

大葉大學

機械工程學系

91~96 學年度

工程數學考古題

大 葉 大 學 九 十 一 學 年 度 碩 士 班 甄 試 試 題 紙

所 別	組 別	考 試 科 目 (中 文 名 稱)	考 試 日 期	考 試 時 間	備 註
			12 月 10 日		

註：備註欄若未註明可攜帶計算機或其他資料作答時，考生一律不准攜帶。

一、 Find two unit vectors orthogonal to $\vec{V}_1 = 3i + 4j - 2k$ and $\vec{V}_2 = -3i + 4j + k$ (15 %)

二、 Solve the following equation (i.e. find x_1 , x_2 and x_3) by using Gauss-elimination.

$$\begin{cases} 3x_1 + 6x_2 - 6x_3 = 9 \\ 2x_1 - 5x_2 + 4x_3 = 6 \\ -x_1 + 16x_2 - 14x_3 = -3 \end{cases} \quad (15 \%)$$

三、 Calculate $\int F(x) \cdot dx$, where $F(x, y) = xyi + ye^x j$ along the path C, where C is the rectangle joining the points (0,0) , (2,0) , (2,1) and (0,1) counter clock wise. (15 %)

四、 Given $f(x) = \begin{cases} x+1 , & -1 \leq x \leq 0 \\ -x+1 , & 0 \leq x \leq 1 \end{cases}$ $f(x+2) = f(x)$ (i.e. period = 2) (20 %)

Find its Fouries series.

五、 Show that $\int \frac{e^z}{(z^2 - 1)^2} dz = \frac{\pi i}{e}$, where $|Z| = 2$, where $Z = X + iY$ (15 %)

六、 Compute $\int_0^{\infty} \frac{dx}{1+x^6}$ (20 %)

大葉大學九十二學年度碩士班甄試試題紙

所別	組別	考試科目 (中文名稱)	考試日期	考試時間	備註
電機系/電訊		工程數學	12月9日	9:00 - 10:30	共乙頁

註：備註欄若未註明可攜帶計算機或其他資料作答時，考生一律不准攜帶。

(1). (20%) Find the following inverse Laplace transforms. (a) $L^{-1}\left\{\frac{1}{s-4}e^{-2s}\right\}$

$$L^{-1}\left\{\frac{s}{s^2+9}e^{-2s}\right\} \quad (c) \quad L^{-1}\left\{\ln\left(1+\frac{a^2}{s^2}\right)\right\} \quad (d) \quad L^{-1}\left\{\frac{1}{s^2(s^2+a^2)}\right\}.$$

(2). (20%) Evaluate $\oint_C (3y - e^{\sin x})dx + (7x + \sqrt{y^4 + 1})dy$, where C is the circle $x^2 + y^2 = 9$.

(3). (20%) Solve $u_{xx}(x,y) = 4u_y(x,y)$.

(4). (20%) Find the general solution of the differential equation

$$3x \frac{dy}{dx} + y = -x^2 y^4$$

(5). (20%) Find the Fourier series of $f(x) = x + \pi$ when $-\pi < x < \pi$ and $f(x) = f(x + 2\pi)$.

大葉大學九十三年學年度碩士班甄試試題紙

所 別	組別	考 試 科 目 (中 文 名 稱)	考 試 日 期	考 試 時 間	備 註
電機工程學系		工程數學	12月8日	9:00~10:30	共乙頁

註：備註欄若未註明可攜帶計算機或其他輔助工具作答時，考生一律不准攜帶。

(1) . Solve the following first order differential equations (where $y' = \frac{dy}{dx}$). (26%)

(a). $y' + y = (x+1)^2$, $y(0) = 0$

(b). $y' - x^3 y = -4x^3$, $y(0) = 6$

(2) . Solve the following Second order differential equations (where $y' = \frac{dy}{dt}$, $y'' = \frac{d^2y}{dt^2}$). (26%)

(a). $y'' + 2y' + 2y = \cos t$, $y(0) = 1.2$, $y'(0) = 1.4$

(b). $ty'' - y' = (3+t)t^2 e^t$, $y(-1) = 2 + e^{-1}$, $y'(-1) = -2 - e^{-1}$

(3) . Find the following complex integration. (30%)

