提要 358：勞倫級數(Laurent Series)之應用(2)

茲再以一例說明勞倫級數（Laurent Series）之應用。

範例一
試以 $z=0$ 為中心點，將函數 $f(z)=\dfrac{z^2+1}{z(z^2+6iz-1)}$ 作勞倫級數展開。

【解答】

本題應引用幾何級數展開的觀念求解。由題意知，需以 $z=0$ 為中心點作勞倫級數展開，故：

$$
\begin{aligned}
f(z) &= \frac{z^2+1}{z(z^2+6iz-1)} \\
&= \frac{z^2+1}{z}\frac{1}{z-(-3+2\sqrt{2})i}\frac{1}{z-(-3-2\sqrt{2})i} \\
&= \frac{z^2+1}{z}\frac{1}{(-3+2\sqrt{2})i}\frac{1}{1-\dfrac{z}{(-3+2\sqrt{2})i}}\frac{1}{(-3-2\sqrt{2})i}\frac{1}{1-\dfrac{z}{(-3-2\sqrt{2})i}} \\
&= \left(\frac{1}{z}+z\right)\frac{1}{(-3+2\sqrt{2})i}\left[1+\frac{z}{(-3+2\sqrt{2})i}+\cdots\right]\frac{1}{(-3-2\sqrt{2})i}\left[1+\frac{z}{(-3-2\sqrt{2})i}+\cdots\right] \\
&= \frac{1}{(9-8)i^2}\left(\frac{1}{z}+z\right)\left[1+\frac{z}{(-3+2\sqrt{2})i}+\cdots\right]\left[1+\frac{z}{(-3-2\sqrt{2})i}+\cdots\right] \\
&= -\left(\frac{1}{z}+z\right)\left[1+\frac{z}{(-3+2\sqrt{2})i}+\cdots\right]\left[1+\frac{z}{(-3-2\sqrt{2})i}+\cdots\right] \\
&= -\left(\frac{1}{z}+z\right)\left[1+\frac{z}{(-3+2\sqrt{2})i}+\frac{z}{(-3-2\sqrt{2})i}+\cdots\right] \\
&= -\frac{1}{z}-\left[\frac{1}{(-3+2\sqrt{2})i}+\frac{1}{(-3-2\sqrt{2})i}\right]-z-\left[\frac{1}{(-3+2\sqrt{2})i}+\frac{1}{(-3-2\sqrt{2})i}\right]z^2-\cdots
\end{aligned}
$$

以上所示即為問題之解。