第二類習題：拉氏轉換

1. 試求下列階梯函數之拉普拉斯轉換(Laplace transform) :

$$f(t)=\begin{cases}0, & 0\le 1\le 2\\ 1, & 2\le t\le 5\\ -1, & 5\le t\end{cases}$$

【92 中山環工 20%】

2. Find the Laplace transform of the staircase function which is formed by the successive addition of unit step functions as 0, b, $2b$, $3b$, $\ldots$, etc.【94 台大環工 15%】

3. 試求：(1) $\int_a^b \delta(t-t_0)f(t)dt$，$a<\mathrm{t}_0<\mathrm{b}$；(2) $L[\delta(t-t_0)]$，$t_0>0$；(3) $L\left[\delta(a-t)\right]$，$a>0$。【94 台大環工 15%】

4. Find the integral $\int_0^{10} e^x\delta[(x-1)(x-2)(x-3)]dx$.【88 清大物理 8%】

5. 試證：(1) $\Gamma(x+1)=x\Gamma(x)$ 或 $\Gamma(x)=\dfrac{1}{x}\Gamma(x+1)$；(2) $\Gamma(1)=1$；(3) $\Gamma(n+1)=n!$，$n=0, 1, 2, \cdots$。【91 元智工工 20%】

6. $\Gamma(x)=\int_0^\infty e^{-1}t^{x-1}dt$, then find $\dfrac{\Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{11}{3}\right)}$.【94 高科光電 10%】

7. If n is a positive integer and $x-n\neq 0,-1,-2,....$, evaluate $\dfrac{\Gamma(x+n)}{\Gamma(x-n)}$, where $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$ is the Gamma function.【93 清大物理 10%】

8. Evaluate (1) $\int_0^\infty e^{-ax^2}dx$ (2) $\int_0^\infty xe^{-ax^2}dx$ (3) $\int_0^\infty x^2e^{-ax^2}dx$, $a>0$.【92 交大應化 15%】

9. Solve $y''+2y'+2y=\delta(t-2)$, $y(0)=y'(0)=0$.【92 台科機械 10%】

10. Solve $y'+y=tu(t-3)$, $y(0)=3$.【92 中央環工 30%】

11. Solve (1) $I_1=\int_0^\infty (x+1)^2e^{-x^3}dx$ (2) $I_2=\int_0^\infty \frac{x^c}{c^x}dx$, $c<0$.【91 成大機械 8%】

12. We would like to evaluate an integral involving the derivative of the Dirac

δ-function. Find the general formula for $\int_{-\infty}^{\infty} x(t)\delta'(t\text{-}t_0)\,dt$.【91 中山電機 20%】

13. $\mathrm{f(t)}=\begin{cases}1, 0<t<1\\0, 1<t<2\end{cases}$, $\mathrm{f(t+2)=f(t)}$, find $\mathrm{L[f(t)]}$.【92 台北化工 15%】

14. Show that $\int_0^\infty e^{-x^4}dx=\left(\frac{1}{4}\right)!$.【92 中山材料 20%】

15. If $f(t)$ can be expressed as

$$f(t)=(t-1)[u(t-1)-u(t-2)]+[u(t-2)-u(t-4)]-(t-5)[u(t-4)-u(t-5)].$$

(1) Draw the figure of $f(t)$ versus t (use t as $x-axis$).
(2) Find the Laplace transform of $f(t)$.【92 海洋電機 10%】

16. 若$L[f(t)]=F(s)$，則$L[f(at)]=\frac{1}{a}F\left(\frac{s}{a}\right)$，$L^{-1}\left[F\left(\frac{s}{a}\right)\right]=af(at)$，$a<0$。【91 元智工工 10%】

17. Given that the Laplace transform $L\left\{\frac{2}{1}\left[1-\cos(t)\right]\right\}=ln\left(\frac{s^2+1}{s^2}\right)$, please find the

value of $L\left\{\frac{1}{t}[1-\cos t(2t)]\right\}$.【94 雲科電機 10%】

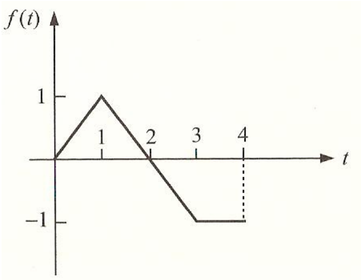
18. Solve $y''-6y'+9y=L[xe^{3x}]$, by Laplace transform.【94 高科電機 15%】

19. Solve $y''+2y'+5y=e^{-x}\cos x$, $y(0)=y'(0)=2$ using the Laplace Transform.【94 師大工數 15%】

20. Solve $y''+9y=f(t)$, $y(0)=y'(0)=1$, $f(t)=\cos t$, $t>\pi$.【94 成大電機 18%】

21. If $f(t)=\begin{cases} 0, & \mathrm{t}<0 \\ 2\mathrm{t}, & 0\le \mathrm{t}<3 \\ t^2, & 3\le \mathrm{t} \end{cases}$, find the Laplace transform of the given function $f(t)$.【94 中原機械 15%】

22. (1) Find the Laplace transform of the following function.



(2) Find the inverse Laplace transform of the following function:

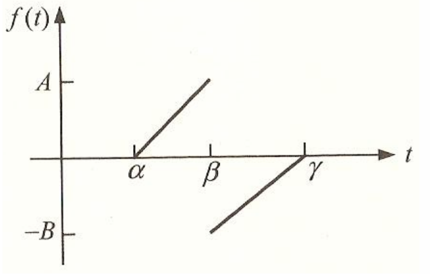
$$\frac{s+12}{s^2+10s+35}$$

【94 中山電機 10%】

23. Solve the following ordinary differential equation by using the Laplace transform: $y''+2y'+2y=\delta(t-3)$, $y(0)=y'(0)=0$, where δ is the Dirac delta function.【93 淡江電機 20%，93 台大化工 10%】

24. Solve $y''+3y'+2y=\begin{cases} 0, & t<0 \\ e^{-3t}, & t>0 \end{cases}$.【94 高科通訊 20%】

25. Write the function $f(t)$ whose graph is shown in the following figure in terms of the Heaviside function, and find its Laplace transform.