(a). $\int_c \frac{6z^2 - 4z + 1}{(z-2)(1+4z^2)} dz$, $c: |z| = 1$

(b). $\int_0^{2\pi} \frac{\cos \theta}{17 - 8 \cos \theta} d\theta$

(c). $\int_{-\infty}^{\infty} \frac{x}{(4+x^2)^2} dx$

(4) . Consider the saw tooth function.

$$F(x) = \begin{cases} x+1, & -1 \leq x \leq 0 \\ -x+1, & 0 \leq x \leq 1 \end{cases}, f(x+2) = f(x) \quad \text{Find its Fourier series. (18\%)}$$

大葉大學九十四學年度碩士班甄試試題紙

所 別	組別	考試科目 (中文名稱)	考試日期	考試時間	備註
電機工程研究所		工程數學	12月13日	9:00 ~ 10:30	共乙頁

註：備註欄若未註明可攜帶計算機或其他輔助工具作答時，考生一律不准攜帶。

Part I. Ordinary Differential Equations

Solve the following differential equations, please write down all the steps in details. (40%)

(1). $y' = \frac{x(1-y^2)}{1+x^2}$

(2). $y' = 3x^2 - \frac{y}{x}$

(3). $e^x \sin y - 2x + (e^x \cos y + 1)y' = 0$

(4). $xy' = \frac{y^2}{x} + y$

(5). $x^2y'' + 3xy' + 10y = 0$

(6). $y'' - 6y' + 9y = 5e^{3x}$

Part II. Laplace Transform

Solve the following initial value problems by Laplace transform, please write down all the steps in details. (20%)

(7). $y'' + y = t; \quad y(0) = 1, \quad y'(0) = 0$

(8). $y'' + 4y = f(t); \quad y(0) = y'(0) = 0$

in which $f(t) = \begin{cases} 0 & \text{for } t < 3 \\ t & \text{for } t \geq 3 \end{cases}$

Part III. Fourier Transform

(9). Let K be a positive number and let $f(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq K \\ 0 & \text{for } t > K \end{cases}$

Find the Fourier sine and Fourier cosine transform of the above function. (20%)

Part IV. Complex Integration

(10). Evaluate the complex integration, (20%)

$\oint_C (z + \bar{z} + \frac{1}{z} + z\bar{z} + x + y) dz$, where $z = x + iy$, The closed path defined by $C: |z| = 1$

系 所 別	組 別	考 試 科 目 (中文名稱)	考 試 日 期	節 次	備 註
電機工程		工程數學	12月19日	第一節	共二頁

註：考生可否攜帶計算機或其他資料作答，請在備註欄註明（如未註明，一律不准攜帶）

- (10%) Find the general solution of the first order differential equation

$$\frac{dy}{dx} - \frac{2}{x}y = x^2 \cos 3x.$$
- (10%) The solution of the second order differential equation $y'' - ay' + by = 0$ with initial condition $y(0) = 1$ and $y'(0) = 3$ is $y(t) = Ae^{-t} \cos 2t + Be^{-t} \sin 2t$ where a, b, A, B are constants. Find the values of a, b, A, B
- (12%) Solve the integral-differential equation by the method of Laplace transform

$$y'(x) = 1 - \int_0^x y(t)e^{-2(x-t)} dt; y(0) = 1$$
- (18%) Let $H(\omega), F(\omega), G(\omega)$ be the **Fourier Transform** of $h(t), f(t)$ and $g(t)$ respectively
 - (6%) (1) If $g(t)$ is related to $f(t)$ as follows:

$$g(t) = 2f(t) + \frac{1}{2}[f(t-2) + f(t+2)]$$
 What is the relationship between $F(\omega)$ and $G(\omega)$.
 - (12%) (2) If $g(t)$ be the convolution of $f(t)$ and $h(t)$. If $f(t) = e^{i\omega_1 t} + e^{-i\omega_2 t}$, express $g(t)$ in terms of ω_1, ω_2 , and $H(\omega)$.
- (12%) Using (a) direct calculation (b) Green's theorem in the plane evaluate

