [f(x+ct)] + [g(x-ct)], for a fixed constant c. (25%)

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1. Solve the following differential equations:
   (a) $ydx - 2xdy = 0$ (5%)
   (b) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \cos 3x$ (10%)
   (c) $x\frac{dy}{dx} + 2y = xy^3$ (10%)

2. If matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$

   (a) Find eigenvalues of A (5%)
   (b) Find eigenvectors of A (10%)
   (c) Find the diagonalized form of A (10%)

3. Let the matrix $\mathbf{A} = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}$.

   (a) (5%) Find the eigenvalues of **A**.

   (b) (5%) Find the eigenvectors of **A**.

   (c) (5%) Find the inverse of **A**.

   (d) (5%) Find a matrix **P** such that $\mathbf{P}^{-1}\mathbf{AP}$ is a diagonal matrix.

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4. Consider the function $f(x, y) = 2x^2y^3 + 6xy$.

(a) (5%) Find the gradient of $f(x, y)$ at the point (1, 1).

(b) (5%) Find the directional derivative of $f(x, y)$ at the point (1, 1) in the direction of a unit vector whose angle with the positive x-axis is $\pi/3$.

(c) (5%) Find an equation of the tangent plane to the graph of $z = f(x, y)$ at the point (1, 1, 8).

5. (15%) Solve the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = 4\frac{\partial^2 u}{\partial t^2}, \quad 0 < x < \pi, \ t > 0$$

subject to $u(0,t) = 0$, $u(\pi,t) = 0$, $u(x,0) = 0$, $\left.\frac{\partial u}{\partial t}\right|_{t=0} = \sin 2x - \sin 3x$.

國立雲林科技大學
96 學年度碩士班入學招生考試試題

系所：機械系
科目：工程數學

1. Find constants c_1 and c_2 such that the set of functions $\{x, x^2, x+c_1x^2+c_2x^3\}$ is orthogonal with respect to the weight function $w(x)=1$ on the interval $[-2,2]$. (10%)

2. Find a continuous solution satisfying

$$\frac{dx}{dt}+2x=\begin{cases}0, & 0\le t<1\\ -\int_1^t x(\tau)d\tau, & t\ge 1\end{cases}$$

and the initial condition $x(0)=1$. (20%)

3. Consider the initial-value problem $\frac{d^2x}{dt^2}+2x=2\cos t$, $x(0)=0$, $x'(0)=0$. Find a function $h(t)$ such that $x(t)$ equals the convolution of $h(t)$ and $\cos t$, that is, $x(t)=h(t)*\cos t=\int_0^t h(\tau)\cos(t-\tau)d\tau$. (10%)

4. The Fourier series of $f(t)=\begin{cases}0, & -\pi<t<0\\ \sin t, & 0\le t<\pi\end{cases}$ is given by

$$f(t)=\frac{1}{\pi}+\frac{1}{2}\sin t+\frac{1}{\pi}\sum_{n=2}^{\infty}\frac{(-1)^n+1}{1-n^2}\cos nt.$$

Let us define a function $g(t)=f(t)+t$ on the interval $(-\pi,\pi)$. Expand $g(t)$ in a Fourier series. (10%)

國立雲林科技大學
96學年度碩士班入學招生考試試題

系所：機械系
科目：工程數學

5. (30%)

(a) Please show that the following integral is independent of any path C between (-1,0) and (3,4), and evaluate it.

$$\int_C (y^2 - 6xy + 6)dx + (2xy - 3x^2)dy$$

(b) Please find the work done by $\vec{F} = x\vec{i} + y\vec{j}$ along the curve C traced by

$\vec{r}(t) = \cos(t)\vec{i} + \sin(t)\vec{j}$ from t = 0 to t = π.

(c) Please evaluate the double integral (as shown below) over the region bounded by the graphs of y = 1, y = 2, y = x and y = –x + 5.

$$\iint_R e^{x+3y} dA$$

6. (20%)

Please show the area of triangle defined by two vectors $\vec{A}$ and $\vec{B}$,

which belongs to R^2 space is $\frac{1}{2}\sqrt{|\vec{A}|^2|\vec{B}|^2 - (\vec{A}\cdot\vec{B})^2}$

國立雲林科技大學
97 學年度碩士班入學招生考試試題

系所：機械系
科目：工程數學

1.(25%)
Please find the Laplace transform for the function f(t) shown in the figure 1.