【93 高科機械 15%】

26. Solve $y''+3y'+2y=r(t)$, $r(t)=4t$, $0<t<1$, $r(t)=1$, $t>1$.【94 中山材料 16%】

27. Suppose $f(t)$ satisfies the difference-differential equation $\dfrac{df(t)}{dt}+f(t)-f(t-I)=0$, $t\geq 0$, and the initial condition, $f(t)=f_0(t)$, $-1\leq t\leq 0$ where $f_0(t)$ is given. Show that the Laplace transform of $f(t)$ satisfies

$$F(s)=\frac{f_0(0)}{1+s-e^{-s}}+\frac{e^{-s}}{1+s-e^{-s}}\int_{-1}^{0}e^{-u}f_0(t)dt$$

Find $f(t)$, $t\geq 0$ when $f_0(t)=1$.【93 交大電信】

28. Suppose Laplace transformation $F(s)=L\{f(t)\}$ exists for $s\geq a\geq 0$. Show that if a and b are constants with $a>0$, then inverse Laplace transformation

$$L^{-1}(F(as+b))=\frac{1}{a}e^{\frac{bt}{a}}f\left(\frac{t}{a}\right).$$

【91 中原物理 10%】

29. 一函數$f(t)$如圖所示，且已知$f(t)$之拉普拉斯(Laplace)轉換為 $L[f(t)]=F(s)$，求$L\left[f\left(\dfrac{t-a}{b}\right)\right]$，其中$a,b>0$，$a,b$均為常數，$f(t)=0$ for $t<0$。【90 台科營建 15%】

30. 使用 Laplace 轉換方法計算下述系統之反應$y(t)=?$

$$\frac{d^2y}{dt^2}+y=U\left(t^3-7t^2+14t-8\right)，y(0)=\frac{dy(0)}{dt}=0$$

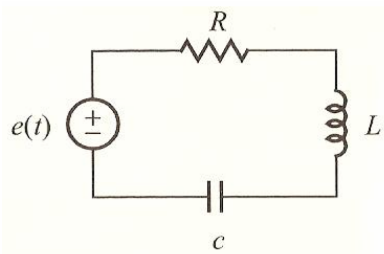
式中 U 為單位步階函數(unit step function)。【90 嘉義土木 15%】

31. By Laplace transform, solve $y''+4y'+4y=t^2e^{-2t}$, $y(0)=1$, $y'(0)=0$.【92 中正機械 15%】

32. Find $L[f(t)]$, $L[g(t)]$, $f(t)=\sin at\cosh bt$, $g(t)=t^2u(t-1)$.【92 海洋電機 15%】

33. Determine the current in the circuit:

$$R=20,\ L=0.1,\ c=1.5625\times10^{-3},\ e(t)=160t,$$
$$0<t<0.01,\ e(t)=1.6,\ t>0.01,\ i(0)=i'(0)=0.$$



【92 大同電機 18%】

34. Solve $y''+2y=r(t)$, $y(0)=y'(0)=0$, $r(t)=1$, $0<t<\pi$, $r(t)=0$, $\pi<t<2\pi$. $r(t)=\sin t$, $t>2\pi$.【92 台大電機 15%】

35. Find $L^{-1}\left[\dfrac{se^{-s}}{(s+1)^2(s^2+2s+2)}\right]$.【94 雲科電機 10%】

36. Given that $L\left[\dfrac{1}{t}\sin t\right]=\tan^{-1}\dfrac{1}{s}$, find $L\left[\dfrac{1}{t}\sin at\right]$.【94 大同電機 10%】

37. Find $L\left[te^{-t}\sinh 2t\right]$.【94 中山電機 20%】

38. Solve $y''+3y'+2y=f(t)$, $y(0)=y'(0)=0$, $f(t)=4t$, $0<t<1$, $f(t)=8$, $t>1$.【94 海洋電機 10%】

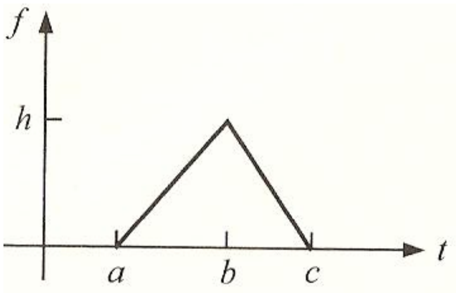
39. Solve $y''+4y'-21y=2e^{-2t}\sin 3t$, $y(0)=1$, $y'(0)=0$.【92 元智電機 20%】

40. Solve $y''+y'=1+\delta(t-2)$, $y(0)=0$, $y'(0)=3$.【93 台科化工 15%】

41. Solve $mx'' + cx' + kx = \alpha\delta(t)$, $x(0) = x'(0) = 0$, m, c, k, a are constants.【93 清大物理 20%】

42. Find $L[e^{-3t} f(t)]$, $f(t) = 0$, $t < 6$ and $f(t) = t^2 - 3$, $t \geq 6$.【93 清大電機 10%】

43. Write the function whose graph is shown below in terms of the Heaviside function, and find its Laplace transform.

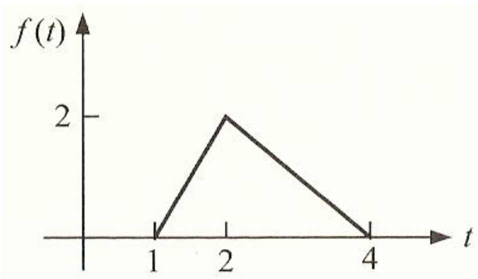


【92 高科機械 10%】

44. Consider the differential equation $\ddot{y}(t) + 2\dot{y} + 2y = f(t)$.

(1) Let $f(t) = e^{-t}\sin t + \cos 2t$. Solve the above differential equation.

(2) Let $f(t)$ be described as shown in figure. Solve the above differential equation.



【92 清大動機 20%】

45. Using Laplace transform to solve the following equation

$$\frac{dy(t)}{dt} + \int_0^t y(x)dx = tH(t-2),\ y(0) = 1,$$

where $H(t)$ is the Heaviside step function.【92 暨南土木 15%】

46. Using Laplace transform to solve the following equations for $y(t)$.

$$\frac{d^2y}{dt^2} + 4y = f(t)$$

where $f(t) = 1$ for $0 < t < 1$ and $f(t) = 0$ everywhere else. The initial condition for y are $y(0)=0$ and $\dfrac{dy(0)}{dt}=0$.【94 清大動機 15%】

