暨南大學

土木所

91~96 學年度
工程數學考古題

國立暨南國際大學九十一學年度碩士班研究生入學考試試題

第 1 節工程數學　適用:(土木所大地 471 土木所結構 481 土木所水利 491　)　　(本試題共 一頁，第 一頁)

考生注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回。(餘請詳閱試場規則)

1. Solve the following differential equation

$$y''-2y'-8y=\int_0^t f(t-\tau)e^{-3\tau}d\tau,\ y(0)=1,\ y'(0)=0 \quad (15\%)$$

2. Find the general solution of the following differential equation in terms of Bessel function. (15%)

$$9x^2y''+9xy'+(4x^{2/3}-16)y=0$$

3. Find a stationary function y(x) for the integral satisfying the given conditions (15%)

$$\int_1^3[x(y')^2-y]dx,\quad y(1)=3,\ y(3)=4$$

4. Evaluate $\int_0^{2\pi}\frac{d\theta}{(5-3\sin\theta)^2}$ (15%)

5. Solve the boundary value problem using Fourier Transform in x. (15%)

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}(-\infty<x<\infty, t>0)$$

$$u(x,0)=f(x)(-\infty<x<\infty)$$

6. Evaluate $\iint_S F\bullet N dA$, where $F=xi+yj+zk$, S is the surface bounding the cone $z=\sqrt{x^2+y^2}$ for $0\le z\le 1$ and N is the unit outer normal vector. (15%)

7. Show that the Eigenvalues of a real, symmetric matrix are real. (10%)

國立暨南國際大學九十二學年度碩士班研究生入學考試試題

第 1 節工程數學適用:(土木所大地組 471 土木所結構組 481 土木所水利組 491) (本試題共 / 頁，第 / 頁)

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3. 試題隨卷繳回。(敬請詳閱試場規則)

1. Solve the following nonhomogeneous differential equation: [15%]

$$x^2\frac{d^2y(x)}{dx^2}-5x\frac{dy(x)}{dx}+8y(x)=x^5\cos x$$

2. Solve the following nonhomogeneous differential equation: [15%]

$$\frac{d^2y(x)}{dx^2}+8\frac{dy(x)}{dx}+16y(x)=x\cos(2x)$$

3. Use Laplace transform to solve the following equation [15%]

$$\frac{dy(t)}{dt}+\int_0^t y(x)dx=t\mathrm{H}(t-2),\quad y(0)=1$$

where H(t) is the Heaviside function.

4. Evaluate the surface integral $\iint_S \mathbf{F}\cdot\mathbf{n}dS$ where $\mathbf{F}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ and S is the surface of the sphere $x^2+y^2+z^2=4$ lying between the planes $z=1$ and $z=2$. [20%]

5. Evaluate the integral $\int_{-\infty}^{\infty}\frac{\cos x}{x^2-2x+5}dx$ [15%]

6. Solve the problem of the vibrating rectangular membrane: [20%]

$$\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=0,\ \text{for}\ 0<x<\pi,\ 0<y<A,\ t>0$$

$u=0$, for $x=0,\ x=\pi,\ y=0,\ y=A$

$u(x,y,0)=f(x,y)$,

$\frac{\partial u}{\partial t}(x,y,0)=g(x,y)$

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3. 試題隨卷繳回。(餘請詳閱試場規則)

1. Find the general solution of the differential equation: [15%]

$$\frac{d^2 y(x)}{d x^2} - 2\frac{dy(x)}{dx} + y(x) = 2x + 25\sin(2x)$$

2. Find the general solution of the differential equation: [15%]

$$x^2 \frac{d^2 y(x)}{d x^2} - 4x\frac{dy(x)}{dx} + 4y(x) = 9x^2 + 6x + 6$$

3. Find the minimum value of $x^2 + 4y^2 + 16z^2$ under the constraint $xy = 1$ and locate the corresponding points. [15%]

4. Evaluate the surface integral $\iint_S f(x)dS$ where $f(x) = 4x^2$ and S is the portion of the plane $x + y + z = 1$ inside the cylinder $x^2 + y^2 = 1$ [20%]

5. Evaluate the integrals $\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+4)(x^2+9)} dx$ [15%]

6. Solve the following problem [20%]

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2},$$

$$u(0,t) = u(L,t) = 0,$$

$$u(x,0) = x, \qquad \frac{\partial u}{\partial t}(x,0) = 0$$

科目：工程數學 適用：土木所結構組 土木所大地組 土木所水利組 地震所

編號：481 491 501 511

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本試題 共壹頁 第壹頁

1. For the nonhomogeneous equation

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 6x\frac{\mathrm{d}y}{\mathrm{d}x} + 12y = 16\ln(x).$$

Please find

(a) the homogeneous general solution y_h, and [10%]

(b) the homogeneous particular solution y_p. [10%]

2. Please use Laplace Transformation to solve the initial value problem [20%]

$$y'' + 5y' + 6y = f(t); \quad y(0) = y'(0) = 0;$$

with

$$f(t) = \begin{cases} -2 & \text{for } 0 \le t < 3 \\ 0 & \text{for } t \ge 3 \end{cases},$$

3. Please find the eigenvalues and eigenvectors of [20%]

$$A = \begin{pmatrix} -3 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 0 \end{pmatrix},$$

where all the eigenvectors are of length 1.

