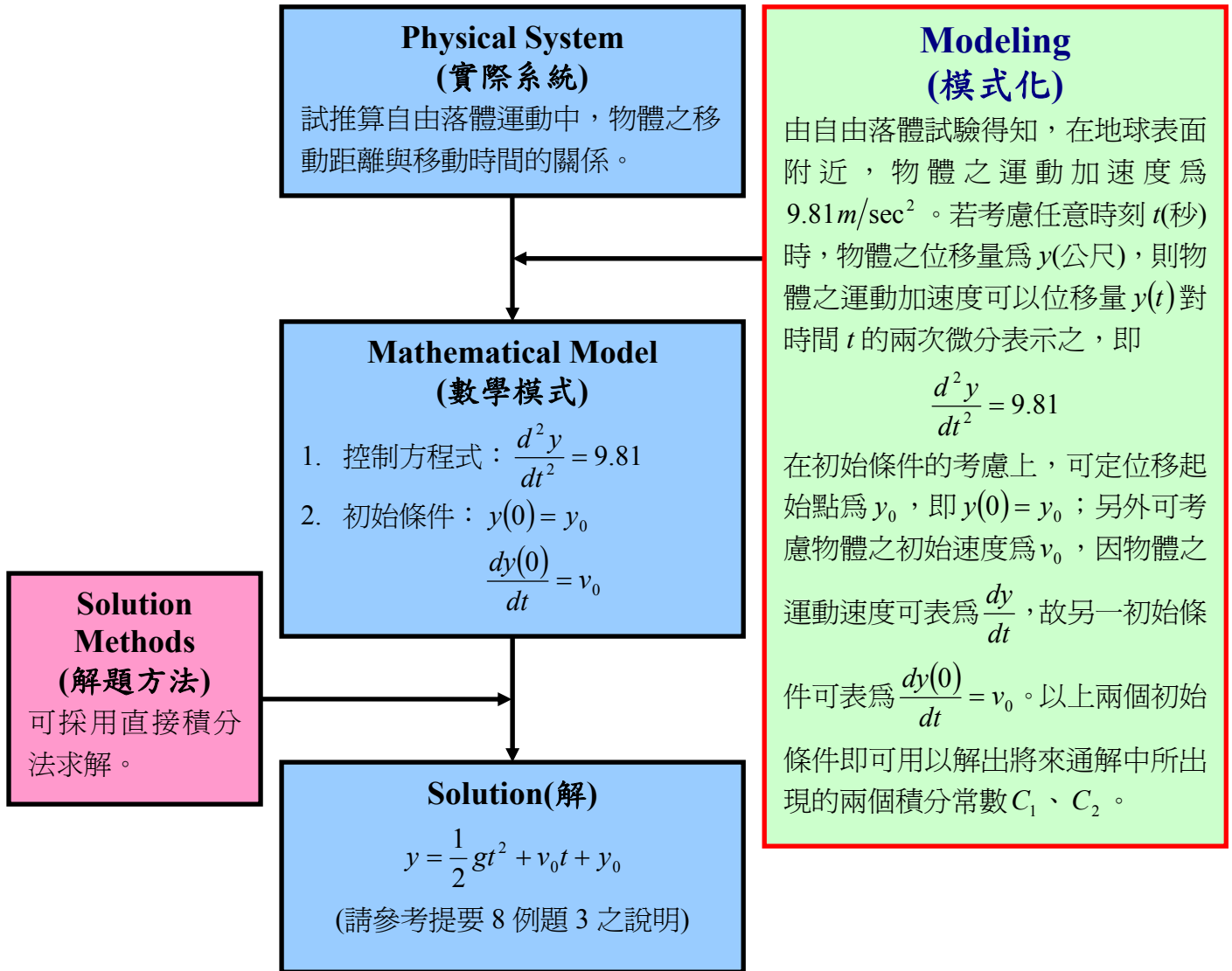


提要 6：如何建立數學模式？(五)

茲以自由落體運動說明之，請參考紅色框線內之說明。



Notice:

1. 在地球表面上，物體自由落下之運動加速度約為 $9.81 m/sec^2$ 。
2. $\frac{d^2 y}{dt^2} = 9.81$ 為自由落體運動定律的化身。
3. 本問題之數學模式為二階(2次微分)之常微分方程式，故其解會出現兩個積分常數。
4. 因有兩個積分常數會出現在通解中，故需安排兩個初始條件解之。

習題

1. Air containing 0.06% CO_2 is pumped into a room whose volume is $8,000 \text{ m}^3$. The rate at which the air is pumped in is $2,000 \text{ m}^3/\text{min}$, and the circulated air is then pumped out at the same rate. If there is an initial CO_2 concentration of 0.2%, find the differential equation for $A(t)$, the amount (in m^3) of CO_2 in the room at time t . 【90 交大物理所 15%】
2. Find a curve in xy plane that passes through $(0,3)$ and whose tangent line at a point (x,y) has slope $2x/y^2$. 【88 成大製造所 10%】
3. Suppose that a mothball loses volume by evaporation at a rate proportional to its instantaneous area. If the diameter of the ball decreases from 2 cm to 1 cm in 3 months, how long will it take until the ball has practically gone, say, until its diameter is 1 mm . 【87 中原化工所 20%】
4. An oil tanker of mass M is sailing in a straight line. At time zero it shut off its engines and coasts. Assume that the water tends to slow the tanker with a force proportional to $[v(t)]^m$, in which $v(t)$ is the velocity at time t and m is constant.
 - (a) Derive a differential equation for $v(t)$ and solve it. (Let $v = v_0$ at time zero.)
 - (b) Does the tanker eventually come to a full stop? If so, find the time required for the tanker to stop. If not, why?Hint: Solve and discuss the problem by considering cases on m . (i) $0 < m < 1$ (ii) $m = 1$ (iii) $m > 1$. 【89 台大化工所 15%】