第三類習題：矩陣與行列式

1. If A is real matrix and $A^T A = 0$, find A.【90 東華電機 10%】

2. 設 $A = \left[a_{ij}\right]_{m\times n}$ ， $B = \left[b_{ij}\right]_{n\times m}$ ，證明：
   (1) $tr(AB) = tr(BA)$
   (2) $tr(A) = tr(U^{-1}, AU)$【90 中央土木 10%，90 台大電機 20%】

3. Let A be a 2×3 matrix such that multiplication by A transforms $X_1 = [4,7,3]^t$ onto $[1,3]^t$, $X_2 = [1,1,0]^t$ onto $[1,4]^t$, and $X_3 = [1,0,0]^t$ onto $[1,1]^t$. Determine what multiplication by A transforms onto $[1,6,3]^t$.【91 高科通訊 10%】

4. The complete solution to $Ax = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$ is $x = \begin{bmatrix} 3 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $c \in R$. Find A.【91 雲科電機 10%】

5. Consider a square matrix of the form $M = \begin{bmatrix} A & O \\ P & B \end{bmatrix}$, where A is $p\times p$ and B is $q\times q$. Verify the following statements.
   (1) If A is singular, so is M.
   (2) If M is nonsingular, so are A and B.
   (3) If A and B are invertible, M^{-1} is given by the formula:

$$M = \begin{bmatrix} A^{-1} & 0 \\ -B^{-1}P\,A^{-1} & B^{-1} \end{bmatrix}$$

   Note: Do not use statement (3) to verify statements (1) and (2). 【90 暨南資訊 30%】

6. 設點 (x_1, y_1) 爲點 (x_0, y_0) 繞原點逆時針旋轉 θ 角所得，且 $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = A\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ ，求 A ， A_n 。【89 交大電子 10%】

7. 直角座標中，繞第二軸順時鐘方向轉 α 角度。
   (1) 試求 3×3 的旋轉矩陣。
   (2) 證明該矩陣爲正交矩陣。【91 中央土木 20%】

8. In R^3, $P = \{(x,y,z) | x + 3y - 2z = 0\}$, $(X,Y,Z)^t = T(x,y,z)^t$.
   (1) T is a reflection of R^3 about P, find T.
   (2) T is a projection of P along the line perpendicular to P, find T.【87 台大資工 10%】

9. (1) Given $B=\begin{bmatrix}12 & 11 & -32\\ -5 & 9 & 30\\ 32 & -18 & 15\end{bmatrix}$, write B as a sum of a symmetric and a skew-symmetric matrix.

(2) Prove that B^T and B is symmetric.

10. 【是非題】

A, B, C are $n\times n$ real matrices and A^{-1}, B^{-1} exist, then

(1) $A^{-1}=A^T$ if A is symmetric.

(2) det $(AB)=(\det A)(\det B)$.

(3) $(AB)^{-T}=B^{-T}A^{-T}$. [Note: $(\bullet)^{-T}=\left((\bullet)^{-1}\right)^T$]

(4) $AC=0$ implies $C=0$.

(5) Similarity matrices $\hat{A}=B^{-1}AB$ and A have the same eigenvalues and eigenvectors.【90 交大土木 10%】

11. (1) 已知A、B、C三個矩陣之關係爲$AC=BC$，是否可由此推論出$A=B$?

(2) 若A與B皆爲對稱(symmetric)矩陣，某同學以下列步驟欲證明AB亦爲對稱矩陣：$\because (AB)^T=B^TA^T=A^TB^T \therefore AB$爲對稱。請說明上述步驟何者有誤?並舉出一個$AB$爲對稱的例子

(3) 若是A與B皆爲正交(orthogonal)矩陣，則AB是否亦爲正交矩陣？若是，則試證明之；若否，則試舉出一反例。【90 雲科營建 15%】

12. For an unknown 3×3 matrix A, the mapping relationship as follows: $A(3,2,1)^t=(6,5,4)^t$, $A(6,5,4)^t=(9,8,7)^t$, what is $A(9,8,7)^t$?【89 成大醫工 10%】

13. Let L: $R^3\to R^2$ be a linear transformation for which $L(1,0,0)=(2,-1)$, $L(0,1,0)=(3,1)$, $L(0,0,1)=(-1,2)$. Find $L(-3,4,2)$.【89 成大製造 5%】

14. (1) 點(x_1,y_1)爲點(x_0,y_0)在直線$y=mx$上之投影點，且$\begin{bmatrix}x_1\\ y_1\end{bmatrix}=B\begin{bmatrix}x_0\\ y_0\end{bmatrix}$，求$B$。

(2) 點(x_1,y_1)爲點(x_0,y_0)對稱於$y=mx$之點，且$\begin{bmatrix}x_1\\ y_1\end{bmatrix}=C\begin{bmatrix}x_0\\ y_0\end{bmatrix}$，求$C$。【86 中正電機 10%】

15. 請計算行列式$\begin{vmatrix}4 & 5 & 1 & 0\\ -3 & 8 & 9 & 1\\ 0 & 2 & 6 & -1\\ 2 & 0 & -4 & 6\end{vmatrix}$之值。【91 高科環安 20%】

16. Find the determinant of the matrix $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$.【91 清大資訊 5%】

17. 是非題(每小題答對給 2 分，答錯扣兩分，不答 0 分，本題總分≥ 0)

If A and B are two $n \times n$ matrices.

(1) $\det\ A^{-1} = (\det A)^{-1}$

(2) $(A+B)^{-1} = A^{-1} + B^{-1}$

(3) $\det(A+B) = \det A + \det B$

(4) $(AB)^T = A^T B^T$

(5) $\det(ABC)^T = \det A^T \det B^T \det C^T$ 【91 中央資訊 10%】

18. Which of the following is a root of the function $f(t)$ listed below?

(1) 0　(2) $\pi/3$　(3) $\pi/2$　(4) π

$$f(t) = \det \begin{bmatrix} 7 & 1 & -2 & 0 & 0 \\ -1 & 2 & 1 & 0 & 0 \\ 4 & 3 & 2 & 3 & 4 \\ 2 & 2 & 2 & 2 & 2 \\ 2 & 1 & 0 & \cos t & 2 \end{bmatrix}$$

【91 台大電機 5%】

19. Evaluate the given determinant $\begin{vmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & 2 & 3 & 6 \end{vmatrix}$.【91 雲科機械 10%】

20. 試證：

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ \alpha & \beta & \gamma & \delta & \varepsilon \\ \alpha^2 & \beta^2 & \gamma^2 & \delta^2 & \varepsilon^2 \\ \alpha^3 & \beta^3 & \gamma^3 & \delta^3 & \varepsilon^3 \\ \alpha^4 & \beta^4 & \gamma^4 & \delta^4 & \varepsilon^4 \end{vmatrix}$$
$$= (\beta-\alpha)(\gamma-\alpha)(\delta-\alpha)(\varepsilon-\alpha)(\gamma-\beta)(\delta-\beta).(\varepsilon-\beta)(\delta-\gamma)(\varepsilon-\gamma)(\varepsilon-\delta)$$

【91 台科電子 15%、91 北科電機 10%】

21. (1) Please find A^{40} with $A=\begin{bmatrix}-1 & 4\\ 0 & 3\end{bmatrix}$.

(2) Please show the following determinant

$$\begin{vmatrix}1 & a_1 & a_1^2 & ..... & a_{1^{n-1}}\\ 1 & a_2 & a_2^2 & ..... & a_2^{n-1}\\ \vdots & \vdots & & & \vdots\\ \vdots & \vdots & & & \vdots\\ 1 & a_n & a_n^2 & ..... & a_n^{n-1}\end{vmatrix}\neq 0 \text{ where } a_1\neq a_2\neq a_3.....\neq a_{n-1}\neq a_n\neq 0.$$

【91 中央通訊 20%】

22. A polynomial $p(t)$ of degree $n-1$ is defined as $p(t)=c_0+c_1t+c_2t^2+...+c_{n-1}t^{n-1}$, where $c_0,c_1,c_2,....c_{n-1}$ are n real numbers. Given n arbitrary real numbers $y_1,y_2,...,y_n$ and n distinct real numbers $x_1,x_2,...,x_n$. Show that there exists one and only one polynomial $p(t)$ of degree $n-1$ such that $p(x_1)=y_1$, $p(x_2)=y_2$, … , $p(x_n)=y_n$.【90 中正資訊 10%】

23. Find the eigenvalues of $A=\begin{bmatrix}2 & 4 & 1 & -1 & 2\\ -1 & -2 & 3 & 0 & -2\\ 0 & 0 & 1 & 8 & -4\\ 0 & 0 & 0 & -1 & 1\\ 0 & 0 & 0 & -4 & 3\end{bmatrix}$.【90 中興機械 15%】

24. Show that $\det\begin{bmatrix}I_m & B\\ -C & I_n\end{bmatrix}=\det(I_n+CB)=\det(I_m+BC)$.【91 北科電機 15%】

25. Given the following $2n\times 2n$ partitioned matrix $M=\begin{bmatrix}O & A\\ A & O\end{bmatrix}$. When A is an $n\times n$ nonsingular matrix, and O is the $n\times n$ zero matrix.

(1) Perform a sequence of row operations on M such that $EM=\begin{bmatrix}I & O\\ O & A\end{bmatrix}$, where I is the $n\times n$ identity matrix. What is E?

(2) Obtain $\det(M)$ in terms of $\det(A)$.【88 交大電信 15%】

26. 求行列式$\begin{vmatrix}-5 & 4 & 1 & 7\\ -9 & 3 & 2 & -5\\ -2 & 0 & -1 & 1\\ 1 & 14 & 0 & 3\end{vmatrix}$。【89 北科環境 10%】

27. 下列等式之右式爲矩陣的行列值(determinant)，試求a,β,γ的值爲多少?