47. Find the inverse transform of the function $\ln\left(1+\dfrac{\omega}{s^2}\right)$.【94 暨南電機 10%，94 高科機械 10%】

48. Find the inverse Laplace transform of the function $F(s)=\tan^{-1}\left(\dfrac{2}{s}\right)$.【92 雲科電機 10%】

49. Find $L[t\cos 2t]$.【94 清大電機 5%】

50. Find the Laplace transform of each of the following functions:

(1) $\cos(t)u(t-1)$

(2) $te^{-3t}\sin 2t$【92 海洋電波 10%】

51. 求 $F(s)=\dfrac{(s+2)^2-4}{\left[(s+2)^2+4\right]^2}$ 之反拉氏轉換 $F(s)=L^{-1}\left[\dfrac{(s+2)^2-4}{\left[(s+2)^2+4\right]^2}\right]$。【94 高應電機 16%】

52. Solve the differential equation $\ddot{x}+16x=f(x)$ with the initial values $x(0)=0$ and $\dot{x}(0)=1$, where $f(t)=\begin{cases}\cos(4t), & 0\le t\le\pi\\ 0, & \mathrm{t}\ge\pi\end{cases}$.【94 台科電機 15%，94 北科自動化 20%】

53. Find $L\left[\dfrac{1}{t}\left(e^{-at}-e^{-bt}\right)\right]$.【94 成大船舶 5%】

54. 求 $F(s)=\dfrac{(s+2)^2-4}{\left[(s+2)^2+4\right]^2}$ 之反拉氏轉換 $f(t)=L^{-1}\left[\dfrac{(s+2)^2-4}{\left[(s+2)^2+4\right]^2}\right]$。【94 高應電機 16%】

55. Solve the differential equation $\ddot{x}+16x=f(t)$ with the initial values $x(0)=0$

and $\dot{x}(0)=1$, where $f(t)=\begin{cases}\cos(4t), & 0\le t\le \pi \\ 0, & t\ge \pi\end{cases}$.【94 台科電機 15%，94 北科自動化 20%】

56. Find $L\left[\frac{1}{t}\left(e^{-at}-e^{-bt}\right)\right]$.【94 成大船舶 5%】

57. Using the Laplace transform to solve the given initial value problem.

$$y''+y=f(t),\ \ y'(0)=1,\ \ y(0)=0\text{, where }\ f(t)=\begin{cases}1, & 0\le t\le \pi/2 \\ \sin t, & t\ge \pi/2\end{cases}.$$

【94 中興機械 15%，93 台大電機 7%】

58. 若$L[f(t)]=F(s)$，$L[g(t)]=G(s)$，則試證：

$$L\left[\int_0^t f(t-\tau)g(\tau)d\tau\right]=L\left[\int_0^t f(\tau)g(t-\tau)d\tau\right]=F(s)G(s)$$

【94 元智機械 10%，93 北科機電 15%】

59. Find $\left(e^{-t}-e^{-2t}\right)*e^{-t}$.【94 清大電機 5%】

60. Apply the convolution of Laplace transform, find the solution of

$$y''+y=3\cos 2t\,;\ y(0)=0,\ \ y'(0)=0.$$

【93 中山物理 12%】

61. Solve the following differential equation by the method of Laplace transform.

$$\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=\frac{1}{1+t^2},\ \ x=0,\ \ \frac{dx}{dt}=0\ \text{ for }\ t=0.$$

【93 中興機械 10%】

62. Show that $y(x)=c_1\cos x+\sin x+\int_0^x f(s)\sin(x-s)ds$ is a general solution to the differential equation $y''+y=f(x)$, where $f(x)$ is a continuous function on $(-\infty,\infty)$.【92 交大電信 10%】

63. (1) Find the inverse Laplace transform of $\dfrac{1}{s\cosh(as)}$ where $\cosh(x)=\dfrac{e^{x}+e^{-x}}{2}$.

(Hint: $\cosh(z)=\cos(iz)$ where $i=\sqrt{-1}$)【93 台科化工 15%】

(2) Find $L^{-1}\left[\dfrac{1}{\sqrt{s}}\dfrac{1}{s-1}\right]$.【92 北科化工 10%】

64. Using the Laplace transform to solve Bessel's equation of order zero.

$$ty''+y'+ty=0,\quad y(0)=1.$$

【92 台科高分子 15%】

65. Find the Laplace transform of the given function:

$$\int_0^t \frac{\sin\tau}{(t-\tau)}d\tau$$

【93 交大機械 17%】

66. Solve the difference equation

$$3y(t)-4y(t-1)+2y(t-2)=t,$$

Using the Laplace transform if $y(t)=0$ for $t=0$.【93 清大動機 10%】

67. Find $\lim\limits_{t\to0^{+}}g(t)$ and $\lim\limits_{t\to\infty}g(t)$ if $L[g(t)]=\dfrac{16s^{3}+72s^{2}+216s-128}{(s^{2}+2s+5)^{2}}$.【94 暨南電機 15%】

68. (1) Let $y(t)$ be the solution of $\dfrac{d^{2}y}{d^{2}t}+w_0^{2}y=(A/m)\cos(\omega t)$, with

$y(0)=\dfrac{dy}{dt}(0)=0$. Assuming that $\omega\neq\omega_b$, find $\lim\limits_{\omega\to\omega_0}y(t)$.

(2) How does this limit compare with the solution of $\dfrac{d^{2}y}{d^{2}t}+\omega_0^{2}y=(A/m)\cos(\omega_0 t)$,

with $y(0)=\dfrac{dy}{dt}(0)=0$.【94 交大物理 25%，94 海洋電機 15%】

69. (1) Solve $y''+4y=f(x),\quad y(0)=y'(0)=0$.

(2) By convolution theorem, find $h(t)$ if $H(s)=L[h(t)]=\dfrac{1}{s(s-2)^2}$.【94 元智電機 10%】

70. By convolution theorem, find $L^{-1}\left[\dfrac{1}{(s^2+9)^2}\right]$.【94 中興材料 10%】

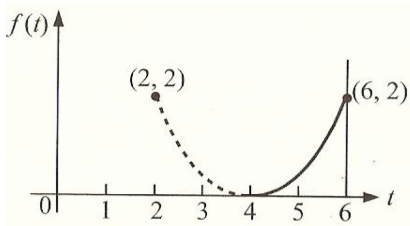
71. $L\left[\dfrac{1}{t}\left(1-e^{-t}\right)\right]=?$【94 清大材料】