$$\oint_C [(3x^2 + y)dx + 4y^2 dy]$$

C is the boundary of the triangle with vertices $(0,0), (1,0), (0,2)$ in counterclockwise
- (10%) $f(x) = 2x, g(x) = 3 + cx, 0 \leq x \leq 1$
 - (1) What is the value of c so that $f(x)$ and $g(x)$ are orthogonal?
 - (2) What are the normalized functions of $f(x)$ and $g(x)$ respectively?
- (10%) Solve the eigenvalue problem

$$y'' + \lambda y = 0, y(0) = y(1) = 0$$
- (8%) Find the determinant of the $n \times n$ matrix \mathbf{B} that has p 's on the diagonal and q 's elsewhere:

$$\mathbf{B} = \begin{bmatrix} p & q & \cdots & q \\ q & p & \cdots & q \\ \vdots & \vdots & \ddots & \vdots \\ q & q & \cdots & p \end{bmatrix}$$
- (10%) Let λ and μ be two distinct eigenvalues of a real square matrix \mathbf{A} , and let \mathbf{x} and \mathbf{y} be corresponding eigenvectors of \mathbf{A} , Show that if in addition, \mathbf{A} is symmetric, then \mathbf{x} and \mathbf{y} are orthogonal

大葉大學 96 學年度 研究所碩士班甄試 招生考試試題紙

系 所 別	組 別	考 試 科 目 (中文名稱)	考 試 日 期	節 次	備 註
電 機 工 程		工 程 數 學	12 月 23 日	第 一 節 8:30~10:00	莫乙夏

註：考生可否攜帶計算機或其他資料作答，請在備註欄註明（如未註明，一律不准攜帶）

1. Please solve the following first-order differential equations:

(a) $(x^2 + y^2 + x)dx + (xy)dy = 0$ (10%)

(b) $(1 + x^2)(dy - dx) = (2xy)dx$ (10%)

2. A second-order differential equation with boundary conditions is given as follows:

$x^2y'' - xy' + y = x \ln x$ (10%)

B.C. $y(1) = 0$ and $y'(1) = 0$

If the solution with a form likes $y(x) = (c_1 + c_2 \ln x)x + c_3x \ln^3 x$, please find the constants c_1, c_2 and c_3 ?

3. Laplace transform:

(a) Use **convolution theorem** to find the inverse of the following Laplace's equation:

$F(s) = \frac{1}{(s^2 + 1)^2}$ (10%)

(b) Use Laplace transform to solve the following integral equation:

$y'(t) + 5 \int_0^t \cos 2(t - \tau)y(\tau)d\tau = 10 \quad y(0) = 2$ (10%)

4. Sturm-Liouville boundary value problem and expansion of Fourier series:

(a) Solve the following Sturm-Liouville boundary value problem (SL-BVP): (15%)

$y'' + \lambda y = 0 \quad y(x) = y(x + 2\pi) \text{ and } -\pi \leq x \leq \pi$

B.C. $y(-\pi) = y(\pi)$

$y'(-\pi) = y'(\pi)$

Please discuss the following conditions in detail, i.e., $\lambda > 0$, $\lambda = 0$ and $\lambda < 0$, respectively.

(b) Please expand the following function $f(x)$ in a Fourier series: (15%)

$f(x) = \begin{cases} -1 & \text{if } -\pi \leq x < 0 \\ +1 & \text{if } 0 \leq x \leq \pi \end{cases}, f(x) = f(x + 2\pi)$

5. Complex integral and Laurent series

(a) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for $0 < |z+1| < 2$ (10%)

(b) Find the complex integration $\frac{1}{2\pi i} \oint_C \frac{dz}{(z^{50} + 1)(z - 3)}$ and $C: |z| = \infty$ (10%)

大葉大學

機械工程研究所
機電自動化研究所
車輛工程研究所

91~95 學年度
工程數學考古題

大 葉 大 學 九 十 一 學 年 度 碩 士 班 甄 試 試 題 紙

所 別	組 別	考 試 科 目 (中 文 名 稱)	考 試 日 期	考 試 時 間	備 註
機械工程研究所 機電自動化研究所 車輛工程研究所	甲乙丙丁 甲 甲	工程數學	12月10日	8:30~10:00	

註：備註欄若未註明可攜帶計算機或其他資料作答時，考生一律不准攜帶。

注意：(1) 答題應詳列運算步驟，否則不予計分。

(2) 答案卷務必標明題號，無題號之計算視為草稿，不予計分。

1. (15 分)

(a) Is the following differential equation exact or not? (5 分)

$$y(x+y)dx + (x+2y-1)dy = 0$$

(b) Solve the above differential equation. (10 分)

2. (15 分) Solve the differential equation

$$y'' + 5y' + 6y = 3e^{-2x} + e^{3x}$$

3. (15 分) Find the inverse Laplace transform of $\frac{1}{s(s-4)^2}$.