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = \alpha a + \beta b + \gamma c$$

【88 台科營建 15%】

28. 求行列式$(\det A)$的微分（$\frac{d}{dx}(\det A)$在$x=0$的值）：

$$\det A = \begin{vmatrix} a-b & 3x & 4 & 2 \\ 6 & 26 & 7 & 5 \\ 13 & 4\sin x + 9 & 1 & 10 \\ 3 & 3 & 4 & 2 \end{vmatrix}$$

【88 中央土木 10%】

29. $A = \begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}$, $\det A = 5$. Find

(1) $\det(-4A)$ (2) $\det(A^{-1})$ (3) $\det(A^2)$ (4) $\det((3A^{-1})^T)$ (5) $\det\begin{bmatrix} t & r & s \\ w & u & v \\ z & x & y \end{bmatrix}$

【87 交大電子 12%】

30. (1) Assume $A = (-A)^t$, prove that $\det(A) = 0$ when the order of A is odd.
(2) Assume $A = A^{-1}$, prove $\det(A) = \pm 1$.【87 台科電子 15%】

31. Determine the general solution:

$$\begin{aligned} x_1 - x_2 + x_3 - x_4 &= -2 \\ -2x_1 + 3x_2 - x_3 + 2x_4 &= 5 \\ 4x_1 - 2x_2 + 2x_3 - 3x_4 &= 6 \end{aligned}$$

【89 成大土木 15%】

32. $A = \begin{bmatrix} -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 2 & 1 \\ 1 & 2 & -3 & -7 & -2 \end{bmatrix}$, $X = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 1 \\ -8 \end{bmatrix}$

(1) 試求此非齊性聯立方程式$AX = b$之通解。
(2) 試求齊性聯立方程式$AX = 0$之通解。

(3) 假設 $X_1=(1,2,3,4,5)^T$ 為 $AX=(2,18,-42)^T$ 之一特解，試求非齊性聯立方程式 $AX=(2,18,-42)^T$ 之通解。【89 交大土木 15%】

33. Solve $x+2y+3z=9$, $4x+7y+6z=24$, $2x+7y+12z=40$ by the use of Gauss elimination.【89 高科大營建 20%】

34. 矩陣 $A=\begin{bmatrix}1&5&2\\2&1&1\\1&2&1\end{bmatrix}$，$B=\begin{bmatrix}3\\3\\b\end{bmatrix}$，$x=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$。若聯立方程式 $Ax=B$ 有解(solution)，試求常數 b 之值？【90 中興土木 13%】

35. 求解：$\begin{cases}w_1+2w_2+3w_3=3\\2w_1+5w_2+7w_3=7\\3w_1+8w_2+10y_3=11\end{cases}$，$\begin{cases}x_1+2x_2+3x_3=4\\2x_1+5x_2+7x_3=9\\3x_1+8x_2+10x_3=13\end{cases}$，

$\begin{cases}y_1+2y_2+3y_3=5\\2y_1+5y_2+7y_3=12\\3y_1+8y_2+10y_3=18\end{cases}$，$\begin{cases}z_1+2z_2+3z_3=1\\2z_1+5z_2+7z_3=3\\3z_1+8z_2+10z_3=5\end{cases}$。【台大化工 8%】

36. Let $A=\begin{bmatrix}1&3&8\\2&4&11\\1&2&5\end{bmatrix}$ and $B=\begin{bmatrix}-3&5\\1&5\\3&4\end{bmatrix}$. Compute $A^{-1}B$ without computing A^{-1}.【89 中央資訊 10%】

37. Let $A=\begin{bmatrix}1&4&3\\-1&-2&0\\2&2&3\end{bmatrix}$, $x=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$, $b=\begin{bmatrix}12\\-12\\8\end{bmatrix}$ and $Ax=b$.

(1) Find the inverse of A (that is A^{-1}).
(2) Solve for the solution of x.【90 中原電子 10%】

38. Find the inverse of $A=\begin{bmatrix}1&1&0\\-2&1&1\\0&1&1\end{bmatrix}$. 【91 雲科機械 15%】

39. Under what condition b_1,b_2,b_3 is the following system solvable? Find all solutions.

$$\begin{aligned}x_1+2x_2-2x_3&=b_1\\2x_1+5x_2-4x_3&=b_2\\4x_1+9x_2-8x_3&=b_3\end{aligned}$$

【90 雲科電機 10%】

40. Using the Cramer's rule, solve the following equations.

$$\begin{cases} x - y + 2z = -5 \\ -x + 3z = 0 \\ 2x + y = 1 \end{cases}$$

【90 雲科電機 10%】

41. For a linear system of n equations in the same number of unknowns $x_1, \ldots, x_n$.

$$\begin{gathered} a_{11}x_1 + a_{12}x_2 + \ldots. + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots. + a_{2n}x_n = b_2 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots. + a_{nn}x_n = b_n \end{gathered}$$

Use Cramer's rule to analyze the existence and uniqueness of its solutions.【91 彰師機械 15%】

42. 證明，若 A, B 皆爲方陣，則

(1) $(AB)^{-1} = B^{-1}A^{-1}$

(2) $(A)^{-1} = (A^{-1})^T$ 【90 中央土木 10%】

43. Assume that the stated inverses exist, prove

(1) $(A^{-1} + B^{-1})^{-1} = A(A+B)^{-1}B$

(2) $(I + AB)^{-1}A = A(I + BA)^{-1}$ 【86 台科電機 8%】

44. Let A be a nonsingular $n \times n$ matrix with a nonzero cofactor A_{nn} at (n,n) entry and set $\det(A)/A_{nn}$. Show that if we subtract c from (n,n) entry of A, then the resulting matrix will be singular.【91 雲科電機 10%】

45. For $A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 3 & 1 & -2 \end{bmatrix}$, find the determinant and inverse of A.【91 成大製造 10%】

46. A is $n \times n$ matrix

(1) pprove that A is invertible if and only if $adj(A)$ is invertible.

(2) $\det(adjA) = (\det A)^{n-1}$ 【88 成大電機通信數學 15%】

47. Find the inverse matrix of A, where $A=\begin{bmatrix} 0 & 0 & 1 \\ 0 & \frac{1}{2} & 1 \\ \frac{1}{3} & 0 & 0 \end{bmatrix}$.【91 台大工程科學 15%】

48. 求 $A=\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 2 \\ 2 & -2 & 7 \end{bmatrix}$ 之逆矩陣。【91 成大資源 10%】

49. (1) Find the inverse of $A=\begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}$.

(2) Find eigenvalues and eigenvectors of $B=\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.【91 成大醫工 20%】

50. Find A^{-1}, where $A=\begin{bmatrix} 0 & 0 & 1 \\ 0 & \frac{1}{2} & 1 \\ 1/3 & 0 & 0 \end{bmatrix}$.【91 台大工程科學 15%】

51. Find A^{-1}, where $A=\begin{bmatrix} 0 & 1 & 2 \\ 3 & 2 & 2 \\ 1 & 3 & 2 \end{bmatrix}$.【90 台科機械 20%】

52. Solve the following system of linear algebraic equations:

$$\begin{aligned} 6x_1-2x_2+2x_3+4x_4&=16 \\ 12x_1-8x_2+6x_3+10x_4&=26 \\ 3x_1-13x_2+9x_3+3x_4&=-19 \\ -6x_1+4x_2+x_3-18x_4&=-34 \end{aligned}$$

【90 淡江化工 20%】

53. $AP=B$，先求 A^{-1}，再求 P。

$$A=\begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}，B=\begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$$

【91 台科營建 10%】

54. Determine the number of independent solutions for a homogeneous linear system without actually solving the problem.

(1) $k_1+k_2+k_3=0$, $k_1-3k_2+k_3+k_4-k_5=0$, $-2k_1-k_3-k_5=0$.

(2) $k_3+k_4=0$，$k_1+k_2-2k_3=0$，$k_1+k_2-k_3+k_4=0$.

【91 中興機械 15%】

55. Solve the following system by Gauss elimination

$$\begin{aligned}&3w-6x-y-z=0\\&w-2x+5y-3z=0\\&2w-4x+3y-z=3\end{aligned}$$

【90 中興農機 10%】

56. Solve the following system by Gauss elimination

$$\begin{aligned}&4x-8y+3z=16\\&-x+2y-5z=-21\\&3x-6y+z=17\end{aligned}$$

【90 高雄科大營建 10%】

57. Given a matrix equation $Ax=b$, where A is $m\times n$ coefficient matrix, and b is a vector of m dimension. Please explain under what condition(s), the matrix is called: (1)over-determined, (2)determined, and (3)under determined.【89 清大原子 5%】

58. Find the inverse of A, if it exists

$$A=\begin{bmatrix}1 & 2 & -3\\1 & -2 & 1\\5 & -2 & -3\end{bmatrix}$$

【89 成大製造 5%】

59. A 為可逆矩陣，證明 $A(I+A)^{-1}=(I+A)^{-1}A$。【89 中央土木 10%】

60. Solve by Gauss elimination method

$$\begin{aligned}&3x+y+z=8\\&-x+y-2z=-5\\&2x+2y+2z=12\\&-2x+2y-3z=-7\end{aligned}$$

【88 台大環工 15%】

61. A system of linear algebraic equation can be expressed as $Ax=b$. Where A is a 3×3 matrix and b is 3×1 vector.

$$A=\begin{bmatrix}\alpha & \alpha & 3\\ 0 & \alpha & \beta\\ \alpha & 0 & 2\end{bmatrix},\ b=\begin{bmatrix}3\\ 2\\ \beta\end{bmatrix}$$

Determine of what fixed values of α and β (if any) the system possesses the following:

(1) A unique solution.
(2) A one-parameter solution.
(3) A two-parameter solution.
(4)No solution.【87 清大動機 15%】

62. 下述線性方程組

$$\begin{aligned}ay+z&=b\\ ax+bz&=1\\ ax+ay+2z&=2\end{aligned}$$

在下列給定條件之下，參數a及b之值爲何？
(1)唯一解　(2)有一參數解　(3)有二參數解　(4)無解【87 台大土木 15%】

63. Using the augment matrix and elimination method to find the inverse of

$$A=\begin{bmatrix}1 & 4 & 7\\ 2 & 5 & 8\\ 3 & 6 & 10\end{bmatrix},\ A^{-1}=?$$

(Note: there are no score for using other methods)【91 台科電機 10%】

64. A matrix B has column vectors b_1, b_2, b_3, b_4, where $b_1^T=[1,0-1,0]$, $b_2^T=[1,1,0,2]$, $b_3^T=[0,3,1,-2]$, $b_4^T=[0,1-1,-6]$.

(1) Determine whether these vectors in R^4 are linearly independent or linearly dependent.
(2) What is the rank of matrix B, why?【91 交大土木 15%】

65. 下列向量是否線性獨立？
$v_1=(2,-1,0)^t$，$v_2=(-1,1,1)^t$，$v_3=(1,-2,-3)^t$，$v_4=(4,1,6)^t$。【91 海洋商船 10%】

66. Which of the following vectors in R^4 are linear combinations of $v_1=[1,2,1,0]$, $v_2=[4,1,-2,3]$, $v_3=[1,2,6,-5]$, $v_4=[-2,3,-1,2]$, explain your answer.
(1) $[3,6,2,0]$　(2) $[1,0,0,0]$　(3) $[3,6,-2,5]$　(4) $[0,0,0,1]$　【91 雲科電機 16%】

67. Let $A=\begin{bmatrix}1 & 2\\ 1 & 3\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}$, $C=\begin{bmatrix}-1 & -2\\ -3 & -5\end{bmatrix}$, $D=\begin{bmatrix}-1 & -2\\ 0 & -2\end{bmatrix}$.

(1) Are A, B, C, D linearly dependent?

