提要 36：線性相關與線性獨立(二)

y_1 與 y_2 互為線性相關(*Linear Dependence*)與線性獨立(*Linear Independence*)的定義有三種，這裏介紹第二種。

- 線性相關(*Linear Dependence*)

若 y_1 與 y_2 互為線性相關，則 $y_1/y_2 = C$，C 為常數，反之亦然。

- 線性獨立(*Linear Independence*)

若 y_1 與 y_2 互為線性獨立，則 $y_1/y_2 = u(x)$，$u(x)$ 為與變數 x 相關之函數，反之亦然。

1. $y_1 = e^x$ and $y_2 = x$. Are y_1, y_2 linearly independent?【88 交大電子所 5%】

2. Which of the following sets of functions, each defined on $x \in R$, are linear dependent?
    (a) $\sin x$, $\cos x$ (b) $\cos^2 x$, $\sin^2 x$, $\cos 2x$ (c) $1, x, x^2$ 【93 高應電機所 15%】

3. Show that x, $\sin x$ and $\cos x$ are linear independent.【94 師大機電所 14%】

4. Find three linear independent solutions of $y''' + 3y'' - y' - 3y = 0$. Justify that the solutions are independent.【91 元智電機所控制組 5%】

5. Show that the quantities 1, x, and x^2 are linearly independent.【87 成大製造所 10%】

6. 以下那一組函數是線性相依（linear independent）？
    (a) e^{2x}, e^{-2x}, $\sinh(2x)$
    (b) $\sinh(2x)$, $\cosh(2x)$, $2x$
    (c) e^x, e^{2x}, e^{3x}
    (d) x^2, x^4, $x^6 - 2x^2$
    (e) e^{3x}, e^{-3x}, e^{2x} 【88 台大電機所 5%】

7. Let $u_1(x)$ and $u_2(x)$ be two independent solutions of the differential equation $\dfrac{d^2 u(x)}{dx^2} + v(x) u(x) = 0$. Show that $u_1 u_2' - u_2 u_1'$ is a constant, where $'$ is the symbol denotes differentiation with respect to x. Take the normalization $u_1 u_2' - u_2 u_1' = 1$ and $\psi = u_2 / u_1$. Show that $\dfrac{\psi'''}{\psi'} - \dfrac{3}{2}\left(\dfrac{\psi''}{\psi'}\right)^2 = 2v(x)$.【91 台師大物理所 20%】