4. (15 分) Find the eigenvalues and the solution of the Sturm-Liouville problem

$$y'' + \lambda y = 0; \quad y(0) = y\left(\frac{\pi}{2}\right) = 0$$

5. (15 分) If $\mathbf{F} = xy\mathbf{i} + y^2z\mathbf{j} + z^3\mathbf{k}$, use the divergence theorem of Gauss to evaluate $\iint_S (\mathbf{F} \cdot \mathbf{n}) dS$, where S is

the surface of the unit cube defined by $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$. (本題限用高斯散度定理作答)

6. (10 分) Solve the integral equation for $Y(x)$ ($0 \leq x < \infty$)

$$\int_0^{\infty} Y(x) \sin(xt) dx = \begin{cases} 1 & 0 \leq t < 1 \\ 2 & 1 \leq t < 2 \\ 0 & t \geq 2 \end{cases} \quad (\text{Hint: Use Fourier sine transform pair.})$$

7. (15 分) Given a matrix $\mathbf{A} = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$ and polynomials $p(x) = x^4 - 4x^3 + 6x^2 - x - 3$, $q(x) = x^{100}$,

(a) find the eigenvalues and corresponding eigenvectors of the matrix \mathbf{A} .

(b) find $\mathbf{P}(\mathbf{A}) = \mathbf{A}^4 - 4\mathbf{A}^3 + 6\mathbf{A}^2 - \mathbf{A} - 3\mathbf{I}$ and $\mathbf{Q}(\mathbf{A}) = \mathbf{A}^{100}$.

大 葉 大 學 九 十 二 學 年 度 碩 士 班 甄 試 試 題 紙

所 別	組 別	考 試 科 目	考 試 日 期	考 試 時 間	備 註
機械工程研究所 機電自動化研究所 車輛工程研究所	甲、乙、丙、丁 甲 甲	工程數學	12月9日	09:00~10:30	不准 攜帶計 算機

注意：(1) 答題應詳列運算步驟，否則不予計分。(2) 答案卷務必標明題號，無題號之計算視為草稿，不予計分。

1. (12 分) Solve the differential equation

$$x^2 y'' + 7xy' + 9y = 0$$

2. (15 分) Solve the following initial value problem. (You may apply any method you wish to solve this problem.)

$$y'' - 2y' + 2y = 17e^{5x}, \quad y(0) = 1, \quad y'(0) = 6$$

3. (15 分) Solve the initial value problem by the **Laplace transform**

$$y' + 4y = 20, \quad y(0) = 2$$

4. (16 分) For the given function $f = 2x^2 + y^2 + 3z$, find the following expressions:

a. ∇f b. $\nabla^2 f$ c. $\nabla \times \nabla f$

d. directional derivative of f in the direction of $\bar{b} = 2\bar{i} + \bar{j}$

5. (12 分) Evaluate the line integral $\int_C \bar{F} \cdot d\bar{r}$, with $\bar{F} = [-y, x, -1]$ and

C: $\bar{r} = [\cos t, \sin t, 2t]$, from $(1,0,0)$ to $(1,0,4\pi)$.