(2) Find the dimension and a basis for the space spanned by A, B, C, D.【87 交大資工 4%】

68. Show the three vectors $u=(1,-2,0)^T$, $v=(0,1,4)^T$ and $w=(0,-1,-3)^T$ are linearly independent and express $p=(-1,2,3)^T$ as linear combination of u, v and w.【90 高雄科大電控 12%】

69. Prove that if set B is a basis of linear space V, then B is minimal-generating and maximum-independent in the following sense.
(1) If A is a proper subset of B (that is $A \subset B$, $A \neq B$) then A is not generating.
(2) If $B \subset C \subset V$ and B is a proper subset of C, then C is not linearly independent.【中正電機 10%】

70. Find the value of k so that the vectors $\begin{bmatrix}3-k & -1 & 0\end{bmatrix}^T$, $\begin{bmatrix}-1 & 2-k & -1\end{bmatrix}^T$, and $\begin{bmatrix}0 & -1 & 3-k\end{bmatrix}^T$ span a two-dimensional space.【90 交大資料 7%】

71. (1) Assume T is a linear transformation in 3-tuple real vectors and $T(a_1,a_2,a_3)=(a_1-xa_2-a_3, ya_1+5a_3, -2a_1+4a_2+za_3)$. If the null space of T is $\{(a,a,a) \mid a$ is any real number$\}$, find x, y and z.
(2) Given $M_1=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, $M_2=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ and $M_3=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, show that the span of $\{M_1, M_2, M_3\}$ is the set of all 2 by 2 symmetric matrices.【90 台大電機 10%】

72. (1) Show that the vectors $\begin{Bmatrix}1\\1\\0\end{Bmatrix}$, $\begin{Bmatrix}0\\1\\1\end{Bmatrix}$, $\begin{Bmatrix}1\\0\\1\end{Bmatrix}$ form a basis for the vector space R^3.
(2) Use the basis given in part (1) to construct an orthonormal basis for the same vector space R^3 with the Gram-Schmidt orthonormalization process.【88 清大動機 20%】

73. Show that the set W of polynomials in P_2 such that $P(1)=0$ is a subspace of P_2.
(1) Make a conjecture about the dimension of W.
(2) Confirm your conjecture by finding a basis for W.【91 暨南資訊 10%】

74. Determine whether or not the following are subspaces of R^3

(1) $\{(x_1,x_2,x_3) \mid x_1+x_3=1\}$
(2) $\{(x_1,x_2,x_3) \mid x_1=x_2=x_3\}$
(3) $\{(x_1,x_2,x_3) \mid x_3=x_1+x_2\}$

(4) $\{(x_1, x_2, x_3) | x_3 = x_1^2 + x_2^2\}$

【89 交大資工 8%】

75. Let U be the subspace of R^3 spanned by the vectors $(3,-1,2)$ and $(1,0,4)$, and let V be the subspace of R^3 spanned by the vectors $(4,-1,6)$ and $(1,-1,-6)$. Show that $U = V$.【86 北科電腦通訊 10%】

76. Let K be the subset of all skew-symmetric matrices in $M_{3\times3}(R)$, and G be the subset of all orthogonal matrices in $M_{3\times3}(R)$. Judge if K and G are subspaces of $M_{3\times3}(R)$ and for each subset judged to be subspace, find a basis for it. Justify your answers.【87 台大電機 20%】

77. (1) Find a basis for the subspace W_1 of vectors $[a,b,c,d]^T$ with $a+c+d=0$.
(2) Find a basis for the subspace W_2 of vectors $[a,b,c,d]^T$ with $a+b=0$ and $c=2d$.
(3) What is the dimension of the intersection?【91 雲科電機 18%】

78. Let the subspace V be generated by $v_1 = (1,3,2,-1)$, $v_2 = (0,-1,2,1)$, $v_3 = (2,7,2,-3)$. And the subspace W be generated by $w_1 = (1,4,3,0)$, $w_2 = (1,5,0,-3)$, $w_3 = (1,0,5,4)$. Find a basis for $V \cap W$.【89 台大資訊 15%】

79. Let W be a subspace of R^n and let $W^\perp$ be the orthogonal complement of W. Show that $W^\perp$ is a subspace of R^n.【89 中央資訊 10%】

80. Show that if V_1 and V_2 are subspace of R^n. We define $S^\perp$ be the set of all vectors $w \in R^n$ such that $x \cdot w = 0$ for all $w \in S$. Let $S = \{[1,3,1,-1]^t, [2,6,0,1]^t, [3,9,1,0,]^t\}$. Find an orthogonal basis for $S^\perp$.【91 高科通訊 10%】

81. Let S be the subspace of R^4 containing all vectors with $X_1 + X_2 + X_3 + X_4 = 0$ and $X_1 + X_2 - X_3 - X_4 = 0$. Find a basis for the space $S^\perp$ ($S^\perp$= containing all vectors orthogonal to S).【91 台大資訊 10%】

82. 已知R爲向量空間，A爲其子集合，請問在何種條件下，A也爲向量空間？【91 中央環工 4%】

83. 下列各項量何者線性獨立?
(1) e^x, e^{-3x}, xe^{-3x}
(2) $(1,0,-2,4)$, $(0,2,-3,1)$, $(3,1,2,-1)$【90 台科化工 10%】

84. Determine whether the vectors, $v_1 = (1,1,-2,-2)^t$, $v_2 = (2,-3,0,2)^t$,

$v_3 = (-2,0,2,2)$ and $v_4 = (3,-3,-2,2)^t$, are linearly dependent or linearly independent in R^4. Show that the three vectors are linearly independent.【88 中央土木 10%】

85. Which are linearly independent set in $c[0,1]$?

(1) $[\cos \pi x, \sin \pi x]$

(2) $\left[x^{\frac{3}{2}}, x^{\frac{5}{2}}\right]$

(3) $\left[1, e^x + e^{-x}, e^x - e^{-x}\right]$

(4) $\left[e^x, e^{-x}, e^{2x}\right]$【92 交大電子 8%】

86. True or false with explanation.

If u_1, u_2, u_3 in R^3 are linearly independent, then the vectors $w_1 = u_1 + u_2$, $w_2 = u_1 + u_3$, $w_3 = u_2 + u_3$ are also linearly independent.【91 清大微機電 5%】

87. Let S be the subspace of R^3 spanned by a vector $x = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

(1) Find a basis for $S^\perp$, where $S^\perp$ is the orthogonal complement of S.

(2) Give a geometrical description of $S^\perp$.【88 交大電子 3%】

88. Let $A = \begin{bmatrix} 1 & -3 & 2 & -4 & 8 \\ 3 & -9 & 6 & -12 & 24 \\ -2 & 6 & -5 & 11 & -18 \\ 1 & -3 & 6 & -16 & 16 \end{bmatrix}$.

(1) Find a basis for the row space of A.

(2) Find a basis for the column space of A.

(3) Find a basis for the null space of A.

(4) Find a basis for $\ker(A)$.【91 北科自動化 15%】

89. Let the 4×5 matrix be defined as

$$A = \begin{bmatrix} 1 & -2 & 3 & 0 & 0 \\ 3 & -7 & 9 & -2 & -3 \\ 1 & 3 & 3 & 4 & 9 \\ 1 & 1 & 3 & 7 & 10 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \end{bmatrix}$$

(1) Compute the rank of the matrix A. Hint: Compute the reduced row echelon form the matrix A.

(2) We can use the column vectors of the matrix A to form a basis for the column space of the matrix A. The set of all column vectors is denoted as the set S, where $S = \{a_1, a_2, a_3, a_4, a_5\}$. Find a set $V \subset S$, where V contains

several vectors from the set S. This subset can form a basis for the column space of the matrix A. Hint: The selection can be deduced from the reduced row echelon form of the matrix A.

(3) Compute the dimension of the nullity of the matrix A.【90 交大電子 15%】

90. Given a 4×4 matrix T as $T=\begin{pmatrix} 0 & -2 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 2 & 0 & 2 & -2 \\ 0 & 2 & 0 & 0 \end{pmatrix}$.

(1) Let S be the consisting of all vectors $x\in R^4$ such that $Tx=0$. Show that S is a subspace of R^4.

(2) Find the dimension and a basis for the set S discussed in (1).

(3) Find the transformation matrix P that diagonalizes T.

(4) Find a general solution for the system of differential equations:

$$\frac{dX}{dt}=TX$$

Hint: Set $PY=X$. Derive the ordinary differential equation for Y and solve it.
【91 台大機械 20%】

91. The null space of a matrix A consists of all vectors such that $Ax=0$. The row space of a matrix consists of all combinations of the row vectors. Let

$$A=\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

(1) Find a basis for the null space of A and verify that it is orthogonal to the row space.

(2) Given $x=[2,3,-1]^T$, split it into a row space component x_R and a null space component x_n.【90 台大機械 9%】

92. Let $A=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix}$. Please find a basis of all x's that satisfy $Ax=0$.

Also find a basis of all b's for which $Ax=b$ has feasible solutions, and all solutions.【91 台科電機 10%】

93. Prove that a square matrix A is invertible if and only if its columns (or rows) are linearly independent.【91 台科電機 12%】

94. Prove that $rank(AB)\le \min\{rank(A), rank(B)\}$.【90 中央機械 25%】

95. Let A be an $m\times n$ matrix, show that if and only if A has linearly independent

column vectors, then $N(A)=\{0\}$.【88 台科電機 10%】

96. $A=\left[a_{ij}\right]_{m\times n}$ ，$m\neq n$ ，證明：

(1) 若 A 具有左逆矩陣，則 A_L^{-1} 非唯一，且不存在右逆矩陣。

(2) 若 A 具有右逆矩陣，則 A_R^{-1} 非唯一，且不存在左逆矩陣。【85 成大資訊 16%】

97. Let V be a finite-dimensional vector space over a field F. Let $\beta=\{x_1,\cdots,x_n\}$, be an ordered basis for V. Let Q be $n\times n$ invertible matrix with entries from F. Define $x_j'=\sum_{i=1}^{n}Q_{ij}x_i$ for $1\le j\le n$, and set $\beta'=\{x_1',\cdots,x_n'\}$. Prove that β' is a basis for V.【87 台大電機 10%】

98. Determine the rank of the following matrix. Show the steps of how you arrive at the answer. $A=\begin{bmatrix}1&0&3&0\\0&2&4&6\\3&0&5&1\\2&3&0&1\end{bmatrix}$【91 中正機械 5%】

99. (1) Let A be a 6×5 matrix, if the dimension of nullespace of A is 2, then what is rank A.

(2) If B is a 3×5 matrix, then does $Bx=0$ have nontrivial solutions? How many? Explain.【90 海洋機械 10%】

100. $A=\begin{bmatrix}1&0&3&0\\0&1&0&3\\1&0&3&1\end{bmatrix}$. Find $\operatorname{rank}(A)$, $\operatorname{nullity}(A)$.【89 清大材料 10%】

101. 若 $rank(A)=2$ ，則 x 應如何？$A=\begin{bmatrix}5-x&4&-2\\4&5-x&-2\\-2&-2&3-2x\end{bmatrix}$【86 台科營建 15%】

102. $A=\begin{bmatrix}1&0&3\\0&1&1\\1&5&8\end{bmatrix}$. Find bases for $csp(A)$, $rsp(A)$, $N(A)$.【88 成大電機 15%】

103. 已知$\begin{bmatrix}1&-1&2\\2&1&-3\\4&-1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}4\\-2\\6\end{bmatrix}$，即 $Ax=b$ 。

(1) 求解

(2) 求 A 之階數(rank)與零數(nullity)

(3) 求 A 之列空間維數

(4) 判斷並解釋 b 是否屬於 A 之行空間。【中央土木 15%】

104. True or false with explanation.

(1) A matrix A has a vector $(1,2,3)$ in row space and a vector $(-3,1,1)^t$ in null space.

(2) $A=\begin{bmatrix} 3+i & 2-i \\ 2+i & 3-i \end{bmatrix}$ is Hermitian.【92 交大電子 5%】

105. (1) After three experiments, we obtain $(1,2)$, $(4,2)$ and $(9,4)$ points. To fit all these three points to one line $y=ax+b$ in the least mean square error sense, find optional a and b.