4. Please solve the Dirchlet problem [20%]

$$\begin{aligned} \nabla^2 u(x,\, y) &= 0 \quad \text{for } 0 < x < L, \ 0 < y < K, \\ u(x,\, 0) &= 0 \quad \text{for } 0 \le x \le L, \\ u(0,\, y) = u(L,\, y) &= 0 \quad \text{for } 0 \le y \le K, \\ u(x,\, K) &= (L-x)\sin(x) \quad \text{for } 0 \le x \le L. \end{aligned}$$

5. Please find the Fourier coefficients and the Fourier series of function $f(x) = e^{-4x}$ for $-2 \le x \le 2$. [20%]

科目：工程數學 適用：土木所防災 土木所應力 地震所

編號：451 471 491

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本試題共 / 頁 第 / 頁

1 (18%) Please solve the following initial value problem.

$$y'''-y''-y'+y=4e^{x},\quad y(0)=2,\quad y'(0)=1,\quad y''(0)=2$$

2 (18%) Please solve the following initial value problem.

$$y'+3x^2y=e^{-x^3}/x^2,\quad y(1)=0$$

3 (20%) The convolution $f*g$ of functions f and g is defined by

$$(f*g)(x)=\int_{-\infty}^{\infty}f(p)g(x-p)dp=\int_{-\infty}^{\infty}f(x-p)g(p)dp.$$

And the definition of Fourier transform of $f(x)$ is $\mathcal{F}(w)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-iwx}dx$.

Suppose that $f(x)$ and $g(x)$ are piecewise continuous, bounded and absolutely integrable on the x-axis. Then please prove that $\mathcal{F}(f*g)=\sqrt{2\pi}\,\mathcal{F}(f)\mathcal{F}(g)$.

4 (24%) A curve is defined as $\vec{r}(t)=[a\cos t, a\sin t, ct]$. Please find

(a) $\vec{r}(s)$, where s is the arc length (4%)

(b) $\vec{u}(s)$, where $u(s)$ is the 'unit tangent vector' (4%)

(c) $\kappa(s)$, where $\kappa(s)$ is the curvature of curve' (4%)

(d) $\vec{p}(s)$, where $p(s)$ is the 'unit principal normal vector' (4%)

(e) $\vec{b}(s)$, where $b(s)$ is the 'unit binormal vector' (4%)

(f) $\tau(s)$, where $\tau(s)$ is the torsion of curve. (4%)

5 (20%) A mechanical system is governed by the differential equations

$$y_1''=-5y_1+2y_2$$
$$y_2''=\ \ 2y_1-2y_2$$

In vector form, it becomes

$\mathbf{y}''=\begin{bmatrix}y_1\\y_2\end{bmatrix}=\mathbf{A}\mathbf{y}=\begin{bmatrix}-5 & 2\\2 & -2\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix}$. We try a vector solution of the form $\mathbf{y}=\mathbf{x}e^{\omega t}$, which implies to an eigenvalue problem $\mathbf{A}\mathbf{x}=\lambda\mathbf{x}$. Please use the above information to solve and find $y_1(t)$ and $y_2(t)$.

科目：工程數學 適用：土木所耐震 土木所應力 地震所

編號：461 481 491

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本試題 共 / 頁 第 / 頁

1 Please solve the following initial value problem. (20%)

$x^2y''+xy'+4y=0$ with $y(1)=2$, $y'(1)=-1$ and $x>0$.

2 An unit step function is defined as $u(t-a)=\begin{cases}0 & \text{if } t<a\\ 1 & \text{if } t>a\end{cases}$.

For the following initial value problem, $y''+3y'+2y=r(t)$,

with $r(t)=\begin{cases}2 & \text{if } 1<t<2\\ 0 & \text{otherwise}\end{cases}$ and $y(0)=y'(0)=0$ please

(a) express $r(t)$ with unit step function (10%), and

(b) find the solution $y(t)$ by means of Laplace Transformation. (15%)

Remark: $\mathcal{L}[u(t-a)]=e^{-as}/s$, $\mathcal{L}[e^{at}f(t)]=F(s-a)$

$\mathcal{L}[f(t-a)u(t-a)]=e^{-as}F(s)$, $\mathcal{L}\left[t^{n-1}/(n-1)!\right]=1/s^n$ $n=1,2,\cdots$.

3 Please find the real general solution $\mathbf{y}(t)=\begin{bmatrix}y_1(t)\\ y_2(t)\end{bmatrix}$ of the following system

$\begin{cases}y_1'=y_1+y_2\\ y_2'=3y_1-y_2\end{cases}$ (20%).

4 Let $\mathcal{F}(f)$ denote the fourier transform of f, that $\mathcal{F}(f)=\hat{f}(w)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-iwx}dx$.

Please find the fourier transform of $f(x)=k$ if $0<x<a$ and $f(x)=0$ otherwise. (20%)

5 For a real and symmetric matrix, **A**, please show that its eigenvectors associated with distinct eigenvalues are orthogonal. (15%)