

提要 35：線性相關與線性獨立(一)

y_1 與 y_2 互為線性相關(*Linear Dependence*)與線性獨立(*Linear Independence*)的定義有三種，這裏先介紹第一種。

- 線性相關(*Linear Dependence*)

若 y_1 與 y_2 互為線性相關，則恆等式 $k_1 y_1 + k_2 y_2 = 0$ 成立的條件是 k_1 與 k_2 不全為零，反之亦然。

- 線性獨立(*Linear Independence*)

若 y_1 與 y_2 互為線性獨立，則恆等式 $k_1 y_1 + k_2 y_2 = 0$ 成立的唯一條件是 k_1 與 k_2 全為零，亦即 $k_1 = k_2 = 0$ ，反之亦然。

習題

1. $y_1 = e^x$ and $y_2 = x$. Are y_1, y_2 linearly independent? 【88 交大電子所 5%】
2. Which of the following sets of functions, each defined on $x \in R$, are linear dependent?
(a) $\sin x, \cos x$ (b) $\cos^2 x, \sin^2 x, \cos 2x$ (c) $1, x, x^2$ 【93 高應電機所 15%】
3. Show that $x, \sin x$ and $\cos x$ are linear independent. 【94 師大機電所 14%】
4. Find three linear independent solutions of $y''' + 3y'' - y' - 3y = 0$. Justify that the solutions are independent. 【91 元智電機所控制組 5%】
5. Show that the quantities $1, x$, and x^2 are linearly independent. 【87 成大製造所 10%】
6. 以下那一組函數是線性相依 (linear independent) ?
(a) $e^{2x}, e^{-2x}, \sinh(2x)$
(b) $\sinh(2x), \cosh(2x), 2x$
(c) e^x, e^{2x}, e^{3x}
(d) $x^2, x^4, x^6 - 2x^2$
(e) e^{3x}, e^{-3x}, e^{2x} 【88 台大電機所 5%】
7. Let $u_1(x)$ and $u_2(x)$ be two independent solutions of the differential equation $\frac{d^2u(x)}{dx^2} + v(x)u(x) = 0$. Show that $u_1u_2' - u_2u_1'$ is a constant, where ' is the symbol denotes differentiation with respect to x . Take the normalization $u_1u_2' - u_2u_1' = 1$ and $\psi = u_2/u_1$. Show that $\frac{\psi'''}{\psi'} - \frac{3}{2}\left(\frac{\psi''}{\psi'}\right)^2 = 2v(x)$. 【91 台師大物理所 20%】