(2) Generalize the above problem for 50 experiments (x_1,y_1), (x_2,y_2), … , (x_{50},y_{50}).【89 成大電子 10%】

106. 某位學生做物理實驗所測量虎克常數與力的關係為

$$\begin{aligned} 4k &= 3 \\ 7k &= 5 \\ 11k &= 8 \end{aligned}$$

請問 k 的最小平方解是多少？【90 中正電機 10%】

107. 若 x 與 y 之理論關係表示為： $y=Ax^2+B$ 。現已知 x 與 y 有一組試驗數據 $(\tilde{x}_i,\tilde{y}_i)$ ， $i=1,\cdots,n$ 。說明如何用最小平方差法(least square error)決定 A 與 B ？【90 成大資源 16%】

108. Fit a straight line equation to the following experimental data by using least squares:

x	0.0	0.1	0.2	0.3
$f(x)$	0.9	1.9	2.8	4.2

【90 中原醫工 15%】

109. Find an equation of the least-squares line for the data: $P_1(1,1)$, $P_2(2,3)$, $P_3(3,4)$, $P_4(4,3)$ and $P(5,6)$.【89 台大環工 10%】

110. Given a series of points (x_1,y_1), (x_2,y_2), … ,(x_n,y_n), we want to find a straight line $y=ax+b$ that best approximates the data points. This is usually called the Least Square line. Please show the formulation and find the solutions of a and b.【91 中正機械 10%】

111. Define $f(x)=\begin{cases} 0,-\pi\le\pi\le 0 \\ \pi-x,0\le x\le\pi \end{cases}$. Let $g(x)=k_0+k_1\sin(x)+k_2\sin(2x)$, in which k_0, k_1 and k_2 are constants. Find the values of k_0, k_1 and k_2, so that

$\int_{-\pi}^{\pi}\left(f(x)-g(x)\right)^2 dx$ is minimized.【91 雲科機械 10%】

112. 某基樁現地試驗量測到樁頂之荷重與沉陷量如下表所示：

P 荷重(t)	W 沉陷量(m)
100	1.0
200	2.0
300	35
400	5.0
500	6.5
600	9.0
700	10.5
800	13.5

擬採用下列數學函數迴歸 $P=\dfrac{w}{a+bw}$ 。

(1) 求常數 a,b 。

(2) 當沉陷量趨近無限大時，荷重等於多少？【88 台科營建 25%】

113. Let $S=Span\left[(1\quad 3\quad 1\quad 1)^T,(1\quad 1\quad 1\quad 1)^T,(-1\quad 5\quad 2\quad 2)^T\right]$ be a subspace of R^4, and let $b=(4\quad -1\quad 5\quad 1)^T$.

(1) Find an orthonormal basis for S.

(2) Use your answer in (1) to find the projection p of b onto S.

$$\text{Given}\quad A=\begin{pmatrix}1 & 1 & -1\\ 3 & 1 & 5\\ 1 & 1 & 2\\ 1 & 1 & 2\end{pmatrix}$$

Use your answer in (2) to solve the least squares problem $Ax=b$.【91 清大資訊 15%】

114. Consider the vector space $C[-1,1]$ with the inner product

$$\langle f,g\rangle=\int_{-1}^{1} f(x)g(x)dx$$

(1) Write down the Cauchy-Schwarz inequality (with respect to this particular inner product).

(2) Consider the subspace $W=span\{1,x,x^2\}$. Find an orthonormal basis for W.

(3) Find the orthogonal projection of x^3 onto the subspace W.【90 交大資料 14%】

115. Find the projection of the vector $v=\begin{bmatrix}1\\0\\-2\end{bmatrix}$ onto the subspace

$$S = Span\left\{\begin{bmatrix}0\\-1\\1\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix}\right\}.$$

【92 交大電子 8%】

116. 請以最小平方誤差法(least square)，求出與以下 5 點之 best fit parabola $y = a + bx + cx^2$，$(1,-1)$, $(-1,2)$, $(-2,2)$, $(-3,3)$, $(4,-5)$。【87 交大環工 14%】

117. Students have recorded the temperature and resistance measurements as shown in the given table. Careful observation suggests a linear relationship with R denoting resistance and T representing temperature. Find the constants a and b in the following equation to predict R for any given T. $R = aT + b$

$T, °C$	$R, ohms$
20.5	765
32.7	826
51.0	873
73.2	942
95.7	1032

Note that in the constants a and b should be chosen in such a way that the sum of the mean squares prediction errors is minimized.【88 中山機械 15%】

118. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix}-2 & 2 & -3\\2 & 1 & -6\\-1 & -2 & 0\end{bmatrix}$.【90 中央化工 10%，91 成大機械 20%】

119. Let A be a 2×2 real and diaonalizable matrix; i.e., $A \in R^{2\times 2}$. If A has a complex egenvalue $\sigma + jw$ with its corresponding eigenvector $V_R + jV_t$, where $\sigma, w \in R$, and $V_R, V_I \in R^{2\times 1}$, show that
(1) The matrix A has the other complex eigenvalue $\sigma - jw$ with a corresponding eigenvector $V_R - jV_I$.
(2) The matrix A is similar to the real matrix $B = \begin{bmatrix}\sigma & w\\-w & \sigma\end{bmatrix}$; ie., there exists a real 2×2 matrix T such that $T^{-1}AT = B$.【89 交大機械 20%】

120. Find: (1) the characteristic equation, (2) the eigenvalues, and (3) the bases for the eigenspaces of the given matrix A. $A = \begin{bmatrix}5 & 6 & 2\\0 & -1 & -8\\1 & 0 & -2\end{bmatrix}$【91 淡江資訊 20%】

121. 令 A 為 4×4 之實數矩陣，且已知 A 的兩組特徵值及特徵向量分別為

$\lambda_1 = 2, e_1 = \langle 1,2,0,0 \rangle^t$ 及 $\lambda_2 = 2+i, e_2 = \langle 3,1+i,2i,6 \rangle^t$，其中 $i=\sqrt{-1}$。求 Ax 之值，其中 $x = \langle 8,1-i,2i,6 \rangle^t$。【91 台科營建 20%】

122. Suppose the Matrix A has eigenvalues 0, 1, 2 with eigenvectors V_0, V_1, V_2. Solve the following equation for X.
(1) $AX = V_0$
(2) $AX = V_1 + V_2$【91 台大資訊 15%】

123. (1) Find all eienvalues and a basis for the corresponding eigenspace for the matrix

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

(2) Use your answer to compute $A^{100}B$ where $B = [2,2,8]^t$.【91 中正電機 10%】

124. 某地層無地下水，土壤單位重量爲γ，若k表示側向土壓力係數，在地面下深度z處，假設土壤單位立方體所受之正向應力與剪應力表示如下：

$$\sigma_z = \gamma z \text{ ，} \sigma_x = \sigma_y = k\sigma_z \text{ ，} \tau_{xz} = \tau_{zx} = \tau_{xy} = \tau_{yx} = \tau_{zy} = \left(\frac{1-k}{2}\right)\sigma_2$$

求$k = 0.5$時，該土壤單元立方體所受之最大主應力，與$k = 2.0$時，該土壤單元立方體所受之最大主應力。【90 台科營建 20%】

125. 已知矩陣A之部分元素的值及其二個特徵向量$\vec{e}_1$及$\vec{e}_2$：

$$A = \begin{bmatrix} 7.3 & 0.2 & a \\ -11.5 & 1.0 & b \\ 17.7 & 1.8 & c \end{bmatrix} \text{，} \vec{e}_1 = \{-1,3,-1\} \text{，} \vec{e}_2 = \{1,-1,3\}$$

(1) 請求出矩陣A中待定常數a,b,c？
(2) 請求出特徵向量所對應之特徵值λ_1及λ_2？
(3) 求出第三個特徵值λ_3及特徵向量$\vec{e}_3$？【89 中央土木 20%】

126. Matrix $A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$, find eigenvalues and eigenvectors.【91 中央資訊 10%】

127. Find eigenvalues and eigenvectors of A:

$$A=\begin{bmatrix}2 & 0 & -2\\ 0 & 4 & 0\\ -2 & 0 & 5\end{bmatrix}$$

【91 北科機電 10%】

128. Find eigenvalues and eigenvectors of A:

$$A=\begin{bmatrix}1 & -4\\ 1 & 1\end{bmatrix}$$

【90 海洋光電 10%】

129. Find matrix A whose eigenvalues are 1, 2, 3 and eigenvectors are $(2,2,1)^t$, $(1,6,2)^t$, $(3,1,1)^t$ respectively.【90 清大資訊 10%】

130. There exist two values of λ delivering the nontrivial solutions in the linear system:

$$(\lambda-1)x-4y=0\,,\quad -2x+(\lambda-3)y=0$$

Find the corresponding nontrivial solutions.【92 交大電子 8%】

131. Find eigenvalues and eigenvectors of A:

$$A=\begin{bmatrix}1 & 0 & 0\\ -8 & 4 & -6\\ 8 & 1 & 9\end{bmatrix}$$

【92 交大電子 6%】

132. 求矩陣 A 的固有值(eigen value)與特徵向量：

$$A=\begin{bmatrix}5 & -3 & -2\\ 8 & -5 & -4\\ -4 & 3 & 3\end{bmatrix}$$

【90 海洋光電 10%】

133. 求特徵值與特徵向量：

$$A=\begin{bmatrix}0 & 1 & -2\\ 2 & 1 & 0\\ 4 & -2 & 5\end{bmatrix}$$

【91 台科化工 10%】

134. $A=\begin{bmatrix} 2 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 5 \end{bmatrix}$代表應力矩陣，求：

(1) 三個主應力大小。
(2) 三個主應力方向之單位向量。【91 嘉義土木 15%】

135. Find eigenvectors and eigenvalues of A:

$$A=\begin{bmatrix} 4 & 2 & 3 \\ 2 & 1 & 0 \\ -1 & -2 & 0 \end{bmatrix}$$

【87 交大應化 15%】

136. Solve the generalized eigenvalue problem $Ax=\lambda Bx$ where

$$A=\begin{bmatrix} 4k_1+k_2 & k_2 \\ k_2 & 4k_1+k_2 \end{bmatrix},\ B=\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix},\ x=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

【88 台大土木 10%】

137. Which one(s) of the following statements are true?(每小題兩分，答錯到扣兩分)

(1) An $n\times n$ matrices A has a basis of eigenvectors for R^n if and only if A has n distinct eigenvalues.
(2) If the eigenvalues of A matrix are λ_i, then the eigenvalues of A^{-1} are $1/\lambda_i$.
(3) The rank of a matrix A is equal to the rank of its transpose A^{-1}.
(4) The rank of an $m\times n$ matrix A is 0 if and only if all elements of A are zero.
(5) n vectors $x_1,x_2,\cdots,x_n$ are linearly dependent if the rank of the matrix with orw vectors $x_1,x_2,\cdots,x_n$ is greater than n.
(6) A homogeneous linear system $Ax=0$ has non-trivial solutions if and only if the rank of the $n\times n$ matrix A is n.
(7) For a square matrix A, the inverse of A exists if and only if $\det A\neq 0$.
(8) The inverse of an orthogonal matrix is orthogonal.
(9) The sum of two orthogonal matrices is orthogonal.
(10) The eigenvalues of both orthogonal matrices and unitary matrices have absolute value 1.【88 交大機械 20%】

138. $A=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$,

(1) find eigenvalues and eigenvectors;

(2) solve $Ax = \alpha I x + \begin{bmatrix}1 & 0 & -2\end{bmatrix}^t$, α may be any real.【86 台大土木 25%】

139. 求特徵值與特徵向量：

$$A = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & -4 & 0\end{bmatrix}$$

【91 中興土木 25%】

140. $A, B \in R^{n\times m}$ ，S 爲常數，其關係式爲 $A = (I+SB)^{-1}(I-SB)$，求出 λ_A 與 λ_B 之關係，即 $Ax = \lambda_A x$ ，$Bx = \lambda_B x$ 。【89 淡江環工 25%】

141. Let the eigenvalues of the following matrix be $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, find

(1) $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$.