6. (12 分) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

7. Given the periodic function

$$f(x) = \begin{cases} 1 & \text{if } -\pi < x < 0 \\ -1 & \text{if } 0 < x < \pi \end{cases} \quad \text{and } f(x+2\pi) = f(x)$$

a. Plot the function $f(x)$. (2 分)

b. Is $f(x)$ odd, even or neither odd nor even? (1 分)

c. Find the Fourier series of this function. (15 分)

大葉大學九十二學年度碩士班甄試試題紙

所 別	組 別	考 試 科 目	考 試 日 期	考 試 時 間	備 註
機械工程研究所 機電自動化研究所 車輛工程研究所	甲、乙、丙、丁 甲 甲	工程數學	12月9日	09:00~10:30	不准 攜帶計 算機

注意：(1) 答題應詳列運算步驟，否則不予計分。(2) 答案卷務必標明題號，無題號之計算視為草稿，不予計分。

1. (12分) Solve the differential equation

$$x^2 y'' + 7xy' + 9y = 0$$

2. (15分) Solve the following initial value problem. (You may apply any method you wish to solve this problem.)

$$y'' - 2y' + 2y = 17e^{5x}, \quad y(0) = 1, \quad y'(0) = 6$$

3. (15分) Solve the initial value problem by the Laplace transform.

$$y' + 4y = 20, \quad y(0) = 2$$

4. (16分) For the given function $f = 2x^2 + y^2 + 3z$, find the following expressions:

a. ∇f b. $\nabla^2 f$ c. $\nabla \times \nabla f$

d. directional derivative of f in the direction of $\vec{b} = 2\vec{i} + \vec{j}$

5. (12分) Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, with $\vec{F} = [-y, x, -1]$ and

C: $\vec{r} = [\cos t, \sin t, 2t]$, from $(1,0,0)$ to $(1,0,4\pi)$.

6. (12分) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

7. Given the periodic function

$$f(x) = \begin{cases} 1 & \text{if } -\pi < x < 0 \\ -1 & \text{if } 0 < x < \pi \end{cases} \quad \text{and } f(x+2\pi) = f(x)$$

a. Plot the function $f(x)$. (2分)

b. Is $f(x)$ odd, even or neither odd nor even? (1分)

c. Find the Fourier series of this function. (15分)

大葉大學九十三年年度碩士班甄試試題紙

所別	組別	考試科目 (中文名稱)	考試日期	考試時間	備註
機械工程研究所 機電自動化研究所 車輛工程研究所	甲、乙、丙 甲 甲	工程數學	12月8日	9:00 ~ 10:30	不准攜帶計算機或輔助工具 (試題共一頁)

注意：(1) 答題應詳列運算步驟，否則不予計分。

(2) 答案卷務必標明題號，無題號之計算視為草稿，不予計分。

1. (10分) Obtain a general solution of the linear differential equation $y' + 2y = 3xe^x$.
2. (20分) Solve the initial value problem: $(x^2D^2 - 3xD + 3)y = 3\ln x - 1$; $y(1) = 0$, $y'(1) = 1$, where $D = \frac{d}{dx}$.
3. (15分) Use the Laplace transform to solve the initial value problem: $y'' + 4y = f(t)$; $y(0) = 1$, $y'(0) = 0$,

$$\text{where } f(t) = \begin{cases} 0 & 0 \leq t < 4 \\ 3 & t \geq 4 \end{cases}$$

[Hint: First, express the function $f(t)$ in terms of the unit step function (or called Heaviside function).]

4. (15分) (a) Write the mathematical formula for the divergence theorem of Gauss.
 (b) Apply the divergence theorem to $\vec{F} \times \vec{C}$ to show that $\iint_A \vec{n} \times \vec{F} dA = \iiint_V \nabla \times \vec{F} dV$, where \vec{C} is an arbitrary constant vector, $\vec{F}(x, y, z)$ is a differentiable vector function, and \vec{n} is the outer unit normal vector of A .
5. (15分) For the matrix $A = \begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix}$, (a) find its eigenvalues λ_1 and $\lambda_2 (> \lambda_1)$, (b) find the corresponding eigenvectors, (c) find a matrix S and its inverse S^{-1} so that $S^{-1}AS = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$.

6. (10分) Let $f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \pi e^{-x} & \text{if } x > 0 \end{cases}$ and the Fourier integral representation of $f(x)$ be given as

$$f(x) = \int_0^{\infty} [A(\omega)\cos\omega x + B(\omega)\sin\omega x] d\omega. \text{ Find the coefficient } A(\omega).$$

7. (15分) Use the residue theorem to evaluate $\frac{1}{2\pi i} \oint_C \frac{e^{zt}}{z^2(z^2 + 2z + 2)} dz$, where C is the closed curve $|z| = 3$ taken counterclockwise direction, and $i = \sqrt{-1}$. Express the result in terms of t , e^{-t} , and $\cos(t)$.