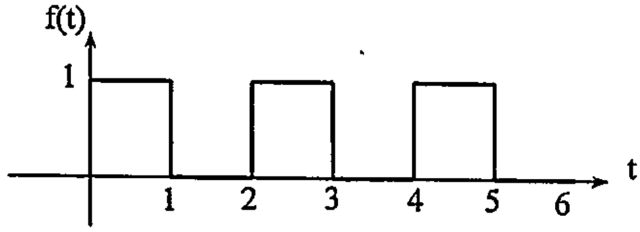

Figure 1.

2.(25%)
Given an equations as below
$y'' + 3y' + 2y = 2f(t)$,
with $y(0)=1.5$, $y'(0) = 0$ and a force function f(t) given in figure 1, please find y(t) **<u>in the range $1 \leqq t \leqq 2$</u>. (Explicit form is required)**

3. Find parametric equations for the line of intersection of

$$x+y-z=1,$$

$$x-2y+z=5.$$ (10%)

4. Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}.$$ (10%)

5. Find the directional derivative of $f(x,y)=2x^3+xy^2$ at $(1,-1)$ in the direction of $(\mathbf{i}-\mathbf{j})$. (10%)

6. Solve the boundary-value problem

$$\frac{\partial^2 u}{\partial x^2}+2=\frac{\partial u}{\partial t},\quad 0<x<1,\ t>0$$

$$u(0,t)=0,\ u(1,t)=1,\quad t>0$$

$$u(x,0)=-x^2+2x+3\sin \pi x,\ 0<x<1.$$ (20%)

雲林科技大學

營建系

91~97 學年度
工程數學考古題

國立雲林科技大學
九十一學年度研究所碩士班入學考試試題

系所：營建系
科目：工程數學

一、 試解下列常微分方程式：

(a) $y''-4y=x^3e^{2x}$ (10%)

(b) $y''+9y=\delta'''(x)$； $y(0)=1,\ y'(0)=1$ (10%)

其中 $\delta(x)$ 為 Dirac-delta function ($\delta(x)=\begin{cases}\infty & x=0\\ 0 & x\neq 0\end{cases}$ 且 $\int_{-\infty}^{\infty}\delta(x)dx=1$)

[(b)小題限採用拉氏變換(Laplace transform)求解]

二、 已知一微分方程式 $y''+5y'+6y=f(x)$ 其中 $f(x)=\begin{cases}b\ , & -a\le x\le a\\ 0\ , & x<-a \text{ and } x>a\end{cases}$

(a) 試以傅立葉積分(Fourier Integral)展開 $f(x)$； (5%)

(b) 試求解此微分方程式。 (15%)

三、 試求下列微分方程式之特徵值(eigenvalues)及特徵函數(eigenfunctions)。

$y''+\lambda y=0$； B.C.： $y(0)=0$， $y(L)+3y'(L)=0$ (10%)

四、矩陣 $\mathbf{A}=\begin{bmatrix}0&1&0\\1&0&0\\1&0&2\end{bmatrix}$，$\mathbf{B}=\begin{bmatrix}6&3&149&1&-3\\0&0&1&0&0\\2&1&267&0&9\\0&2&3&0&-3\\4&2&-78&-2&12\end{bmatrix}$，

(a) 求 $\mathbf{A}$ 之特徵值(eigenvalue)及其對應之特徵向量(eigenvector)； (7%)

(b) 求 $\mathbf{A}^{-5}$ 之特徵值(eigenvalue)及其對應之特徵向量(eigenvector)； (6%)

(c) 若 $\mathbf{B}$ 之特徵值為 λ_1，λ_2，λ_3，λ_4 及 λ_5，則 $\dfrac{1}{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5}=?$ (7%)

五、如果一座山的高程 z 與水平座標 $(x,\ y)$ 之關係為 $z(x,y)=1500-6x^2-4y^2$ (單位：公尺)，且現今你所在山上位置的水平座標為(-10, 10)，

(a) 若你希望往最陡峭的方向前進，則此方向為何？ (5%)

(b) 若你由此位置向山頂方向前進，則須走多少公尺才能攻頂？

提示： $\int\sqrt{x^2+a^2}\,dx=\frac{1}{2}\left[x\sqrt{x^2+a^2}+a^2\log(x+\sqrt{x^2+a^2})\right]$ (7%)

六、解下列偏微分方程式： $\dfrac{\partial u}{\partial t}=4\dfrac{\partial^2 u}{\partial x^2}\ (0<x<1, t>0)$

