彰化師範大學

電子工程學系

93~97 學年度
工程數學考古題

1. Solve $y''-y=f(t)$, where $y(0)=0$ and $y'(0)=1$, and $f(t)=1$ if $0<t<2$, $f(t)=0$ if $t>2$. (15%)

2. Find the inverse Laplace transform ($\mathcal{L}^{-1}(s)$) of the function $\dfrac{a}{s^2(s-a)}$ (10%)

3. Let $A=\begin{bmatrix} 1 & 7 & 1 \\ 0 & 2 & 6 \\ 0 & 1 & -3 \end{bmatrix}$, find the eigenvalues and eigenvectors of A. (10%)

4. Show the following Fourier transform theorems:
   (a) Convolution theorem $\mathcal{F}\{f*g\}=\sqrt{2\pi}\,\mathcal{F}\{f\}\mathcal{F}\{g\}$. (10%)

5. Solve the differential equations
   (a) $x^2y''(x)+xy'(x)-9y(x)=0$. (10%)
   (b) $y''(x)+4y(x)=3\sin x$. (10%)

6. Evaluate the integral $\oint_C \dfrac{z}{(z^2-2)}dz$, C is the circle |z|=2. (10%)

7. Find the tangent to the ellipse $\dfrac{x^2}{4}+\dfrac{y^2}{9}=1$, at P: $(\sqrt{2},\dfrac{3}{\sqrt{2}})$. (10%)

8. Describe the region of integration and evaluate $\int\limits_{0}^{\pi/4}\int\limits_{0}^{y}\sin y\,dxdy$. (15%)

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1. Solve the nonhomogeneous Euler-Cauchy equation.　(20%)

   $$x^3 y''' + 2xy' - 2y = 2x^2$$

2. Solve the initial value problem. (20%)

   $$y'' - 4y' + 5y = 5x, \quad y(0) = 1, \quad y'(0) = 1$$

3. Construct an diagonal matrix from the eigenvectors of　(20%)

   $$\mathbf{A} = \begin{pmatrix} 2 & 6 & 0 \\ 0 & 3 & 2 \\ 0 & 2 & 3 \end{pmatrix}$$

4. $\vec{F} = 2x^2\vec{i} + 4(y^2 - 5z - 2x)\vec{j} + (z^2)\vec{k}$, and S is the circle of $x^2 + (y)^2 = 16$, evaluate $\iint_S (\text{curl } \vec{F}) \bullet \vec{n}\, dS$. Assume that the surface is oriented upward.　(20%)

5. Find the Fourier series of the periodic function $f(x)$ of period 2π.　(20%)

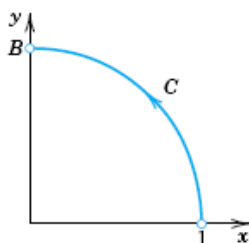 $$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 2, & 0 < x < \pi/2 \\ 0, & \pi/2 < x < \pi \end{cases}$$

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1. Solve the following equations.(20%)

   a. $(2x+3)dy - (2y+1)dx = 0$.

   b. $y' - y = xy^5$

2. Find the value of the line integral, when $\mathbf{F}(\mathbf{r})=[-x, xy]=-x\mathbf{i}+xy\mathbf{j}$, and c is the circular arc below from A to B. (20%)

3. Solve the initial value problem.(20%)

   $$y_1{''}= -2y_1 + 3y_2 \quad y_1(0) = 2, \ y_1'(0) = -1$$
   $$y_2{''}= -6y_1 + 8y_2 \quad y_2(0) = 3, \ y_2'(0) = -3$$

4. Find the Fourier series of the periodic function f(x), of period of P=2L.(20%)

   f(x)=1 (-2<x<-1), f(x)=0 (-1<x<0), f(x)=-1 (0<x<1), f(x)=0 (1<x<2),P=2L=4

5. Inverse Laplace transform.(20%)

   a. $\dfrac{e^{-as}}{s^2(s-3)}$.

   b. $\dfrac{1}{(s^2+4)^2}$.