72. Find $L^{-1}\left[\dfrac{s}{(s^2+a^2)^2}\right]$ by using of convolution theorem.【94 交大環工 12%】

73. $f(t)=t^2+2t+1$, $t\geq 5$, $f(t)=0$, else. Find $L[f(t)]$.【93 北科電機 15%】

74. Find $f(t)=L^{-1}\left[\dfrac{s^3}{s^4+4a^4}\right]$.【94 高應電子 15%】

75. 下圖函數$f(t)$，在$4<t<6$時為拋物線，其他時候為 0，求其拉氏轉換 Laplace transform $F(s)$。



【94 北科光電 10%】

76. Find $L^{-1}\left[\dfrac{s}{(s^2-1)^2}\right]$.【93 海洋電機 10%】

77. Solve $y''+2y'+2y=\delta(t-3)$, $y'(0)=0$.【93 淡江電機 20%】

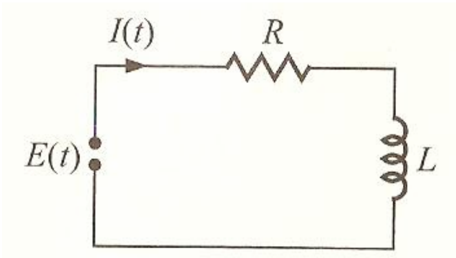
78. Solve $y''+2y'+2y=r(t)$, $r(t)=5\sin 2t$ if $0<t<\pi$ and 0 if $t>\pi$, $y(0)=1$, $y'(0)=-5$.【92 中山物理 15%】

79. Solve $y'+9\int_0^t y(t)\,dt=\cos 4t$, $y(0)=0$.【92 中正機械 20%】

80. Find $L^{-1}\left[\ln\dfrac{s^2+1}{(s-1)^2}\right]$.【92 高科電子 20%】

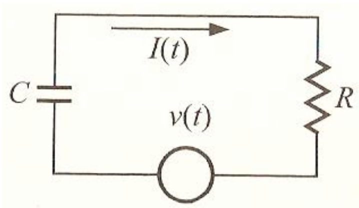
81. Find $L^{-1}\left[\dfrac{8k^3 s}{(s^2+k^2)^3}\right]$.【92 中正電機 8%】

82. Let $R=1$ ohm, $L=1$ henry, and the input $E(t)=1$ volt, when $0<t<3$ sec, and $E(t)=0$, when $t>3$ sec. Find the current $I(t)$, assuming $I(0)=0.5$ ampere.



【93 大同電機 18%】

83. Find the current $I(t)$ in the figure with $R=100$ ohms, $C=0.1$ farad and $v(t)=100$ volts if $1<t<2$ and 0 otherwise, $v_c(0)=0$.



【93 暨南電機 20%、93 師大電機 16%】

84. Find $L[f(t)]$, $f(t)=0$, $0<t<4$, $f(t)=e^{-3t}$, $4\le t<6$, and $f(t)=t+1$, $t\ge 6$.【93 雲科電機 15%】

85. Solve the following differential equation by Laplace Transformation:

$$ty''-2y'+ty=0,\quad y(0)=a.$$

【93 北科化工 20%】

86. Solve $y''-16ty'+32y=14$, $y(0)=y'(0)=0$.【94 雲科光電 15%】

87. Solve $y''+2ty'-4y=1$, $y(0)=y'(0)=0$.【94 成大製造 15%】

88. By applying the Laplace transformation technique to solve the following differential equation:

$$ty''+(4t-2)y'-4y=0,\quad y(0)=1,\quad y'(0)=-2.$$

Please derive its solution. Does there exist a unique solution?

【92 海洋機械 20%、92 台科電子 12%】

89. Using the Laplace transform to solve the equation $t\dfrac{d^2y}{d^2t}+(1-t)\dfrac{dy}{dt}+ny=0$ in which n is any positive integer.

$$(x+y)'' = \sum_{j=0}^{n}\binom{n}{j}x^{n-1}y^{j}$$

(Hint: The binomial expansion formula is useful.)

【95 清大電機 10%、94 海洋電機 20%】

90. Find the solution of a differential equation

$$t\frac{d^2y}{dt^2}-t\frac{dy}{dt}-y=0;\quad y(0)=0,\quad \frac{dy}{dt}(0)=5.$$

【94 台科化工 15%】

91. Given that $t(1-t)y''+2y'+2y=6t$; $y(0)=0$, $y(2)=0$. Please use the Laplace transform to solve the problem.【91 成大電機 20%】

92. Let $u(t)$ denote the unit step function, find the Laplace transform of the following function $f(t)=\sin\left[3\left(4t-\dfrac{\pi}{6}\right)\right]u\left(4t-\dfrac{\pi}{6}\right)$.【94 台科電機 10%】

93. Using Laplace transform to solve the following system equations:

$$\begin{cases} y_1''=-ky_1+k(y_2-y_1) \\ y_2''=-k(y_2-y_1)-ky_2 \end{cases}$$

with $y_1(0)=1,\ y_2(0)=1,\ y_1'(0)=\sqrt{3k},\ y_2'(0)=-\sqrt{3k}$.【93 成大醫工 20%】

94. Using Laplace transform to solve the following linear system:

$$\begin{cases} x'' - 2x' + 3y' + 2y = 4 \\ 2y' - x' + 3y = 0 \end{cases}$$

with the initial conditions $x(0) = x'(0) = y(0) = y'(0) = 0$.【93 海洋通訊導航 15%】

95. Using Laplace transform to solve the deflection $u(x)$ of a fixed-end beam of length l subjected to a concentrated loading P as shown in the following differential equation.

$$EI\frac{d^4u}{dx^4} = P\delta\left(x - \frac{l}{3}\right), \ \ 0 \le x \le l ,$$

with the boundary conditions $u(0) = u(l) = 0$ and $\frac{du(0)}{dx} = \frac{du(l)}{dx} = 0$, where $\delta(\ )$ is the Dirac delta function and the rigidity EI and P are constant.