B.C.: $u(0,t)=u(1,t)=0\ (t>0)$

I.C.: $u(x,0)=x(1-x)\ (0<x<1)$ (18%)

國立雲林科技大學
九十二學年度碩士班入學招生考試試題

系所：營建系
科目：工程數學

一、 試解下列二常微分方程式：

(a) $(x+y^2\sqrt{y^2-x^2})y'=y-xy\sqrt{y^2-x^2}$ (10 分)

(b) $y''+\lambda^2 y=g(x)$ (10 分)

二、 試證明 Laplace Transform 中之 Convolution Theorem。 (15 分)

設 $\mathcal{L}\{f(t)\}=F(s)$、$\mathcal{L}\{g(t)\}=G(s)$

則 $\mathcal{L}\{f*g\}=F(s)G(s)$，其中 $f*g=\int_0^t f(t-\tau)g(\tau)\mathrm{d}\tau$

三、 已知一微分方程式 $y''+6y'+8y=f(x)$

其中 $f(x)=x$, $-p<x<p$ 且 $f(x+2p)=f(x)$

(a) 試以傅立葉級數(Fourier Series)展開 $f(x)$ (5 分)

(b) 試求解此微分方程式 (10 分)

四、 矩陣 $\mathbf{A}=\begin{bmatrix}0&2&0\\2&0&0\\1&0&3\end{bmatrix}$，$\mathbf{B}=\begin{bmatrix}0&-20&0\\-20&0&0\\-10&0&-30\end{bmatrix}$，$\mathbf{C}=\begin{bmatrix}3&2+i&-5i\\2-i&-2&1\\5i&1&0\end{bmatrix}$，

$\mathbf{D}=\begin{bmatrix}0&2&-3\\-2&0&4\\3&-4&0\end{bmatrix}$，$\mathbf{E}=\begin{bmatrix}2/3&e_{12}&e_{13}\\-2/3&e_{22}&e_{23}\\1/3&e_{32}&e_{33}\end{bmatrix}$

(a) 求 **A** 之特徵值(eigenvalue)及其對應之特徵向量(eigenvector)； (6 分)

(b) 求 $\mathbf{A}^{-50}$ 之特徵值(eigenvalue)及其對應之特徵向量(eigenvector)； (4 分)

(c) 求行列式 $|\mathbf{AB}|$ 之值； (5 分)

(d) 下列何者可能為 **C** 的特徵值？何者可能為 **D** 的特徵值？並請說明理由 (5 分)

(1) -7.0, 3.9i, -3.9i (2) –5.0, -1.1, 7.0 (3) 6, 3.2+2i, -3.2+2i (4) 0, 5.4i, -5.4i

(e) 說明如何在矩陣 **E** 中填入未知元素值，使其成為一個 3×3 的 orthogonal 矩陣。 (5 分)

五、空間中有四點：A(3, 0, 0)、B(-3, 0, 2)、C(0, 3, 5)、D(0, -3, 7)，

(a) 求三角形 ABC 之面積； (5 分)

(b) 求 C 點至通過 A 與 B 之直線的最短距離； (3 分)

(c) 若有一段圓形螺旋曲線從 A 點開始，經 B 與 C 點而至 D 點結束，試寫出一參數式以描述此圓形螺旋曲線，並計算此段曲線之總長度； (9 分)

(d) 若有一圓錐曲面以 A 點為頂點、通過 D 點且中心軸垂直於 x-y 平面，試求此曲面在 D 點的單位垂直向量。 (8 分)

國立雲林科技大學

九十三學年度碩士班入學招生考試試題

系所：營建系

科目：工程數學

1. Solve the following ordinary differential equations:
   (a) $y''' - 3y'' + 3y' - y = x^{1/2}e^x$; (10%)
   (b) $xy' = y + \dfrac{x^5 e^x}{4y^3}$, $y(1) = 0$; (5%)
   (c) $y'' + (1 + y^{-1})(y')^2 = 0$. (5%)

2. Solve the following initial value problem by using Lapalce transforms:
   $y'' + y = 3\cos 2t$, $y(0) = 0$, $y'(0) = 0$ (15%)