(2) $\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4$.

(3) $\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\kappa_4$

$$A = \begin{bmatrix}2 & 1 & 0 & 0\\ 1 & 3 & 0 & 0\\ -1 & 1 & 2 & 1\\ 0 & 2 & 1 & 2\end{bmatrix}$$

【89 台科電子 10%】

142. Let $\alpha = [\alpha_1 \ldots \alpha_n]^T$; $\beta = [\beta_1 \ldots \beta_n]$. Define $A = \alpha \cdot \beta$.

(1) What is the rank of matrix A? (Explain or prove your answer as well.)

(2) Fine all eigenvalues of matrix A.【90 中山電機 10%】

143. Assume that $A = \begin{bmatrix}1 & -1 & 1 & -1\\ -1 & 1 & -1 & 1\\ 2 & -2 & 2 & -2\\ -2 & 2 & -2 & 2\end{bmatrix}$,

(1) find the eigenvalues and the eigenvectors of A.

(2) Could the matrix A be diagonalizable? Explain.【91 高科通訊 20%】

144. Find the eigenvalue of matrix A defined below:

$$A = \begin{bmatrix}-1-2i & -1-i & 2+2i\\ -4i & -i & 4i\\ -1-3i & -1-i & 2+3i\end{bmatrix} \quad \text{where } i = \sqrt{-1}$$

For each real eigenvalue, find the corresponding eigenvector.【91 台大化工 20%】

145. (1)矩陣 A 的伴隨矩陣(adjoint matrix)表示爲 $adj(A)$，其行列式值表爲 $|adj(A)|$，請列式計算 $(|adj(A)|)^3$，其中 A 爲 $\begin{bmatrix} 5 & 7 & 3 & 4 & 1 \\ 4 & 8 & 3 & 4 & 1 \\ 4 & 7 & 4 & 4 & 1 \\ 4 & 7 & 3 & 5 & 1 \\ 4 & 7 & 3 & 4 & 2 \end{bmatrix}$。

(2)已知矩陣 $[A]$ 的特徵值爲 1 和 6，所對應的特徵向量分別爲 $\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$ 和 $\begin{Bmatrix} 4 \\ 1 \end{Bmatrix}$，其反矩陣的平方可寫成 $([A]^{-1})^2 = \dfrac{1}{Q}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$，請問 a,b,c,d 和 Q 之值爲何？

【91 中央土木 6%】

146. 求 $\det(A)$：

$$A = \begin{bmatrix} 5 & 3 & 3 & 3 & 3 & 3 & 3 \\ 3 & 5 & 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 5 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 5 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 5 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 & 5 & 3 \\ 3 & 3 & 3 & 3 & 3 & 3 & 5 \end{bmatrix}$$

【89 中央環工 25%】

147. $A = \begin{bmatrix} 0 & 2 & -2 \\ 3 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$，求 A^5 之特徵值。【89 高科營建 15%】

148. Find the eigenvalues of the following matrix

$$A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 \\ 2 & 2 & 1 & 1 \\ 2 & 2 & 1 & 1 \end{bmatrix}$$

【88 北科通訊 15%】

149. Solve the following problems.
Let $A = (a_{ij})$ be an $n \times n$ matrix such that for each $i = 1,\ldots,n$ we have $\sum_{j=l}^{n} a_{ij} = 0$. Show that 0 is an eigenvalue of A.【88 中央機械 8%】

150. Find the determinant:

$$\det\begin{vmatrix} 1+a_1 & a_1 & \ldots & a_1 \\ a_2 & 1+a_2 & \ldots & a_2 \\ \vdots & \vdots & \vdots & \vdots \\ a_n & a_n & \ldots & 1+a_n \end{vmatrix}$$

【87 中央機械 15%】

151. $A=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$

(1) $|A|$

(2) $|A^T A|$

(3) eigenvalues λ

(4) how many linearly independent eigenvectors.【88 成大航太 16%】

152. Find the eigenvalues of A, where

$$A=\begin{bmatrix} 2 & 3 & 2 & 1 \\ -2 & -3 & 0 & 0 \\ -2 & -2 & -4 & 0 \\ -2 & -2 & -2 & -5 \end{bmatrix}$$

【89 交大電控 10%】

153. Let A be a 2×2 matrix and let $P(\lambda)=\lambda^2+b\lambda+c$ be the characteristic polynomial of A. Show that $b=-tr(A)$, $c=\det(A)$.【92 交大電物 10%】

154. $A=\begin{bmatrix} 1 & 5 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$，求 A^3 之特徵值與行列式。【90 中興土木 20%】

155. Diagonalizes A,

$$A=\begin{bmatrix} 5 & 2 & 2 \\ 3 & 6 & 3 \\ 6 & 6 & 9 \end{bmatrix}$$

【90 交大機械 15%】

156. One matrix $A=\begin{bmatrix}5 & 0 & 0\\ 1 & 0 & 3\\ 0 & 0 & -2\end{bmatrix}$.

(1) Please show whether A has inverse. If so, please find it. If not, please explain it.
(2) Please show whether A is diagonalizable, if so, find the matrix P that diagonalizes A and the diagonal matrix D such $D=P^{-1}AP$. If not, please explain it.【91 交大應化 16%】

157. If a matrix A has eigenvalues 1,0 and the corresponding eigenvectors are $(1,0,0)^t$, $(1,1,1)^t$, $(1,0,1)^t$, respectively. Find (1) A; (2) A^{10}.【90 元智機械 15%】

158. Consider the following two matrices,

$$X=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 0\\ 0 & 0 & 2\end{bmatrix} \text{ and } Y=\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 2\\ 0 & 2 & 1\end{bmatrix}.$$

(1) Determine the invert ability of X and Y. Give the reasons.
(2) Determine the diagonalization ability of X and Y. Give the reasons.【91 北科資訊 20%】

159. The matrix D below is not diagonalizable when $\delta=?$

$$D=\begin{bmatrix}\delta & 3 & 2\\ 0 & -10 & -8\\ 0 & 12 & 10\end{bmatrix}$$

【91 台大電機 5%】

160. 設 A 爲矩陣(Matrix)，且 $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$。

試證明 $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$，且 n 爲正整數。【91 屏科機械 20%】

161. Diagonalize A.

$$A=\begin{bmatrix}5 & -4 & 4\\ 12 & 11 & 12\\ 4 & -4 & 5\end{bmatrix}$$

【91 中興土木 15%】

162. $A=\begin{bmatrix}10 & -3 & 5\\ 0 & 1 & 0\\ -15 & 9 & 10\end{bmatrix}$, find eigenvalues, eigenvectors and A^{100}.【91 彰師機械 20%】

163. Diagonalize $A=\begin{bmatrix}7.3 & 0.2 & -3.7\\ -11.5 & 1.0 & 5.5\\ 17.7 & 1.8 & -9.3\end{bmatrix}$.【91 交大機械 25%】

164. Diagonalizes A.

$$A=\begin{bmatrix}2 & 0 & 1\\ 0 & -2 & -2\\ 0 & 2 & 0\end{bmatrix}$$

【89 北科車輛 20%】

165. $A=\begin{bmatrix}2 & -2 & 3\\ 1 & 1 & 1\\ 1 & 3 & -1\end{bmatrix}$

(1) Diagonalizes A.
(2) Find eigenvalues of $f(A)=3A^6+4A^{-4}+5A^2-6A^{-1}-7I$.【89 台大化工 20%】

166. $A=\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix}$, compute A^{15}.【88 台科電子 15%】

167. Assume that $A=\begin{bmatrix}3 & -2\\ 4 & -3\end{bmatrix}$.

(1) Find the eigenvalues of A.
(2) Find the eigenvectors of A.
(3) Let A^{-1} be the inverse of A. By using the results of (1) and (2), find $\left(A^{-1}\right)^{49}$.【91 台大電機 30%】

168. Please diagonalize the matrix $A=\begin{bmatrix}-3 & 1 & 0\\ 0 & -3 & 1\\ -4 & 0 & 0\end{bmatrix}$.【89 中央電機 10%】

169. $A=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$, find e^{At}.【90 成大機械 10%】

170. Solve the equation $X^2 - 5X + 3I = \begin{bmatrix} 1 & -4 \\ 2 & -5 \end{bmatrix}$.【91 成大工程科學 15%】

171. Let $B^2 = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, find B.【90 北科自動化 20%】

172. There are two major cities in a country: cities A and B. In any given year 40% of city A population will move to city B. Similarly, 20% of city B residents will move to city A. The remaining people stay in their place. What is the expected distribution of population in the two cities after a long period of time?【90 中興資訊 10%】

173. A recursive formula is given as $r_{n+1} = 4r_n - t_n$, $t_{n+1} = 2r_n + t_n$ with the initial values, $r_0 = 100$ and $t_0 = 10$. Determine $\lim\limits_{n\to\infty} \dfrac{r_n}{t_n} = ?$【88 中央機械 8%】

174. $x_{n+2} = x_{n+1} + 6x_n$ $x_0 = x_1 = 1$, find x_n.【87 雲科電子 10%】

175. 求矩陣方程式$X^2 + 4X + A = 0$之解，$A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$。【90 高雄科大營建 15%】

176. 若$A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$，求$\sin A$。【88 中央環工 20%】

177. If $p(x) = x^7 - 4x^5 + 6x^2 - x - 3$ and $A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$, please

(1) find eigenvectors of A
(2) diagonalize A and
(3) evaluate $p(A)$【87 台大化工 15%】

178. $A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$, find e^A.【88 台科化工 10%】

179. Solve $X^2 - 4X + 4I = \begin{bmatrix} 4 & 3 \\ 5 & 6 \end{bmatrix}$.【86 成大製造 13%】

180. Define exponential matrix e^A of a $n \times n$ matrix A:

$$e^A = \sum_{n=0}^{\infty} \frac{A^n}{n!}$$

where $A^0 = I$ is a $n \times n$ identity matrix. Evaluate e^A for a given matrix

$A=\begin{bmatrix}1 & 4\\ 1 & 1\end{bmatrix}$.【91 中原物理 20%】

181. Given a matrix $A=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$ and $e^{A}=I+A+\dfrac{A^2}{2!}+\dfrac{A^3}{3!}+\ldots$, evaluate the determinant of e^{A}, where $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$.【90 北科通訊 15%】

182. $X'=AX$, where $A=\begin{bmatrix}5 & -4 & 4\\ 12 & -11 & 12\\ 4 & -4 & 5\end{bmatrix}$ and $X=\begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}^T$.

(1) determine the $rank(A)$.

