提要 38：*Wronskian* 的定義

先從最簡單的情況加以說明。兩個函數 y_1 與 y_2 之 *Wronskian* 係定義爲：

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \tag{1}$$

依此類推，三個函數 y_1 、 y_2 與 y_3 之 *Wronskian* 係定義爲：

$$W(y_1, y_2, y_3) = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} \tag{2}$$

同理，n 個函數 y_1 、 y_2 、 y_3 、…、 y_n 之 *Wronskian* 係定義爲：

$$W(y_1, y_2, y_3, \cdots, y_n) = \begin{vmatrix} y_1 & y_2 & y_3 & \cdots & y_n \\ y_1' & y_2' & y_3' & \cdots & y_n' \\ y_1'' & y_2'' & y_3'' & \cdots & y_n'' \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_1^{(n-1)} & y_2^{(n-1)} & y_3^{(n-1)} & \cdots & y_n^{(n-1)} \end{vmatrix} \tag{3}$$

習題

1. Let y_1 and y_2 be linear independent solutions of $x^2y''+2xy'+(x-2)y=0$. Then the Wronskian of y_1 and y_2 can be defined as $W(x)=y_1y_2'-y_1'y_2$. Given $y_1(1)=0$, $y_1'(1)=1$, $y_2(1)=2$, and $y_2'(1)=3$, find $W(x)$.【88 台科電機所 10%】

2. If $u=u(x)$ and $v=v(x)$ are solutions of $\frac{d^2y}{dx^2}+p(x)\frac{dy}{dx}+q(x)y=0$, then the Wronskian is defined as $W(x)=uv'-vu'$.
   (a) Show that $W'(x)=-p(x)W(x)$.
   (b) Solve the differential equation $W'(x)=-p(x)W(x)$.
   (c) Show that if $W(x_0)\neq 0$ for some point x_0 in $a<x<b$, then $W(x)\neq 0$ for all points in $a<x<b$.【88 北科電腦通訊所 15%】