3. For matrices $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 3 & 4 \\ -2 & 99 & 0 & -3 \\ 3 & 921 & -1 & 2 \end{bmatrix}$, $\mathbf{b}_1 = \begin{Bmatrix} 7 \\ 7 \\ -11 \\ 11 \end{Bmatrix}$, and $\mathbf{b}_2 = \begin{Bmatrix} -8 \\ -9 \\ 104 \\ 921 \end{Bmatrix}$
   (a) find the determinant $|\mathbf{A}|$; (6%)
   (b) compute $(\lambda_1\lambda_2\lambda_3\lambda_4)^3$ if λ_1, λ_2, λ_3, λ_4 are the four eigenvalues of **A**; (4%)
   (c) solve the three linear systems of equations $\mathbf{Ax} = \mathbf{b}_1$, $\mathbf{Ax} = \mathbf{b}_2$, and $\mathbf{Ax} = 2\mathbf{b}_1 - \mathbf{b}_2$ where $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$. (10%)

4. For three points A(1, −1, 2), B(−1, −1, 0), and C(0, 1, 3) in the *x-y-z* coordinate space,
   (a) determine the equation for the plane passing through A, B, and C; (6%)
   (b) what is the value of angle $\angle$BAC? (3%)
   (c) what is the area of the circle Γ passing through A, B, and C? (3%)
   (d) evaluate the integral $\int\left[y^2z(3x^2+z^2)\mathrm{d}x + 2xyz(x^2+z^2)\mathrm{d}y + xy^2(x^2+3z^2)\mathrm{d}z\right]$ from B to C along Γ. (8%)

5. If $f(x)$ is a periodic function with a period of 2 and $f(x) = \left|e^{-x}\right|$ for $-1 < x < 1$,
   (a) find the Fourier series of $f(x)$; (10%)
   (b) use the result of (a) to prove that $\sum_{n=1}^{\infty}\dfrac{1-(-1)^n e^{-1}}{1+n^2\pi^2} = \dfrac{e^{-1}}{2}$; (5%)
   (c) use the result of (a) to solve the following partial differential equation:

$$\frac{\partial u}{\partial t} = \frac{1}{\pi^2}\frac{\partial^2 u}{\partial x^2} \quad \text{with B.C.'s} \begin{cases} \dfrac{\partial u}{\partial x}(0,\ t) = 0 \\ \dfrac{\partial u}{\partial x}(1,\ t) = 0 \end{cases} \text{for all } t \text{ and I.C. } u(x,\ 0) = e^{-x} \text{ for } 0 \le x \le 1.$$

   (10%)

國立雲林科技大學
94學年度碩士班入學招生考試試題

系所：營建系
科目：工程數學

一、 試求解下列微分方程式:

(a) $4y'' + 36y = \csc 3x$ (10 分)

(b) $(y + x^2 y^4)dx + 3xdy = 0$ (10 分)

二、 若 $Q = x_1^{\ 2} + 5x_2^{\ 2} + x_3^{\ 2} + 2x_1x_2 + 2x_2x_3 + 6x_1x_3 = 5$

(a) 找出一對稱矩陣 **A** 使得 $Q = \mathbf{x}^T\mathbf{A}\mathbf{x}$，其中 $\mathbf{x}^T = [x_1 \ \ x_2 \ \ x_3]$ (3 分)

(b) 求 **A** 之特徵值及其相對應之特徵向量 (7 分)

(c) 利用(b)之結果將 Q 經由座標轉換成 $Q = \mathbf{y}^T\mathbf{D}\mathbf{y}$，其中 **D** 為對角(diagonal)矩陣。(5 分)

三、 試計算 $\iint_S x^3 dydz + x^2 ydxdz + x^2 zdxdy$

其中 S 不是封閉曲面，而是一圓柱之側面與底面。該圓柱之方程式為:($x^2 + y^2 = 1$，$0 \le z \le 1$)。 (15 分)

四、 已知 $f(x) = \pi - x$ ，$0 \le x \le \pi$；試分別以 Taylor's series、Fourier periodic series、Fourier sine series、Fourier cosine series 四種方式繪圖表示 $f(x)$，圖形展開的範圍為 $-3\pi \le x \le 3\pi$。（本題不需要寫出級數） (12 分)

五、 試求 $\dfrac{e^{-as}(4s+7)}{s^2+8s+25}$ 之逆拉氏變換(Inverse Laplace Transform)。 (10 分)

六、 試求解下列偏微分方程式： (28 分)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \sin x \ ; \quad t \ge 0 \ , \ 0 \le x \le \pi$$