(2) Find the eigenvalues and eigenvecters of matrix A

(3) Write the general solution of the system $X'=AX$.【91 台師大電機 12%】

183. 請將$\dfrac{d^4y}{dx^4}-2\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+y=x$改寫成一階微分方程式的聯立方程式的形式，並以矩陣的形式呈現。【91 中央環工 10%】

184. Solve the simultaneous differential equations:

$$\dot{x}(t)=x-y+e^{t},\quad \dot{y}(t)=2x-2y+\sin(2t)e^{-t}$$

【91 清大動機 15%】

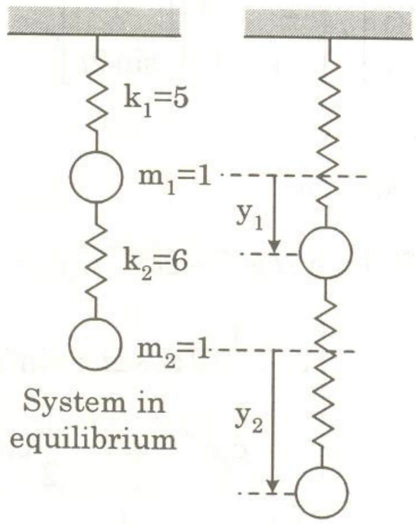
185. Consider the mass-spring system in which y_1 and y_2 measure displacements of masses m_1 and m_2, respectively, from equilibrium positions. The spring constants are $k_1=5$ and $k_2=6$ as shown, and we choose $m_1=m_2=1$.
Assume no damping and no external driving forces.
The motion is governed by:

$$y_1''=-(k_1+k_2)y_1+k_2y_2,\quad y_2''=+k_2y_1-k_2y_2$$

(1) Find the eigenvalue of the system in Figure.

(2) Find the eigenvector of he system in Figure.

(3) Find the general solution of the system in Figure.



【90 成大製造 30%】

186. $x_1'' = -3x_1 + 2(x_2 - x_1)$, $x_2'' = -2(x_2 - x_1)$.

If a solution x is assumed of the form, $x = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} e^{wt} = ke^{wt}$, solve the ODE.【91 交大機械 25%】

187. Solve $X' = \begin{bmatrix} 2 & 1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix} X$.【90 中央電機 15%】

188. Find the general solution of following equations:

$$\begin{cases} y_1' = -y_1 + y_2 \\ y_2' = -y_1 - y_2 \end{cases}$$

【91 中山機電 15%】

189. Solve $x' = Ax + b$, $A = \begin{bmatrix} 5 & 8 \\ -6 & -9 \end{bmatrix}$, $b =\geq \begin{bmatrix} 1 \\ t \end{bmatrix}$.【91 成大土木 20%】

190. Solve $x' = x - 2y$, $y' = 5x - y$, $y(0) = 2$, $x(0) = 1$.【91 北科製造 12%】

191. Solve $y' = AY + g$, $A = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$, $g = \begin{bmatrix} -b \\ 2 \end{bmatrix} e^{-2t}$.【91 成大機械 20%】

192. Solve $4x'' = -2x + y$, $3y'' = 2x - 2y$.【89 成大土木 15%】

193. Solve $y_1' = 4y_1 + 3y_2 - 8e^{-2t}$, $y_2' = 2y_1 - y_2 + 2e^{-2t}$.【91 中興材料 20%】

194. $A=\begin{bmatrix}1&0&1\\0&3&2\\0&0&2\end{bmatrix}$，$y'=A^3y$，$y(0)=(2,-1,1)^t$，求特徵值及特徵向量，並解 y。

【89 中興土木 20%】

195. Solve $x''+3x+y=\sin^2 t$, $y''+2y+2x=\cos^2 t$.【88 清大動機 15%】

196. Given $\begin{cases} y_1''=-3y+2(y_2-y_1) \\ y_2''=-2(y_2-y_1)\end{cases}$ …..(a)

where y_1 and y_2 are functions of t. The initial conditions are $y_1(0)=1$, $y_2(0)=2$, $y_1'(0)=-2\sqrt{6}$, $y_2'(0)=\sqrt{6}$.

(1) If equations of (a) are expressed as $Y''=AY$. What are the Y and A?
(2) Determine the eigenvalues and eigenvectors of A.
(3) If $Y=Xe^{wt}$ is the solutions of (a), find X.
(4) Find the solution of (a).【87 台大環工 20%】

197. Solve

$$x'=-4x+y+z,\quad x(0)=9;$$
$$y'=x+5x-z,\quad y(0)=7;$$
$$z'=y-3z,\quad z(0)=0.$$

【91 台大電機 10%】

198. Given a forced-vibration system described by

$$\begin{cases} \ddot{x}_1+2x_1-x_2=a\sin\Omega t \\ \ddot{x}_2+2x_2-x_1=b\sin\Omega t \end{cases} \text{…….(a)}$$

(1) If we assign $a=0$, $b=1$, and $\Omega=2$ in equations (a), and the initial conditions are $x_1(0)=x_2(0)=0$ and $\dot{x}_1(0)=\dot{x}_2(0)=0$, find the solutions of equations (a).
(2) If the forcing frequency Ω is assigned as $\Omega=\sqrt{3}$, what is the relation between a and b in order that the solutions of equations (a) with zero initial conditions are bounded for all times?【91 台大應力 30%】

199. 已知 $\dfrac{d\bar{X}}{dt}=\begin{bmatrix}0&1\\-16&0\end{bmatrix}\bar{X}$，$\bar{X}(0)=\begin{bmatrix}1/2\\0\end{bmatrix}$，試求解 $\bar{X}$。【91 中興環工 10%】

200. 下列微分方程系統可表示爲矩陣 $X'=AX+G$ 的形式：

$$x_1'=2x_1+x_2-2x_3-2$$
$$x_2'=3x_1-2x_2+5e^{2t}$$
$$x_3'=3x_1+x_2-3x_3+9t$$

(1) 求矩陣 A 的 eigenvalues。
(2) 求矩陣 A 的 eigenvectors。
(3) 今存在一矩陣 P，使 $P^{-1}AP$ 爲以 A 的 eigenvalues 爲主對角線元素之對角線矩陣，求 P 的反矩陣 P^{-1}。
(4) 以變數轉換 $Z = P^{-1}X$ 求上列微分方程系統的通解。【91 北科環境 25%】

201. 將三階 ODE $y''' - 4y'' - 8y' - 10y - \cos t$，改寫爲三個一階線性聯立方程式。【91 北科製造 12%】

202. Let a matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$, find

(1) A^{10}
(2) $A^5 - 2A^4 + 2A^3 + A^2 - 3A + 5I$【90 高雄科大電子 20%】

203. Given $A = \begin{bmatrix} 0 & 2 & -2 \\ 0 & 1 & 0 \\ 1 & -1 & 3 \end{bmatrix}$, calculate: (1) A^{-1}, and (2) e^{At}.【91 輔仁電子 10%】

204. $A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & -2 & 3 & -1 \end{bmatrix}$

(1) What is the characteristic equation of A?
(2) Compute A^{100}.【90 中興電機 12%】

205. 已知一矩陣 A 之特徵值爲 1、2、3，求下式中之 α、β、γ 值。

$$A^4 = \alpha A^2 + \beta A + \gamma I \text{，其中 } A \text{ 爲 } 3\times 3 \text{ 矩陣。}$$

【90 中央土木 25%】

206. $A = \begin{bmatrix} 0.97 & 0.15 \\ 0.03 & 0.85 \end{bmatrix}$, find $\lim_{n\to\infty} A^n$.【91 北科電機 20%】

207. Let A be the following matrix, compute A^{100}.

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

【90 交大機械 25%】

208. 已知矩陣 A 之部分元素的值：

$$[A]=\begin{bmatrix}1 & -1 & a\\ 3 & 2 & -1\\ 2 & 1 & b\end{bmatrix}$$

又已知 $A^{-1}=\frac{1}{6}\left(-A^2+2A+5I\right)$，請求出矩陣 A 中之常數 a 、 b ？【89 中央土木 15%】

209. Let A be an $n\times n$ diagonalizable matrix with characteristic equation

$$\lambda^n+a_{n-1}A^{n-1}+\ldots+a_1=0\lambda+a_0$$

Prove that

$$A^n+a_{n-1}A^{n-1}+\ldots.+a_1A+a_0I=0_n$$

【91 北科光電 10%，89 交大機械 17%】

210. Let $A=\begin{bmatrix}-1 & 1 & 1\\ 2 & -2 & -3\\ -2 & 2 & 3\end{bmatrix}$, and $a=\sin(1)$, find $\sin\left(A^{2001}\right)=?$【91 北科電機 20%】

211. Compute A^{20} where A is the matrix given as $A=\begin{bmatrix}1 & -3\\ 1 & 1\end{bmatrix}$.【91 北科通訊 10%】

212. Let $A=\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 0 & 0 & 8 & 1\\ 0 & 0 & 0 & 8\end{bmatrix}$, please find $A^{\frac{1}{3}}$.【90 成大電機 10%】

213. $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, find e^A.【91 清大通訊 10%】

214. If $A=\begin{bmatrix}2 & 6\\ 1 & 3\end{bmatrix}$, find the matrix function e^{At} where t is a scalar variable.【91 北科電機 15%】

215. Use the <u>matrix exponential</u> to solve the following initial value problems:

$$\frac{d}{dt}Y(t)=AY(t) \text{ ， } Y(0)=Y_0 .$$

(1) $A=\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$, $Y_0=\begin{bmatrix} -4 \\ 2 \end{bmatrix}$, and $Y_{(t)}=\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$.

(2) $A=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$, $Y_0=\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$, and $Y(t)=\begin{pmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{pmatrix}$.【92 交大電物 20%】

216. Use the Laplace transform to solve y_1 and y_2 from the non-homogeneous linear differential equation systen $\begin{bmatrix} y_1' \\ y_2' \end{bmatrix}=\begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}+\begin{bmatrix} -6 \\ 2 \end{bmatrix}e^{-2t}$, with their initial conditions given by $y_1(0)=y_2(0)=0$.【89 清大電機 8%】

217. Solve the following differential equations:
If $Y(t)=\left[y_1(t)y_2(t)\ldots y_n(t)\right]$ is known, where y_i are the n linearly independent solutions to the homogeneous linear system: $y'(t)=A(t)y(t)$, where $A(t)$ is an $n\times n$ time-varying matrix, find the general solution to the forced linear system $y'(t)=A(t)y(t)+g(t)$.【89 清大電機 15%】

218. $A=\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find A^{299}.【90 北科車輛 20%】

219. 計算 $f(A)=A^3-4A^2-A+4I$ ， $A=\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$。【90 雲科環安 10%】

220. Find e^A, $A=\begin{bmatrix} 2 & -3 \\ 2 & -5 \end{bmatrix}$.【88 清大電機 10%】

221. $A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, find e^A.【85 台大電機（10%）】

222. 用 Cayley-Hamilton 定理，求 A^{-1}。【86 中央環工 10%】

223. Let $A=\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Find e^{At} by using the formula $L\left[e^{At}\right]=(sI-A)^{-1}$.【87 成大製造 15%】

224. $A=\begin{bmatrix}3 & -1\\ 1 & 1\end{bmatrix}$, find: $(1)\, e^{At}$, $(2)\sin A$.【91 南師資訊 15%】

225. Consider a matrix $A=\begin{bmatrix}1 & 2\\ 3 & 0\end{bmatrix}$.

(1) Find the eigenvalues.

(2) Calculate $A^4-A^3-4A^2-A$.【91 台科控制 20%】

226. 若矩陣$[A]=\begin{bmatrix}0 & -1 & -1\\ -3 & -1 & -2\\ 7 & 6 & 5\end{bmatrix}$，試求：

(1) 矩陣$[A]$之秩（rank）；

(2) 將$[A]$化為喬登正則式（Jordan canonical form）。

（提示：1 為其特徵值）【89 北科土木 30%】

227. Find general solution of the equation

$$\frac{dy}{dt}=\begin{pmatrix}7 & 4 & 4\\ -6 & -4 & -7\\ -2 & -1 & 2\end{pmatrix}y$$

where $y=(y_1,y_2,y_3)^T$.【90 中興土木 15%】

228. Given the matrix $A=\begin{bmatrix}1 & 1 & 1\\ 2 & 1 & -1\\ -3 & 2 & 4\end{bmatrix}$, find

(1) the determinant of $\left(A^T\right)^{-1}$;

(2) the eigenvalues and corresponding eigenvectors of A.