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1. Solve the following differential equations.

   a. $y''+2y'-3y=2e^x$. (10%)

   b. $y'=2\dfrac{y\ln y}{x}$. (10%)

2. Find the Laplace transform of the following function.

   a. $t\sin(t+a)$. (10%)

   b. $t^3\sinh 2t$. (10%)

3. Solve the following equation.

   $y(t)-\int_0^t y(\tau)\sin(t-\tau)d\tau =1$. (20%)

4. Evaluate the surface integration $\iint\limits_S F\cdot ndA$ with $\mathbf{F}=[2x, z, 3y]$, S:x+y+z=2, x≥0, y≥0, z≥0. (20%)

5. Find the Fourier series of the periodic function f(x), of period of P=2π

   f(x)=x+π/2, −π<x<π. (20%)

1.　Solve the following initial value problem. (20%)

$$y''+7y'+10y = 5e^{-t}u(t-2)\,,\quad y(0)=0,\quad y'(0)=1$$

2.　Solve the following integral, (20%)

$$\int_0^\infty te^{-3t}\cos(\,at\,)\,dt$$

3.　Find the inverse of the following matrix. (20%)

$$A=\begin{bmatrix} -2 & 1 & 2 \\ 3 & 1 & -2 \\ 2 & 4 & 4 \end{bmatrix}$$

4.　Find the Fourier cosine series of the following function, (20%)
$$f(x)=x \qquad (0<x<2)$$

5.　Evaluate the integral, (20%)

$$\int_{-\infty}^{\infty}\frac{\cos(x/4)}{x^2+\pi^2}\,dx$$

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彰化師範大學

積體電路設計研究所

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請寫下計算過程，否則不予計分。

一、 (25%)計算下列積分結果。

(1) $\int_0^\infty (e^{-st} \cdot e^{it})dt$?　$i = \sqrt{-1}$　(5 %)

(2) $\int_0^\infty (e^{-st} \cdot \sin t)dt$? (5 %)　　　Hint: $e^{it} = \cos t + i\sin t$

(3) $\int_0^\infty (\dfrac{e^{-st} \cdot \sin t}{t})dt$? (10%)　　Hint: $L\left\{\dfrac{f(t)}{t}\right\} = \int_s^\infty \hat{f}(u)du$, $L\{f(t)\} = \hat{f}(s)$

(4) $\int_0^\infty (\dfrac{\sin t}{t})dt$? (5%)

二、 (30%)求解下列微積分方程式。

(1) $(3x^2 + y\cos x)dx + (\sin x - 4y^3)dy = 0$　(10 %)

(2) $y'' - 3y' + 2y = e^{2x} + 4x$,　$y(0) = 0$,　$y'(0) = 1$　(10 %)

(3) $y' + y \cdot \tan x = \sin 2x$,　$y(0) = 1$　(10 %)

三、 (10%)若週期函數 $f(x) = \begin{cases} x, & 0 < x < p \\ -x, & -p < x < 0 \end{cases}$, $f(x+2p) = f(x)$，試將 $f(x)$ 展開成傅立葉級數。

四、 (20%)若向量場 $\vec{F} = (x + 2y + az)\vec{a}_x + (bx - 3y - z)\vec{a}_y + (4x + cy + 2z)\vec{a}_z$ 為非旋向量場 (irrotational field)，

(1) 決定常數 a、b、c。(10 %)

(2) 決定一純量場 $\phi(x, y, z)$，使得 $\nabla\phi = \vec{F}$ 。(10%)

五、 (15%)設有一力場 $\vec{F} = (2x - y + z)\vec{a}_x + (x + y - z^2)\vec{a}_y + (3x - 2y + 4z)\vec{a}_z$，當某質點在 x-y 平面上，以原點為圓心且半徑為 3 之圓運動一週，求該力場對質點所作之功。

Show the details of your answers

1. Solve the differential equations. (30%)

    (1) $dy - (e^{3x} + 2y)dx = 0$ (10 %)

    (2) $xy' - y = 0$ (10 %)

    (3) $x^3 y''' + x^2 y'' - 2xy' + 2y = 0$，$y(1) = 2$，$y'(1) = 6$，$y''(1) = 2$ (10 %)

