

## 單元 6 複數平面之 Green's 定理

### 【例題 1】

求  $\int_C 3z^2 dz$ ， $C$  之三頂點分別為  $0$ 、 $1$ 、 $i$  的三角形邊界。【91 台師大機電】

【參考解答】  $\oint_C 3z^2 dz = 0$

### 【例題 2】

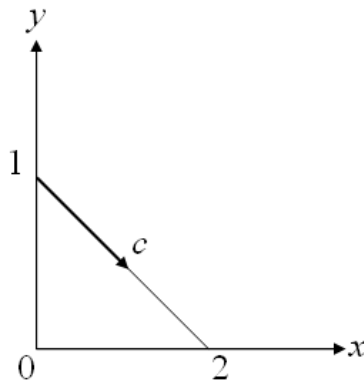
Evaluate  $\int_0^{3+i} z^2 dz$ ,

(1) along the line  $y = \frac{x}{3}$ .

(2) along the real line to 3 and then vertically to  $3+i$ . 【91 清大電機】

【參考解答】 (1)  $\int_0^{3+i} z^2 dz = \frac{1}{3}(18 + 26i)$  (2)  $\int_C z^2 dz = 6 + \frac{26}{3}i$

### 【例題 3】



Knowing that the  $ML$  bound describes  $|\int_C f(z) dz| \leq ML$  where  $|f(z)| \leq M$  on  $C$  and  $L$  is the length of  $C$ . Find the  $ML$  bound of the following integral  $I = \int_C \frac{e^z}{z^2} dz$ ,  $C$  is the straight line as shown.

【參考解答】 當  $\frac{d|f(z)|}{dy} = 0$ ， $|f(z)|$  有極值， $y = 0$ 、 $3/5$ 。

當  $\begin{cases} y = 0, |f(z)| \doteq 1.85 \\ y = 3/5, |f(z)| = 2.23 \end{cases}$ ， $\therefore M = \frac{1}{4}e^2, L = \sqrt{5}$ ，

$$\therefore |I| \leq \frac{1}{4} e^2 \cdot \sqrt{5} .$$

**【例題 4】**

(1) Find  $\frac{d}{dx} x^x$  at  $x = e$ , where  $e$  is the base of natural logarithm.

(2) Find  $\frac{d}{dz} z^{(3+4i)}$  at  $z = 3 + 4i$ .

**【91 中央光電】**

**【參考解答】** (1)  $\frac{dx^x}{dx} = x^x \cdot (\ln x + 1)$

(2)  $\frac{dz^{3+4i}}{dz} = 5^3 e^{-4(\theta+2n\pi)} e^{i(4\ln 5+3\theta)}, n = 0, \pm 1, \pm 2, \dots$