(3) Solve the system $\frac{dy}{dt}=Ay$ where y is a column vector.【91 中興化工 22%】

229. If A is a square matrix, prove that $\sin^2 A+\cos^2 A=I$.【91 北科電機 20%】

230. Find the fundamental matrix e^{At} for the system.

$$X'=AX, \text{ where } A=\begin{bmatrix}1 & 0 & 0\\ 1 & 3 & 0\\ 0 & 1 & 1\end{bmatrix}.$$

【88 交大電信 15%】

231. Solve the following system.

$$x' = 3x - y - 1,\ \ y' = x + y + 4e^t .$$

【91 成大工程科學 14%】

232. Solve $x_1'' + 2x_1 - x_2 = 0$, $x_2'' + 4x_1 + 6x_2 = 0$.【86 中央土木 20%】

233. Let A be a 2×2 matrix defined by $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Find necessary and sufficient conditions for A to be diagonalizable.【88 交大電子 5%】

234. Using matrix method to solve the differential equation.

$$\dot{X} = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 4 & 0 \\ 0 & 1 & 1 \end{bmatrix} X + \begin{bmatrix} 0 \\ -4e^{3t} \\ -4e^t \end{bmatrix}$$

【87 交大電子 10%】

235. Solve $\dot{X} = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} X$.【87 台科化工 15%】

236. Solve $\dot{X} = \begin{bmatrix} 0 & -10 \\ 2.5 & -10 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.【87 北科電力 20%】

237. $A = \begin{bmatrix} 2 & -3 & 1 \\ 7 & 0 & 2 \\ 12 & 4 & 3 \end{bmatrix}$，將 A 化為 Jordan form。【88 政大資訊 20%】

238. Let A be the following matrix, compute A^{200}.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

【90 交大機械 25%】

239. Let A be an $n\times n$ matrix.

(1) Suppose that $A^3 + A^2 - 2A - 2I = 0$. Show that A is invertible, and find A^{-1}.

(2) Let $A^3 = 0$（the zero matrix）, while $A \neq 0$. What is $(I - A)^{-1}$? (Hint：

Consider a combination of I, A and A^2.)【89 中興資訊 10%】

240. Suppose A is an $n\times n$ matrix from the field of complex number.

(1) Show that A is of rank 1 if and only if $A = xy^T$, where x and y are non zero $n\times 1$ column vectors.

(2) Show that if A is of rank 1, then there exist a scalar β such that $A^2 = \beta A$.

(3) Show that if A is of rank 1 and the β in $(2) \neq -1$, then there exists a scalar α such that $(I_n + A)^{-1} = I_n - \alpha A$.【87 交大資料 6%】

241. Determine $\lim\limits_{m\to 0} A^m$, if exist.

$$(1)\quad A=\begin{bmatrix} 2 & -0.5 & -1 \\ 1 & 0.5 & -1 \\ 1 & -0.5 & 0 \end{bmatrix}\qquad (2)\quad B=\begin{bmatrix} 1 & 0 & -1 \\ -2 & 2 & 0 \\ 2 & -1 & -1 \end{bmatrix}$$

【90 清大電機、通訊 10%】

242. $B=\begin{bmatrix} 1 & 0 & 0 \\ 9 & -8 & 9 \\ 6 & -6 & 7 \end{bmatrix}$.

(1) Find eigenvalues of B.

(2) Find a matrix *C, D* and a real constant a that satisfy the following equation: $B^n = C + D\alpha^n, n \geq 1$, n is integer.【90 交大電子 10%】

243. A square matrix A is called idempotent if $A^2 = A$.

(1) Is $I - A$ idempotent?

(2) $(I-2A)^{-1} = I - 2A$, true or false.【91 師大資訊 8%】

244. (1) Let A be an $n\times n$ real matrix and $\alpha \in R, \alpha \neq 0$. If $I - \alpha A$ is nilpotent matrix, show that A is invertable.

(2) Let A be an $n\times n$ real matrix and $A^2 = A$, show that $(A+I)^k = I + (2^k - 1)A, \forall k \in N$.【91 中正資訊 10%】

245. Compute e^A for $A=\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$.【91 成大資訊 10%】

246. Compute the ranks of A, A^2, A^3, and A^4, where $A=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$.【90 成大土木 15%】

247. $A=\begin{bmatrix}-5&3&1\\-4&2&1\\-4&3&0\end{bmatrix}$, find e^{At}. Hint: $\lambda=-1, -1, -1$【88 北科通訊 15%】

248. Suppose that $A^2=4A$. Show that the eigenvalues of A is either 0 or 4.【88 台科電子 12%】

249. If $v\in R^n$ and I_n is the identity matrix, $H=I_n-2\dfrac{vv^T}{v^Tv}$, show that -1 is an eigenvalue of H.【88 北科電電機能源 10%】

250. (1) Show that $(\lambda-4)^2(\lambda-2)=0$ is the characteristic equation for both the matrices

$$A=\begin{bmatrix}6&2&-2\\-2&2&2\\2&2&2\end{bmatrix},\quad B=\begin{bmatrix}6&2&2\\-2&2&0\\0&0&2\end{bmatrix}$$

(2) Compute $A^2-6A+8I$ and $B^2-6B+8I$.
(3) Show that the matrix A has two linearly independent eigenvectors corresponding to $\lambda=4$, but that the matrix B does not have two linearly independent eigenvectors corresponding to $\lambda=4$. Also explain the results.【86 交大電子 18%】

251. $A=\begin{bmatrix}5&4&2\\4&5&2\\2&2&2\end{bmatrix}$, find e^A.【87 中原電機 20%】

252. $A=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$, find minimal polynomial.【91 高科機械 10%】

253. Show that the eigenvalues of A are real if α, β and γ are real numbers.

$$A=\begin{bmatrix}\alpha&\beta\\\beta&\gamma\end{bmatrix}$$

【91 北科車輛 20%】

254. (1) For a matrix $A=\begin{bmatrix}2&1&1\\1&2&1\\1&1&2\end{bmatrix}$, find the eigenvalues and eigenspace.

(2) Is it possible to find all the eigenvectors in (1) to be orthogonal？Please explain.【90 交大應化 15%】

255. 已知$K=\begin{bmatrix}2&-1&0\\-1&2&-1\\0&-1&2\end{bmatrix}$。

(1) 試求K之特徵值與特徵向量，特徵向量之正規化請依據向量長度為 1 的條件。

(2) 令前項特徵向量所組成之矩陣為S，請以高斯消去法求X^{-1}。

(3) X是否為正交矩陣（orthogonal matrix）？請根據正交矩陣之定義驗證之！【91 交大土木 30%】

256. Consider a 3×3 matrix $A=\begin{bmatrix}7&4&-4\\4&-8&-1\\-4&-1&-8\end{bmatrix}$.

(1) Find the eigenvalues and correspond eigenvectors of matrix A.

(2) Find an orthogonal matrix P and a diagonal matrix D such that $P^{-1}AP=D$.【90 清大動機 20%】

257. Show the following conditions are equivalent for an $n\times n$ matrix P.

(1) P is invertible and $P^{-1}=P^{T}$, where P^{T} is transpose of P.

(2) The rows of P are orthonormal.

(3) The columns of P are orthonormal.【88 淡江資訊 20%】

258. Find a unitary matrix P such that $P^{*}AP$ is a diagonal matrix, where

$$A=\begin{bmatrix}3&2+i\\2-i&7\end{bmatrix}$$

【90 北科通訊 15%】

259. $A=\begin{bmatrix}2&1&0\\1&2&0\\0&0&3\end{bmatrix}$.

(1) Find a matrix P such that $P^{-1}AP=D$ is diagonal.

(2) Find a matrix B such that $BB^{t}=A$.

(4) Find a matrix C such that $C^{2}=A$.【89 元智資訊 15%】

260. Determine whether the statement is true or false explain why or given an example that proves your answer.
(1) There exist skew-symmetric orthogonal 3×3 matrices.
(2) Let A, B, C be $n\times n$ matrices, Then if $A\neq 0$, $AB = AC$ general implies $B = C$.【91 元智機械 20%】

261. Prove that eigenvectors of a symmetric matrix corresponding to different eigenvalues are orthogonal. Give an example.【91 成大水利 15%】

262. Let A be an $n\times n$ real matrix. Suppose A is skew-symmetric; i.e., $A^T = -A$.
(1) Show that the diagonal elements of A are all zero.
(2) Show that $\det A = 0$ if n is odd.
(3) Show that if A has an eigenvalue, the eigenvalue must be zero. (You can get partial credits by assuming $n = 3$.)
(4) Show that $I + A$ is nonsingular (invertible) where I is the $n\times n$ identity matrix. (You can get partial credits by assuming $n = 3$.)【91 台大應力 35%】

263. A linear transformation which maps a vector x to another vector y may be represented by $y = Ax$, where both x and y are $n\times 1$ real matrices and A is an $n\times n$ real matrix.
(1) What property does the matrix A must have for the (Euclidean) norms of x and y to be equal, i.e., $\|x\| = \|y\|$. Why? Give a geometrical interpretation on the linear transformation.
(2) What common property do the eigenvalues of the matrix A have in order for $\|x\| = \|y\|$?
(3) What property may the eigenvector matrix of the matrix A have in order for $\|x\| = \|y\|$?
(4) If the matrices A, x and y are complex instead of real and if $\|x\| = \|y\|$, then what are A, the eigenvalues of A, and the eigenvector matrix of A?【91 台大土木 15%】

264. 矩陣R定義如下：$R(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$。試證：
(1) $R(\alpha)R(\beta) = R(\alpha+\beta)$
(2) $R^{-1}(\alpha) = R(-\alpha) = R^T(\alpha)$【91 交大土木 15%】

265. Given $B = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
(1) Is B singular or nonsingular when $\phi = 22.5^\circ$?
(2) If $\phi = 120^\circ$, find B^{-1}.【92 交大電子 6%】

266. Given $A=\begin{bmatrix} 1/2 & -1/2 & -1/2 & -1/2 \\ -1/2 & 1/2 & -1/2 & -1/2 \\ -1/2 & -1/2 & 1/2 & -1/2 \\ -1/2 & -1/2 & -1/2 & 1/2 \end{bmatrix}$.

(1) Please calculate $\left(A^{-1}\right)^{10}$.

(2) Please calculate A^{101}.【92 交大電子 5%】

267. Suppose A and B are both $n\times n$ real symmetric matrices, and x_1, x_2, … , x_k eigenvectors of the equation $(A-\lambda B)x=0$, corresponding, respectively to the distinct eigenvalues λ_1, λ_2, … , λ_k, $k\le n$.

(1) Show that $x_i^t A x_j = 0$ and $x_i^t B x_j = 0$, $i\ne j$.

(2) Show that x_1, x_2, … , x_k are linearly independent.【89 台大應力 25%】

268. Ture or false. No explanation is required.

(1) Every invertible square matrix can be diagonalized.

(2) Every diagonalizable square matrix can be inverted.

(3) Let A be $m\times n$, x $n\times 1$ and b $m\times 1$. If the columns of A are linearly independent, then the system $Ax=b$ has exactly one solution for every b.

(4) Let A be a square matrix. If B is formed from A by an elementary row operation, then B and A have the same eigenvalues.