【93 成大土木 20%】

96. The *RLC* in-series circuit with $R = -2(\Omega)$, $L = 1(H)$, and $C = 1/5(F)$. Using Laplace transform to solve the loop current, $i(t)$, which the initial conditions are $i(0) = 2(A)$, and $i'(0) = -4(A)$. 【94 海洋電機 10%】

97. Given a mass-spring-damper system, with unknown values of K and C, an impulse function $r(t) = \delta(t)$ generates an output response as $y(t) = e^{-t} - e^{-2t}$. Now if we are given another input function $r(t) = \sin t$, please find the corresponding output response.【94 中正光機電 10%】

98. The initial value problem is given by

$$2x'' + 8(x - y) = 0, \ \ z = x - y, \ \ x(0) = 2, \ \ x'(0) = 0,$$

$$y(t) = 2t^2 \ \text{ for } \ 0 < t < \frac{\pi}{6},$$

$$y(t) = \frac{\pi^2}{18} + \frac{2\pi}{3}\left(t - \frac{\pi}{6}\right) \ \text{ for } \ t < \frac{\pi}{6},$$

where $x = x(t)$, $y = y(t)$, $z = z(t)$ and their first derivatives are continuous functions of t. Determine $z(t)$ for $t \ge 0$ and evaluate $z(\pi/6)$, $z'(\pi/6)$, $z(\pi/3)$ and $z'(\pi/3)$.【95 交大機械 17%】

99. Using Laplace transform to solve the boundary value problem:

$$y'' - 2y' + y(x) = x,\quad y(0) = 0,\quad y'(1) = -2.$$

【95 清大工程科學 11%】

100. Solve $y'' + 2ty' - 4y = 1$, $y(0) = y'(0) = 0$.【91 台科電機 10%、92 中原化工 14%】

101. Consider the following differential equation

$$xy'' - 2(x-1)y' - (3x+2)y = 0$$

where $y(x)$ is piecewise continuous on $[0, \infty)$ and of exponential order for $t > T$.

(1) $Y(s)$ is the Laplace transform of $y(x)$. Please find $\lim_{s \to \infty} Y(s)$. Note that you have to present the calculation procedure to get the score.

(2) If $y(0) = \alpha$ and $y'(0) = \beta$, please find $Y(s)$ in terms of α and β.

(3) Find the inverse Laplace transform of $Y(s)$, i.e., $y(x)$.

(4) How many solutions do you get if α and β are given? Please explain why.

【91 台大電機 20%】

102. Solve $y'' - 4ty' + 4y = 0$, $y(0) = 0$, $y'(0) = 10$.【94 海洋機械 15%】

103. Find $L[y(t)]$, $y'' + y = e^{-t}\int_0^t t\sin 2t dt$.【94 成大機械 12%】

104. $y^{(4)} - 2y'' + y = 1$, $y(0) = y'(0) = y''(0) = y^{(3)}(0) = 0$. Solve by Laplace transform.【94 中央光電 10%】

105. Solve $y' + 2y + 6\int_0^1 z(t)dt = -2u(t)$, $y' + z' + z = 0$, $y(0) = -5$, $z(0) = 6$.【95 台科機械 20%】

106. Solve by Laplace transform:

$$y_1' = -y_2 + 1 - u(t-1),\quad y_2' = y_1 + 1 - u(t-1),\quad y_1(0) = y_2(0) = 0.$$

【93 中山材料 20%】

107. $x' - 4x + 2y = 2t$, $y' - 8x + 4y = 1$, $x(0) = 3$, $y(0) = 5$. Solve by Laplace transform.【93 中興化工 10%】

108. Solve $y''-8ty'+16y=3$, $y(0)=0$, $y'(0)=0$.【93 淡江機械 15%】

109. $xy''-2y'+xy=0$, $y(0)=0$. Solve by Laplace transform.【93 台大電機 15%】

110. By Laplace transform, solve $x'+x-y=2$, $y'-y+2z=0$, $z'+x-y=\cos t$, $x(0)=1$, $y(0)=0$, $z(0)=2$.【93 元智通訊 20%】

111. Solve $ty''+(t-3)y'+2y=0$, $y(0)=0$.【89 台科電子 10%】

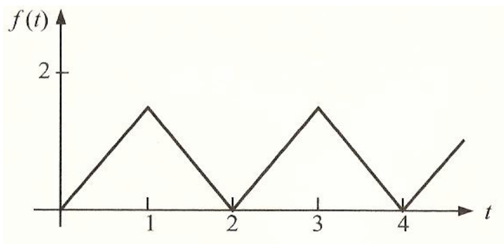
112. Using Laplace transform to solve
$y_1'-2y_2'+3y_3=0$, $y_1-4y_2'+3y_3'=t$, $y_1-2y_2'+3y_3'=1$.【92 成大電機 10%】

113. $y''+\omega_0^2 y=B\sin\omega t$, $y(0)=0$, $y'(0)=0$.

(1) $\omega_0 \neq \omega$

(2) $\omega_0 = \omega$ 【95 交大機械 20%】

114. Find the Laplace transform for the following periodic function.
(Note: In this problem, you should assign: $y=f(t)$, $x=t$ and $t\geq 0$.)



【94 中興化工 10%】

115. 單選題，每題恰有一解，答對一小題給 5 分，答錯或不答，不給分也不扣分。

(1) Define a function $g(t)$ by

$$g(t)=\begin{cases} t^2+n, \text{ if } t\geq 0 \text{ and } 3n\leq t<3n+3, \ n=0,1,2,\cdots \\ 0 \ , \text{ if } t<0 \end{cases}$$

What is the Laplace transform of $g'(t)$? (5%)

(A) $\dfrac{2}{s^2}$ (B) $\dfrac{2-e^{-s}}{s^2-s^2e^{-s}}$ (C) $\dfrac{2-s^2e^{-2s}}{s^2-s^2e^{-2s}}$ (D) $\dfrac{2-2e^{-3s}-s^2e^{-3s}}{s^2-s^2e^{-3s}}$ (E) none.

(2) Let $h(t)=L^{-1}\left\{\frac{(s^2+8)(e^{-s}-e^{-2s})}{s^3-2s^2-8s}\right\}$. $\lim_{t\to 1^+} h(t)=?$

(A) 0 (B) 1 (C) 2 (D) 3 (E) none 【94 交大電機 10%】

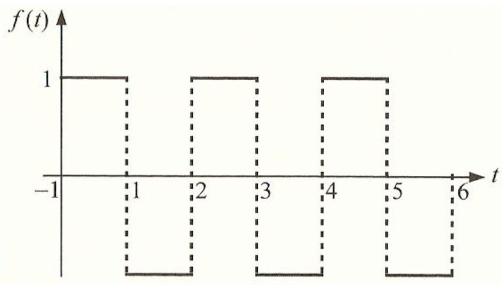
116. 求$|\sin\omega t|$之拉氏轉換$L[|\sin\omega t|]$。【93 北科電機 15%】

117. Given the periodic function $f(t)=\begin{cases}\sin t, & 0<t<\pi \\ 0, & \pi<t<2\pi\end{cases}$. Find the Laplace transform of $[f(t)]$.【94 北科電機 15%，94 清大微電機 10%】

118. Solve the following differential equation with initial conditions given

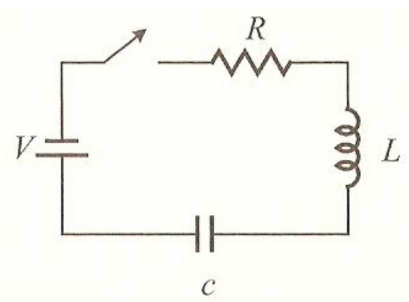
$$y''+2y+10y=f(t),\quad y(0)=1,\quad y'(0)=0,$$

where $f(t)$ is given by the following figure.



