

## 單元 16 z 轉換

### 【例題 1】

Consider a discrete time signal  $x(n]$  If

$x(-2) = 1.5, x(-1) = 0.5, x(0) = 2, x(1) = -2, x(2) = 3, x(3) = -1$ , find two sided  $z$  transform of  $x(n]$ . 【91 海洋電機】

【參考解答】  $X(z) = 1.5z^2 + 0.5z + 2 - 2z^{-1} + 3z^{-2} - z^{-3}$

### 【例題 2】

Invert the Z transform  $X(z) = \frac{1}{(1 - az^{-1})^2}, |z| > a$ . 【91 台科電子】

【參考解答】  $x(n) = \begin{cases} 0, & n = -1, -2, -3, \dots \\ (-a)^n n, & n = 0, 1, 2, 3, \dots \end{cases}$  即為所求

### 【例題 3】

Find the coefficients  $a_{-1}, a_0, a_1$  of the Laurent series  $\sum_{n=-\infty}^{\infty} a_n z^n$  of

$\sec z$  in  $\frac{\pi}{2} < |z| < \frac{3\pi}{2}$ . 【88 清大工程科學】

【參考解答】  $a_1 = 0 + \frac{-4}{\pi^2} + \frac{4}{\pi^2} = 0, a_{-1} = 0, a_0 = 1 - \frac{4}{\pi}$ .

### 【例題 4】

(1) Find all values of  $\sin^{-1} \sqrt{2}$ .

(2) Find the value of the derivative of  $\sin^{-1} z$  at  $z = \sqrt{2}$ . Assume that the principal branch is used. 【91 交大電信】

【參考解答】 (1)  $z = \sin^{-1} \sqrt{2} = \frac{1}{i} \ln(\sqrt{2} \pm 1) + \left( \frac{\pi}{2} + 2k\pi \right), k = 0, \pm 1, \pm 2, \dots$ .

(2)  $\frac{d \sin^{-1} z}{dz} = \frac{1}{\sqrt{1-z^2}},$  at  $z = \sqrt{2}, \frac{d \sin^{-1} z}{dz} = -\frac{1}{i}$ .

### 【例題 5】

Find all of the terms with negative powers and the first two nonzero terms with nonnegative powers in the Laurent series of  $\sin^{-1} z$  about  $z_0 = n\pi$ . The Laurent series is valid in an annular region of the form  $0 < |z - z_0| < R$ . What is the largest value of  $R$  for this series? 【91 交大電信】

【參考解答】 當  $n = -1, -2, -3, \dots$   $\operatorname{Res}(n\pi) = 0, \therefore a_n = 0$

$$\text{當 } n = 0, a_0 = \sin^{-1} n\pi, a_1 = \frac{1}{\sqrt{1 - n^2\pi^2}}$$

事實上  $z_0 = n\pi$  為解析點，取 Taylor series 展開，即為所求。