(5) Suppose the only eigenvectors of a 3×3 matrix A are a multiples of $x=[1,0,0]^T$, then A is not invertible.

(6) A square matrix and its transpose have the same eigenvalues.

(7) A real symmetric square matrix A can be factored into $A=Q^T DQ$, in which Q is orthogonal and D is diagonal.

(8) The eigenvectors associated with different eigenvalues are linearly independent.【90 台大機械 16%】

269. Consider the matrix $A=\begin{bmatrix} 5 & 0 & 8 \\ -1 & 2 & 4 \\ 2 & 0 & 5 \end{bmatrix}$.

(1) Find the eigenvalues of A.

(2) Find three mutually orthogonal eigenvectors of A.

(3) Show that the matrix A is diagonalizable.【89 交大機械 25%】

270. Find an orthogonal matrix to diagonalizable A.

$$A=\begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 2 & 0 \\ \sqrt{2} & 0 & 0 \end{bmatrix}$$

【91 淡江機械 15%】

271. Find an orthogonal matrix to diagonalizable A.

$$A=\begin{bmatrix}5&0&0&0\\0&0&-1&0\\0&-1&0&0\\0&0&0&0\end{bmatrix}$$

【90 北科冷凍 20%】

272. Find an orthogonal matrix P to diagonalizable A.

$$A=\begin{bmatrix}5&-2&0\\-2&6&-2\\0&-2&7\end{bmatrix}$$

【90 淡江土木 20%】

273. Find A^{-1},B^{-1}:

$$A=\begin{bmatrix}1&0&0\\0&\cos\phi&\sin\phi\\0&-\sin\phi&\cos\phi\end{bmatrix},\quad B=\begin{bmatrix}1&0&0\\0&1/\sqrt{2}&-1/\sqrt{2}\\0&-1/\sqrt{2}&1/\sqrt{2}\end{bmatrix}$$

【91 中正電機 10%】

274. 證明 A 爲正交矩陣：

$$A=\begin{bmatrix}\cos\theta&-\sin\theta&0\\\sin\theta&\cos\theta&0\\0&0&1\end{bmatrix}$$

【91 師大機電 10%】

275. Use an orthogonal transformation to diagonalize the matrix $[A]=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$,

i.e., $[Q]=[P]^{-1}[A][P]$ is a diagonal matrix, in which $[P]$ is an orthogonal matrix. Write down the matrices $[Q]$ and $[P]$.【91 台科機械 20%】

276. With an appropriate value of α the complex matrix.

$$A=\begin{bmatrix} \frac{1-i}{\sqrt{3}} & \alpha & \frac{i}{\sqrt{3}} \\ \frac{-1+i}{\sqrt{15}} & \frac{3}{\sqrt{15}} & \frac{2i}{\sqrt{15}} \\ \frac{1-i}{\sqrt{10}} & \frac{2}{\sqrt{10}} & \frac{-2i}{\sqrt{10}} \end{bmatrix}$$

is unitary. (1) What is α? (2) Find A^{-1}; (3) $x=\begin{bmatrix}1 & 0 & i\end{bmatrix}^T$, $y=Ax$, compute the Euclidean norm of y.【87 台大機械 16%】

277. Let A and B be $n\times n$ orthogonal matrices, A' and B' are the transpose matrices for A and B, respectively. "det A" is the determinate of matrix A. Prove:
(1) A' and A^{-1} are orthogonal.
(2) AB is also orthogonal.【88 成大製造 15%】

278. $A=\begin{bmatrix}5 & 3\\ 3 & 5\end{bmatrix}$，求特徵向量矩陣$\phi$，使得$\phi^T A\phi = I$。【86 中興土木 15%】

279. Let A be $n\times n$ normal matrix, prove $tr\left(A^* A\right)=\sum_{i=1}^{n}\left|\lambda_i\right|^2$.【87 台大電機 10%】

280. A be a 4×4 orthogonal matrix, $x=\begin{bmatrix}1 & 2 & 2 & 4\end{bmatrix}'$, $y=\begin{bmatrix}1 1 -1 1\end{bmatrix}'$, find angle between Ax and Ay.

281. (1) Give the definitions of Hermitian matrix and Normal matrix.
(2) Which of the following matrices are Hermitian? Normal?

$$A=\begin{bmatrix}1 & 2-i\\ 2+i & -1\end{bmatrix}\quad B=\begin{bmatrix}\frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}i\end{bmatrix}$$

$$C=\begin{bmatrix}0 & i & 1\\ i & 0 & -2+i\\ -1 & 2+i & 0\end{bmatrix}\quad D=\begin{bmatrix}3 & 1+i & i\\ 1-i & 1 & 3\\ -i & 3 & 1\end{bmatrix}$$

【92 交大電子 12%】

282. $A=\begin{bmatrix}2 & 2 & -2\\ 2 & 5 & 4\\ -2 & -4 & 5\end{bmatrix}$, find a unitary matrix S to diagonalize A.【91 清大資訊 6%】

283. Find out what type of the conic section of following quadratic form represents and transform into principle axis $4x_1x_2+3x_2^2=1$.【91 元智機械 15%】

284. Find out what type of the conic section of following quadratic form represents and transform into principle axis $13x^2-10xy+13y^2=288$.【91 中正機械 10%】

285. 利用座標旋轉標準化二次曲面$7x^2-8y^2-8z^2+8xy-8xz-2yz+9=0$，並求出旋轉後的座標軸與原卡氏直角座標$x$,$y$,$z$軸的關係式（此題即求特徵值與特徵向量者）。【90 中山海環 20%】

286. 將二次曲面$7x^2+7y^2+7z^2+10xy-24yz=20$旋轉至主軸（principal axis），並判斷此曲面爲何種曲面？又此曲面至原點之最短距離爲何？【91 彰師光電 20%】

287. Consider the conic section $2x^2-4xy-y^2-4x-8y=-14$. Use orthogonal diagonalization and completing the squares to find the equation of the curve when it is rotated and translates into standard position.【91 暨南資訊 10%】

288. Find out what type of conic section (or pair straight line) is represented by the given quadratic form. Transform it to principal axes. Express $x^T=\begin{bmatrix}x_1 & x_2\end{bmatrix}$ in terms of the new coordinate vector $y^T=\begin{bmatrix}y_1 & y_2\end{bmatrix}$, $x_1^2+6x_1x_2+9x_2^2=10$.【90 彰師電機 10%】

289. 將二次方程式$5x^2+8xy+5y^2+4xz+4yz+2z^2=100$轉換成 $ax'^2+by'^2+cz'^2=d$之形式，$a+b+c=$？【91 中央環工 6%】

290. $Q=3x_1^2+3x_2^2+4x_3^2+2x_1x_2$

(1) 找一個對稱矩陣A，使得$D=x^*Ax$，D爲對角矩陣。

(2) 將Q經由座標轉換寫成$Q=y^*Dy$。【90 雲科營建 15%】

291. Plot out the curve $17x_1^2-30x_1x_2+17x_2^2=32$.【90 台科電機 20%】

292. 求原點至雙曲線最短距離$x^2+8xy+7y^2=225$。【89 淡江環工 25%】

293. $A=\begin{bmatrix}0 & -1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 2\end{bmatrix}$

(1) Find an orthogonal matrix P that diagonalizes A.

(2) Find A^{50}.

(3) Transform the quadratic surface $f=-2xy+2z^2$ into its standard form.【89

北科冷凍 25%】

294. The ellipse is represented as $9u^2 - 8uv + 3v^2 = 11$ transform into principle axis.【89 交大機械 10%】

295. Transform into principal axis $f = 2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_1x_3 + 2x_2x_3$.【89 交大土木 20%】

296. $f = 5x^2 - 4xy + 5y^2 = 21$, plot out the curve.【90 元智電機 20%】

297. Find the orthogonal matrix that will reduce the quadratic form $4\left(x_1^2 + x_2^2 + x_3^2 + x_4^2\right) - 2\left(x_1 + x_2\right)\left(x_3 - x_4\right)$ to a linear combination of square terms only.【88 交大電子 11%】

298. Give the quadratic form $x^2 - 10\sqrt{3}xy + 11y^2 - 8\sqrt{3x} - 8y - 32 = 0$. Find a change of coordinates so that the resulting equation in a standard form. Plot out this curve and show both old and new coordinate axis.【台大電機 15%】

299. 求a,b,c，使得A為正交矩陣，a,b,c是否為一？【88 海洋電機通訊控制組 15%】

300. An $n \times n$ matrix M is said to be symmetric if $M = M^T$, an $n \times n$ symmetric matrix is said to be positive definite if $x^T Mx$ is positive for any $n \times 1$ column vectors x. Let M be an $n \times n$ symmetric matrix.
(1) If M is positive definite, show that all the eigenvalues of M are positive.
(2) If the eigenvalues of M are all positive, show that M is positive definite.
【90 交大電控 10%，90 清大資訊 10%】

301. Let $A = \begin{bmatrix} 4 & \sqrt{6} \\ \sqrt{6} & 3 \end{bmatrix}$, and $B = A^5 - 3A^4 - 2I$.
(1) Determine the eigenvalues and corresponding eigenvectors of B.
(2) Determine whether B is positive definite.【90 成大土木 20%】

302. Is A positive definite?

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 4 \end{bmatrix}$$

【89 交大電控 10%】

303. (1) Prove that every eigenvalue of $A^T A$, $A \in R^{n\times n}$, is real and non-negative.
(2) Is part (1) still true if $A^T A$ is replaced by symmetric martix B?【90 交大電控光電 10%】

304. Consider the following matrix: $A=\begin{bmatrix}4/5 & 1/5\\ 1/5 & 4/5\end{bmatrix}$.

(1) Find $\lim_{n\to\infty} A^n$.

(2) For any nonzero vector $x \in R^2$, define $a(x)=\dfrac{x^T Ax}{x^T x}$, where T denotes taking transpose. Find the maximum and minimum of $a(x)$ for all nonzero vectors x in R^2.【91 中原電子 15%】

305. Let $A=\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$.

(1) What is the reduced row echelon form of A?
(2) What is the rank of the matrix A?
(3) Find a basis for the null space of A.
(4) Find all the solutions of $Ax=(1\quad 1\quad 1\quad 1)^T$.
(5) Without computing $A^T A$, determine the smallest eigenvalue of $A^T A$.【89 台大機械 25%】

306. An elastic membrane in the x_1x_2-plane with a boundary circle $x_1^2+x_2^2=1$ is stretched so that a point $P:(x_1,x_2)$ goes over into the point $Q:(y_1,y_2)$ given by $y=\begin{bmatrix}y_1\\ y_2\end{bmatrix}=Ax=\begin{bmatrix}5 & 3\\ 3 & 5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$. Please find certain point (x_1,x_2) at the circle, such that the values of $y_1^2+y_2^2$ is maximum.【90 成大醫工 20%】

307. Find the local extreme of the function:

$$f(x_1,x_2,x_3)=35-6x_1+2x_3+x_1^2-2x_1x_2+2x_2^2+2x_2x_3+3x_3^2$$

And justify that these extremes are local maximum or local minimum.【91 成大機械 20%】

308. Consider the function $F(x,y,z)=x^2+xz-3\cos y+z^2$.
(1) Find the Hessian of $F(x,y,z)$ at the point $(0,-\pi,0)$.
(2) Determine whether $(0,-\pi,0)$ is a saddle point of $F(x,y,z)$.【台大電機 16%】