【94 中原機械】

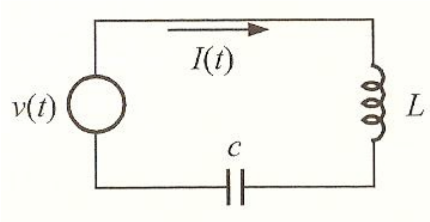
119. Consider the *RLC* circuit shown below. Initially there is no current in the circuit and no charge in the capacitor. At time $t=0$, the switch is closed and left closed for 1 second. At time $t=1$ second, the switch is opened and left open. Find the current in the circuit.



$R = 150\Omega$, $L = 1H$, $C = 0.0002F$, $V = 50V$.

【93 交大電子 12%】

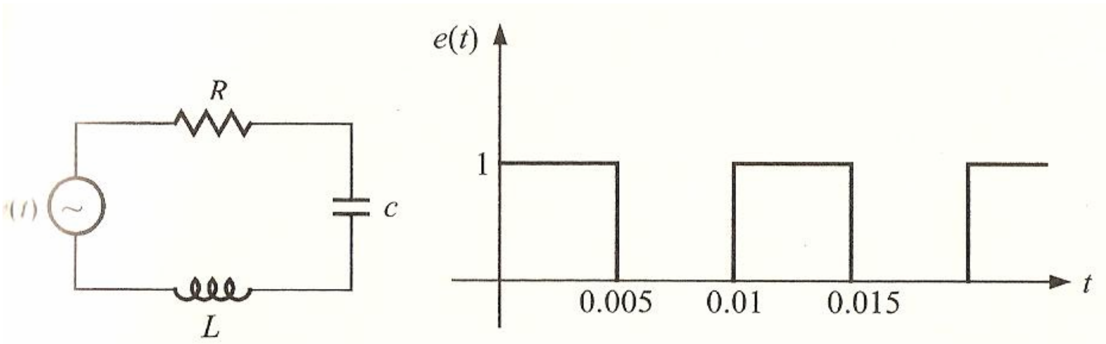
120. Find the current $I(t)$ in the figure with $L = 1$ Henry, $c = 1$ farad, zero initial current and charge on the capacitor, and $v(t) = t$ if $0 < t < 1$ and $v(t) = 1$ if $t > 1$.



【94 師大電機 15%】

121. Solve $y' + y = f(t)$, $t \ge 0$, $f(t) = \delta(t-3)$, $y(0) = 0$, $f(t) = f(t+5)$.【92 台大生物環境 15%】

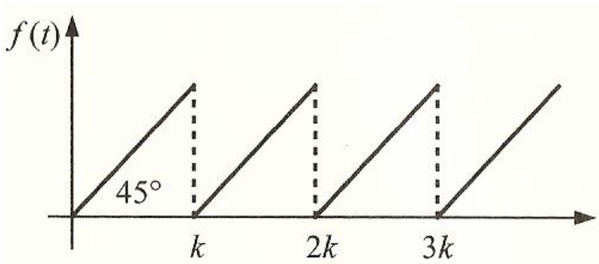
122. Find steady state current of the following circuit.



$R = 250$, $L = 0.02$, $c = 2\times10^{-6}$.【90 中興精密 20%】

123. (1) Find the Laplace transform of the function $f(t)$ as shown.

(2) What is the solution of the equation, if $y_0 = 1$ ant if $f(t)$ is given as in figure with $k = 1$?



【90 中興精密 20%】

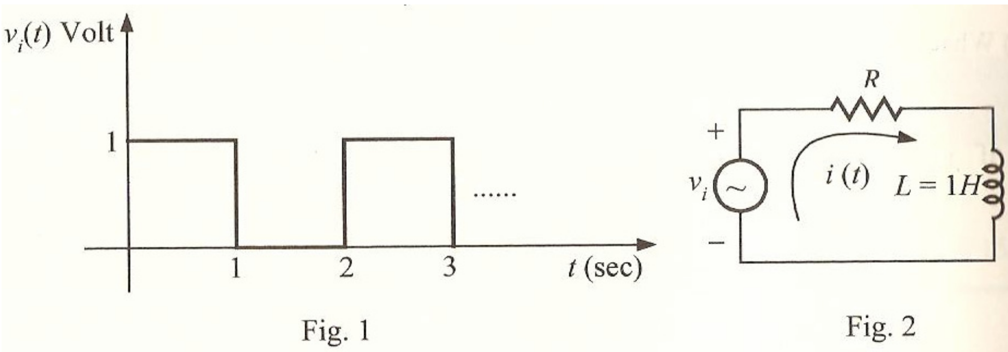
124. Find $L[f(t)]$, $f\left(t+\dfrac{2\pi}{\omega}\right)=f(t)$, $f(t)=0$, $0<t<\dfrac{\pi}{\omega}$, $f(t)=-\sin\omega t$, $\dfrac{\pi}{\omega}<t<\dfrac{2\pi}{\omega}$.【91 暨南電機 10%】

125. (1) Find a Laplace transform of the given infinite-duration pulses in Fig.1.

(2) For a first-order RL circuit in Fig.2 if $v_t(t)$ in part2 is used as an input, using the Laplace transform method, show that

$$i(t)=\sum_{n=0}^{\infty}(-1)^n u(t-n)-\sum_{n=0}^{\infty}(-1)^n e^{-(t-n)}u(t-n)$$

where $u(t)$ is a unit step function.