大葉大學九十四學年度碩士班甄試試題紙

所 別	組別	考 試 科 目 (中 文 名 稱)	考 試 日 期	考 試 時 間	備 註
機械工程研究所 機電自動化研究所 車輛工程研究所	甲、乙、丙 甲 甲	工程數學	12月13日	9:00~10:30	共 乙 頁

註：備註欄若未註明可攜帶計算機或其他輔助工具作答時，考生一律不准攜帶。

1. 求微分方程式之通解：(15%)

$$y'' + 3y' - 4y = 8 \cos 2x + 6 \sin 2x$$

2. 解初始值問題(initial value problem)：(15%)

$$y'' - 2y' + (4\pi^2 + 1)y = 0, \quad y(0) = -2, \quad y'(0) = 6\pi - 2$$

3. 以 Laplace Transform 方法解初始值問題 (其他方法解題者不給分)：(15%)

$$y'' + 2y' + y = e^{-t}, \quad y(0) = -1, \quad y'(0) = 1$$

4. 給予一純量函數 $f(x, y, z) = x \ln y^2 + \cos yz$ ，及一向量函數

$$\vec{V}(x, y) = e^x(\cos y \vec{i} + \sin y \vec{j})$$
，計算下列各式之結果。(15%)

a. ∇f b. $\nabla^2 f$ c. $\nabla \times \vec{V}$ d. $\nabla \cdot \vec{V}$ e. $\nabla \times (\nabla f)$

5. 求線積分 $\int_C \vec{F}(\vec{r}) \cdot d\vec{r}$ 之值，其中 $\vec{F} = [y^2, -x^2]$ ，而 C 為由 (0,0) 到 (1,4) 之拋物線

$$y = 4x^2 \text{。 (10\%)}$$

6. 求週期函數 $f(x) = \begin{cases} -1 & \text{if } -\pi < x < 0 \\ 0 & \text{if } 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$.

之 Fourier series。(15%)

7. 求矩陣 $A = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$ 之特徵值(Eigenvalues)及特徵向量(Eigenvectors)。(15%)

大葉大學 95 學年度 研究所碩士班甄試			招生考試試題紙		
系 所 別	組 別	考 試 科 目 (中文名稱)	考 試 日 期	節 次	備 註
機械工程研究所 機電自動化研究所 車輛工程研究所	甲乙丙 甲 甲	工程數學	12月19日	第一節	芝一頁

註：考生可否攜帶計算機或其他資料作答，請在備註欄註明（如未註明，一律不准攜帶）

1. Find the general solution of the differential equation (15%)

$$y'' + y = t^2 \quad (\text{which } y' = \frac{dy}{dt})$$

2. Using the Laplace transform, solve the following problem (20%)

$$y'' + 3y' + 2y = \delta(t-1), \quad y(0) = 0, \quad y'(0) = 0 \quad (\text{which } y' = \frac{dy}{dt})$$

which $\delta(t-1)$ is Dirac's delta function, by definition $\delta(t-1) = \infty$ for $t=1$,
 $\delta(t-1) = 0$ for $t \neq 1$

3. Please find eigenvalues and corresponding eigenvectors of the following matrix A (15%)

$$A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

4. Find the Fourier series of the function (15%)

$$f(x) = x \quad \text{if } -\pi < x < \pi \quad \text{and} \quad f(x+2\pi) = f(x)$$

5. Scale function $f(x, y, z) = x^2y - yz^2$, vector function $\mathbf{v}(x, y, z) = y\mathbf{i} + z\mathbf{j} + x\mathbf{k}$, find

(a) $\text{div}(\text{grad } f)$ (5%)

(b) $\text{div}(\mathbf{v} f)$ (5%)

(c) $\text{div}(\text{curl } \mathbf{v})$ (5%)

6. Solve the following partial differential equations (PDE) by separation of variables (20%)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad 0 \leq x \leq L, \quad t > 0$$

Boundary conditions: $u(0, t) = 0, \quad u(L, t) = 0$

Initial conditions: $u(x, 0) = f(x)$