邊界條件：$\begin{cases} u(0,t) = 0 \\ u(\pi,t) = \pi \end{cases}$

初始條件：$u(x,0) = \sin x$

國立雲林科技大學
95 學年度碩士班入學招生考試試題

系所：營建系
科目：工程數學

一、 若有一微分方程式 $x^3y''-9xy'+5y+\dfrac{-2+3\ln|x|}{x}=0$，回答下列問題並請說明原因：

(a) 此方程式為 linear 或 nonlinear？ (3 分)

(b) 此方程式為 homogeneous 或 nonhomogeneous？ (3 分)

(c) 若 y_1 及 y_2 均為此方程式的解，請問 $y_3=2y_1-y_2$ 是否亦為此方程式之解？ (4 分)

二、 一微分方程式：$y'''+3y''+4y'+2y=3e^{-t}$

(a) 求此方程式之通解； (10 分)

(b) 若已知初始條件為 $y(0)=3,\ \ y'(0)=-6,\ \ y''(0)=8$，利用 Laplace 轉換求其解。 (15 分)

三、 若空間中有一圓錐面之方程式為 $z=2\sqrt{(x-1)^2+(y-3)^2},\ \ 0\le z\le 4$，

(a) 試寫出此圓錐面之參數式； (5 分)

(b) 求此圓錐面之表面積。 (10 分)

四、 若矩陣 $\mathbf{A}=\begin{bmatrix}2 & -2 & 3\\ -2 & -1 & 6\\ 1 & 2 & 0\end{bmatrix}$，$\mathbf{x}=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$，$\mathbf{b}=\begin{bmatrix}10\\ 11\\ -1\end{bmatrix}$

(a) 求 $\mathbf{A}$ 之特徵值(eigenvalue)及其對應之特徵向量(eigenvector); (6 分)

(b) 求 $\mathbf{A}$ 之反矩陣 $\mathbf{A}^{-1}$； (6 分)

(c) 求 $\mathbf{A}^{-5}$ 之行列式值； (4 分)

(d) 解聯立方程式 $\mathbf{Ax}=\mathbf{b}$； (4 分)

(e) 求 $\mathbf{A}^3-2\mathbf{A}^2-\mathbf{A}$ 之特徵值及其對應之特徵向量。 (5 分)

五、 設 $f(t)=t \quad 0<t<p$ 為週期等於 $2p$ 之函數，試求下列情況下 $f(t)$ 之傅立葉級數(Fourier Series)展開：(a) $f(-t)=f(t)$ (8 分)；(b) $f(-t)=-f(t)$ (7 分)。

六、 函數 $f(x)=e^{-kx}\quad (x>0, k>0)$，試求其傅立葉餘弦積分(Fourier Cosine Integral)。 (10 分)

國立雲林科技大學
96學年度碩士班入學招生考試試題

系所：營建系
科目：工程數學

本試題共六大題，共計 100 分。請依題號作答並將答案寫在答案卷上，違者不予計分。

一、試求解下列常微分方程式：

(a) $y''+y=8\cos^2 x$ (10 分)

(b) $y'+\frac{1}{x}y=3x^2y^3$ (10 分)

二、試以拉氏變換(Laplace Transform)，求解以下微分方程式：

$\frac{dy}{dt}=\cos t+\int_0^t y(\tau)\cos(t-\tau)d\tau$；$y(0)=1$； (12 分)

三、下列各小題的敘述若正確則請證明之，若不正確則請舉反例說明之：

(a) 若 A 與 B 皆為對稱(symmetric)矩陣，則 AB 亦為對稱矩陣；(5 分)

(b) 若 A 為反對稱(skew-symmetric)矩陣，則 A^{-1} 亦為反對稱矩陣；(5 分)

(c) 若 A 與 B 皆為正交(orthogonal)矩陣，則 AB 亦為正交矩陣。(5 分)

四、若矩陣 $A=\begin{bmatrix}5 & -4 & 4\\ 12 & -11 & 12\\ 4 & -4 & 5\end{bmatrix}$、$B=\begin{bmatrix}12 & 921 & 34 & 56\\ 0 & 1 & 2 & 0\\ 12 & 920 & 32 & 56\\ 911 & 119 & 67 & 89\end{bmatrix}$、$C=\begin{bmatrix}98 & 76 & 54 & 32\\ 12 & 34 & 56 & 78\\ 9 & 8 & 7 & 6\\ 1 & 2 & 3 & 4\end{bmatrix}$

(a) 求 A 之特徵值(eigenvalue)及其對應之特徵向量(eigenvector)；(12 分)