【91 交大電子 12%】

126. Solve $f(t)=2t^2+\int_0^t \sin(4\tau)f(t-\tau)\,d\tau$.【93 清大電子 7%、93 台大工程科學 20%】

127. Solve the following integral equation:

$$\varphi(t)+\cos t\int_0^t \varphi(\tau)\cos\tau\,d\tau+\sin t\int_0^t \varphi(\tau)\sin\tau\,d\tau=\sin 2t$$

【94 暨南電機 15%】

128. Solve the following integral equation

$$y(t)=\sin(2t)+\int_0^t y(\tau)\sin\big(2(t-\tau)\big)d\tau.$$

【94 中央電機 10%】

129. Prove that the Beta function:

$B(m,n)=\int_0^t x^{m-1}\cdot(1-x)^{n-1}\,dx=\dfrac{\Gamma(m)\cdot\Gamma(n)}{P(m+n)}$, $m>0$, $n>0$, and Γ is Gamma function.【91 北科機電 15%】

130. Find $L\left[e^{-2t}\int_0^t e^{2a}\cos 3\alpha d\alpha\right]$ by methods below.

(1) Convolution theorem. (10%)

(2) Using $L\left[e^{at}f(t)\right]=F(s-\alpha)$ and $L\left[\int_0^t f(\alpha)d\alpha\right]=\frac{1}{s}F(s)$. (15%)

【94 元智電機 25%】

131. Solve $y(t)=2-3e^{-t}-\int_0^t e^{t-\alpha}y(\alpha)d\alpha$.【94 宜蘭電機 10%】

132. Solve $f(t)=6t^2+\int_0^t f(t-\alpha)e^{-\alpha}d\alpha$.【94 淡江電機 10%、93 台科機械 20%】

133. Find $y(t)$, $y(t)=\sin t+4e^{-t}-2\int_0^t y(\alpha)\cos(t-\alpha)d\alpha$.【93 交大應化 10%】

134. Solve $y(t)=6t+\int_0^t y(t-s)\sin s ds$.【93 清大微機電 10%】

135. $y(t)=1-\sinh+\int_0^t(1+\alpha)y(1-\alpha)d\alpha$ ，求 $y(t)$ 。【93 台大生機 10%】

136. Find $y(t)$, $y(t)=\cos t+e^{-2t}\int_0^t f(\alpha)e^{2\alpha}d\alpha$.【92 中興物理 15%】

137. Solve $y''+(a+b)y'+aby=f(t)$, $y(0)=c$, $y'(0)=d$.【92 成大工程科學 15%】

138. Find $y(t)$, $y=te^t-2e^t\int_0^t e^{-\alpha}y(\alpha)d\alpha$.【92 成大製造 15%】

139. Find $f(t)\neq g(t)$, $f(t)=t$, $g(t)=e^{2t}$.【92 嘉義電機 20%】

140. Solve $y''+y-4\int_0^t y(\alpha)\sin(t-\alpha)d\alpha=e^{-2t}$, $y(0)=1$, $y'(0)=1$.【92 暨南電機 10%】

141. (1) Derive $L[t\sin\beta t]=\dfrac{2\beta}{\left(s^2+\beta^2\right)^2}$ by using $L[tf(t)]=-\dfrac{d}{ds}L[f(t)]$.

(2) Solve $y=2t-4\int_0^t y(\alpha)(t-\alpha)\,d\alpha$.【95 交大機械 10%】

142. Solve by Laplace transform $x\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial t}=xt$, $u(x,0)=0$, $u(0,t)=0$.【94 中山光電 15%】

143. A semi-infinite string at rest along the positive axis with the left end moving in a prescribed fashion. The displacement of the string can be described as following:

$$\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2},\quad y(0,t)=\begin{cases}\sin(2\pi t), & 0\le t\le 1\\ 0, & t>1\end{cases}$$

$$y(x,0)=\frac{\partial y}{\partial t}(x,0)=0\ \ (x>0)$$

Pease find $y(x,t)$ using the Laplace transformation.【93 中興材料 20%】

144. Solve $\dfrac{\partial v}{\partial x}+2x\dfrac{\partial v}{\partial t}=2x$, $v(x,0)=1$, $v(0,t)=1$, by Laplace transform.【94 北科化工 20%】

145. By Laplace transform, solve

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\quad x>0,\ t>0,$$

$$u(x,0)=100,\ u(\infty,t)=100,\ u(0,t)=20,\ 0<t<1,\ u(0,t)=0,\ t>1.$$

Notice: The Laplace transform of complementary error function is

$$L\left[erfc\left(\frac{a}{2\sqrt{t}}\right)\right]=\frac{1}{s}e^{-a\sqrt{s}}.$$【92 淡江化工 25%】

146. Solve $a^2\dfrac{\partial^2 u}{\partial x^2}=\dfrac{\partial^2 u}{\partial t^2}$, $0<x$, $0<t$, $u(x,0)=0$, $u_t(x,0)=0$, $u(\infty,t)$ is finite.
【93 淡江航空 25%】

147. Solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$, $0<x<\pi$, $t>0$, $u(0,t)=u(x,t)=0$, $u(x,0)=\sin x$.【92 中原電機 10%】

148. Solve the following boundary value problem by Laplace transform.

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}(x>0,t>0)$$

$$u(x,0)=A(x>0),\ u(\infty,t)\ \text{ is finite}$$

$$u(0,t)=\begin{cases} B,\ for\ \ 0<t<t_0 \\ 0,\ for\ \ t>t_0 \end{cases}$$

Note: The Laplace transform of $erfc\left(\frac{a}{2\sqrt{t}}\right)$ is $\frac{1}{s}e^{-a\sqrt{s}}$, $erfc(x)=1-efc(x)$.【90 北科光電 20%】

149. Solve $a^2u_{xx}=u_{tt}$, $x>0$, $t>0$, a is a positive constant, $u(x,0)=0$, $u(x,0)=k$, $u(0,t)=0$, $u(\infty,t)$ is bounded, solve u.【94 淡江航空 15%】

150. Find the solution $p(x,t)$ of the following partial differential equation:

$$\frac{\partial^2 p}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}=\delta(x-at),\ \ c>a>0,\ \ 0\le x\le\infty,\ \ 0\le t\le\infty.$$

$p(x,0)=0$, $\partial p(x,0)/\partial t=0$, $p(0,t)=0$, $p(x,t)<\infty$ as $x\to\infty$.
Note: δ denotes the Dirac delta function.【88 交大機械 20%】

151. Using Laplace Transformation (with respect to t) to solve the partial differential equation.

$y_u(x,t)=a^2y_{xx}(x,t)-g$, $x\to\infty$ where a and g are constants.