2. Solve the following problems. (40%)

    (1) Find the value of $\int_0^\infty e^{-at} f(t)dt$, where $f(t) = \begin{cases} 1 & if \quad 0 < t < \pi \\ 0 & if \quad \pi < t < 2\pi \\ \sin t & if \quad t > 2\pi \end{cases}$ (10 %)

    (2) Find the Fourier series or Fourier integral of $f(t)$, where $f(t) = \begin{cases} 2 & if \quad |t| < 1, \\ 0 & if \quad |t| > 1. \end{cases}$ (10 %)

    (3) Evaluate the integral $\int_{-\infty}^\infty \dfrac{\cos 2t}{3^2 + t^2} dt$. (10%)

    (4) Find the Fourier series or Fourier integral of period function $f(t)$, which is assumed to have the period 2π， $f(t) = \begin{cases} |t| & if \ 0 \le t < \pi, \\ -|t| & if - \pi < t < 0 \end{cases}$ (10 %)

3. $x_1, x_2, x_3, x_4 \in R^4$, $V = span \{[x_1, x_2, x_3, x_4] : x_1 + x_2 + x_3 + x_4 = 0, x_2 + x_3 = 0 \} \subseteq R^4$. Determine the dimension of the subspace V. (15 %)

4. Find the directional derivative of $f(x, y, z) = x^3 + 2y^2 + 3z^2$ at the point P :(1, 2, 0) in the direction of the vector $\vec{a} = \hat{i} - \hat{j} + \hat{k}$. (15 %)

1. Solve the differential equations. (30%)

   (1) $y''+4y=8x^2$ (10 %)

   (2) $y'+\dfrac{1}{3}y=\dfrac{1}{3}(1-2x)y^4$ (10 %)

   (3) $y''-\dfrac{3}{x}y'+\dfrac{3}{x^2}y=2x-1$ (10 %)

2. Solve the following problems. (30%)

   (1) Assume $L[f(t)]=\int\limits_{0}^{\infty}e^{-at}f(t)dt$, determine the value of $L[f(t)]$ and $L[f'(t)]$,

   where $f(t)=\begin{cases} 2t & if \ \ 0 \le t \le 1, \\ \\ t & if \ \ \ \ t > 1. \end{cases}$ (20 %)

   (2) Evaluate the integral $\int\limits_{0}^{2\pi}\dfrac{1+4\cos\theta}{17-8\cos\theta}d\theta$. (10%)

3. 試用最小平方法，由已知四點 $(5, 10.0), (10, 8.9), (15, 8.2), (20, 7.0)$ 求出一直線

   方程式. (10 %)

4. Determine the minimum value of function

   $f(x_1, x_2, x_3) = 35 - 6x_1 + 2x_3 + x_1^2 - 2x_1x_2 + 2x_2^2 + 2x_2x_3 + 3x_3^2$ (15%)

5. Let $\vec{r}$ be a position vector, determine $\nabla \bullet (r^{n-1}\vec{r})$. (15 %)

1. Solve the differential equation $2x^2y''-5xy'-4y=0$. (10%)

2. Solve the differential equation $y'' + 4y = 8x^2$. (15%)

3. Diagonalize the matrix A, where

$$A = \begin{bmatrix} 7.3 & 0.2 & -3.7 \\ -11.5 & 1.0 & 5.5 \\ 17.7 & 1.8 & -9.3 \end{bmatrix} \quad (10\%)$$

4. Let S be the subspace of R^4 containing all vectors with $x_1 + x_2 + x_3 + x_4 = 0$ and $x_1 + x_2 - x_3 - x_4 = 0$, find a basis for the space of $S^\perp$. ($S^\perp$ = containing all vectors orthogonal to S). (15%)

5. Find a unit normal vector n of $z^2 = 4x^2 + 9y^2$ at the point p:(2,-1,-5).(15%)

6. Describe the region of the following integration and evaluate. (15%)
$$\int_0^1 \int_x^{2x} xy^2 dydx$$

7. Evaluate the integral(counterclockwise). (20%)

$$\oint_C \frac{3-4z}{(z^3 - z)}dz \qquad C :|z| = 2$$

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