(b) 求行列式 $|B|$ 之值；(4 分)

(c) 求行列式 $|BC|$ 之值。(4 分)

五、已知 $\vec{F}=4xz\vec{i}+xyz^2\vec{j}+3z\vec{k}$，曲面 S 為：$x^2+y^2=z^2$，$0\le z\le 4$，所圍成之封閉曲面；$\vec{n}$ 為曲面 S 之單位法向量。試計算 $\oiint_S \vec{F}\cdot\vec{n}d\vec{A}=?$ (18 分)

六、若已知 $f(x)=\frac{x}{2}$ for $-2<x<2$，且其乃是一個週期為 4 之週期性函數，

(a) 請列出 $f(x)$ 之傅立葉級數(Fourier series)；(12 分)

(b) 以前小題之結果證明 $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+-\cdots\cdots=\frac{\pi}{4}$。(3 分)

國立雲林科技大學
97 學年度碩士班入學招生考試試題

系所：營建系

科目：工程數學

一、 試求解下列常微分方程式：

(a) $xdy-(y+xy^3\ln x)dx=0$ (15 分)

(b) $(3x+2)^2y''+3(3x+2)y'-9y=9x^2+3x-2$ (15 分)

二、 試求下列函數 $f(t)$ 之拉氏變換(Laplace transform) (10 分)。

$$f(t)=\begin{cases}\cos t, & 0<t<\pi\\ 0 &, \pi<t<2\pi\end{cases} \quad \text{且} \quad f(t+2\pi)=f(t)$$

三、 試求下列微分方程式之特徵值(eigenvalue)及特徵函數(eigenfunction)。

$y''+\eta y=0$ ； $x\in[0,L]$ ； B.C.： $y'(0)=0$ ， $y'(L)=0$ (10 分)

四、 矩陣 $\mathbf{A}=\begin{bmatrix}1&2&0\\6&2&-3\\0&2&1\end{bmatrix}$ ， $\mathbf{B}=\begin{bmatrix}987&2&256\\987&1&256\\988&-123&256\end{bmatrix}$ ， $\mathbf{C}=\begin{bmatrix}1&2&3&4\\-2&-4&-3&-7\\3&7&11&12\\-4&-9&-11&-14\end{bmatrix}$ ，

$\mathbf{D}=\begin{bmatrix}1&5&6&7\\5&2&8&9\\6&8&3&10\\7&9&10&4\end{bmatrix}$ ， $\mathbf{E}=\begin{bmatrix}0&2+i&1-2i&-3+2i\\-2+i&-i&0&-4-i\\-1-2i&0&2i&0\\3+2i&4-i&0&-3i\end{bmatrix}$ ，

(a) 求 **A** 之特徵值(eigenvalue)及其對應之特徵向量(eigenvector) (8 分)；

(b) 求行列式 $|\mathbf{A}^3\mathbf{B}^{-1}|$ (6 分)； (c) 求行列式 $|\mathbf{C}|$ (6 分)；

(d) 下列何者可能為 **D** 的特徵值？何者可能為 **E** 的特徵值？請說明理由 (5 分)。

(1) $-4.6, 2.3, 1+3.9i, 1-3.9i$ (2) $5.8i, -6.8i, -3.1i, 2.1i$

(3) $-6.7, -5.6, -3.3, 25.6$ (4) $0, 5.2i, -5.2i, 3.1-2.5i$

五、 若已知 $f(x)=\cos\frac{\pi}{2}x$ for $-1<x<1$ ，且其乃是一個週期為 2 之週期性函數，

(a) 請列出 $f(x)$ 之傅立葉級數(Fourier series) (10 分)；

(b) 以(a)結果證明 $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{n}{(2n-1)(2n+1)}=-\frac{1}{3}+\frac{2}{15}-\frac{3}{35}+\frac{4}{63}-+\cdots\cdots=-\frac{1}{4}$ (5 分)。

六、 若 x-y-z 空間座標系統中有 A(1, 0, 2)、B(−1, 1, 0)及 C(0, 1, 1)三點，

(a) 求三角形 ABC 之面積 (3 分)；

(b) 若 Γ 為連結由 A 至 B、再由 B 至 C 兩段線段的折線，請由 A 至 C 沿著 Γ 進行下列積分： $\int_{\Gamma}\left[y^2z\,dx+2xyz\,dy+xy^2\,dz\right]$ (7 分)。