And $y(x,t)$ satisfies the boundary conditions $y(x,0)=y_t(x,0)=0$, $y(0,t)=0$, $\lim_{x\to\infty}y_x(x,t)=0$.【89 逢甲土木 15%】

152. Solve $\frac{\partial u}{\partial t}+a\frac{\partial u}{\partial z}=-ku$, $u(t,0)=b\sin\omega t$, $u(0,z)=0$.【92 台科化工 15%】

153. Calculate Laplace transforms of real-valued, square-wave function $f(t)$ with period $2\times c$, where

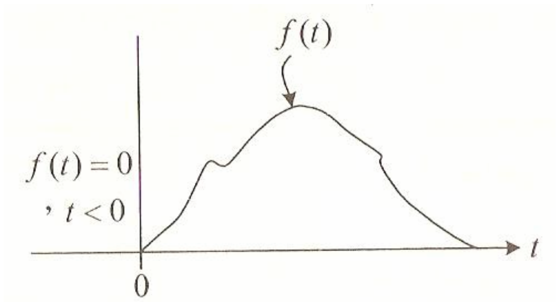
$$f(t)=0 \text{ if } t<0$$
$$f(t)=1 \text{ if } nc\le t\le (n+1)c \text{, and}$$
$$f(t)=-1 \text{ if } (n+1)c<t<(n+2)c \text{, for } n=0,2,4...$$

【89 交大機械 16%】

154. 求$\int_{-\infty}^{\infty}\delta(at-b)\cdot x(t)dt$。【88 台科電子 5%】

155. Find the integral $\int_{0}^{10}e^{x}\delta\left[(x-1)(x-2)(x+3)\right]dx$.【88 清大物理 8%】

156. 一函數$f(t)$如圖所示，且已知$f(t)$之拉普拉斯(Laplace)轉換爲 $L[f(t)]=F(s)$，求$L\left[f\left(\frac{t-a}{b}\right)\right]$，其中$a,b>0$ ，a,b均爲常數。



【90 台科營建 15%】

157. Find $L[\sin at\cdot\cosh at-\cos at\cdot\sinh at]$.【90 海洋光電 10%】

158. By Laplace transform, solve $y''+8y'+16y=t^{2}e^{-4t}$ ，$y(0)=1$ ，$y'(0)=-4$.【90 台大電機 10%】

159. Using the Laplace transform to solve the following differential equation.

$$y''+4y=\begin{cases}0, & 0\le t\le\pi\\ 3\cos(t), & t\ge\pi\end{cases},\quad y(0)=y'(0)=1.$$

【90 雲科電機 10%】

160. Using the Laplace transform to solve the following initial value problem:

$$y'' - 4y' + 4y = \delta(t-1),\ \ y(0) = 0\ ,\ \ y'(0) = -1\ .$$

【90 中山電機 20%】

161. $y'' + 5y' + 6y = 8\delta(t),\ \ y(0) = 3\ ,\ \ y'(0) = 0\ .$
    (1) Find solution of $y(t)$.
    (2) What are $y(0),\ y'(0)$? Using this information, what physical phenomena does delta function model?【90 海洋機械 14%】

162. (1) Solve $y' + 2y = 3u' + 2u$, $y(0) = 2$, where $u(t)$ is a unit step function.
    (2) Check the solution of part (1) if $y(0) = 2$. If not, try to explain.【90 交大機械 25%】

163. (1) From the properties of Dirac delta function, expand $\delta(1-4t^2)$ as the sum of delta functions with simple argument; that is, find the parameters A_n and a_n such that $\delta(1-4t^2) = A_1\delta(t-a_1) + A_2\delta(t-a_2) + \ldots$ holds.
    (2) Solve the following equation by Laplace transform:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = \delta(1-4t^2),\ \ y(0) = 0\ ,\ \ y'(0) = 0\ .$$

    (3) Is the solution $y(t)$ in (2) continuous at $t = \frac{1}{2}$? If not, how are $y(t^+)$ and $y(t^-)$ related? Explain how you can figure out this relationship simply from the equation itself without actually solving for the solution.【88 清大電機 15%】

164. Show that the Laplace transform of $\ln(t)$ is $L[\ln(t)] = \dfrac{\Gamma'(1) - \ln s}{s}$ where the Gamma function is defined as $\Gamma(r) = \int_0^\infty u^{r-1}e^{-u}du$.【87 清大動機 10%】

165. Solve the equation $y'' + 4y' + 4y = f(t),\ \ y(0) = 1\ ,\ \ y'(0) = 2\ .$

$$f(t) = 1,\ \ 0 \le t \le 2$$
$$f(t) = 0,\ \ t \ge 2$$

【90 北科化工 20%】

166. Find the inverse Laplace transform of $\dfrac{e^{-3s}}{(s-3)^2} + \dfrac{e^{-3s}}{(s^2+4s+13)}$.【88 台科電子

10%】

167. Solve the initial value problem:

$y'' + y' + \frac{5}{4}y = g(t)$, $y(0) = 0$, $y'(0) = 0$ where $g(t) = \begin{cases} 1, 0 \le t \le \pi \\ 0, t \ge \pi \end{cases}$.【89 交大土木 25%】

168. Find $f(t)$ if $F(s) = \dfrac{s^2 + 2s}{(s^2 + 2s + 2)^2}$.【90 彰師電機 10%】

169. Find $L^{-1}\left[\dfrac{3s+5}{s(s^2+2s+5)}e^{-3s}\right]$.【90 中原電機 15%】

170. Solve $y'' + 2y' + y = \delta(t-1)$, $y(0) = 2$, $y'(0) = 3$.【90 中山電機 10%】

171. Using Laplace transform, solve

(1) $y'' + 5y' + 6y = u(t-1) + \delta(t-1)$, $y(0) = 0$, $y'(0) = 1$.

(2) $y'' - 2y' + 2y = 8e^{-t}\cos t$, $y(0) = 16$, $y'(0) - 16$.【90 元智機械 20%】

172. Solve the following initial-value ordinary differential equation.

$$\frac{d^2y}{dt} + 2\frac{dy}{dt} + 5y = h(t)$$

$$h(t) = 1,\ 0 < t < \pi$$

$$h(t) = 0,\ t > \pi$$

$$y(0) = y'(0) = 0$$

【90 中興化工 15%】

173. Solve $x'' + 4x = \delta''(t)$.【90 交大環工 15%】

174. Solve $y'' + 2y' + 5y = e^{-x}\cos x$, $y(0) = y'(0) = 2$, using the Laplace Transform.【88 北科電力能源 20%】

175. Using the Laplace Transform to solve $y'' + 2y' + 2y = \cos t\,\delta(t - 3\pi)$ subject to $y(0) = 1$ and $y'(0) = -1$, where $\delta(t)$ is the Dirac delta function.【87 元智化工 15%】

176. Solve the initial vale problem

$$y'' - 4y = f(t);\ y(0) = y(0) = 0$$

where $f(t)=0$ if $t<3$ and $f(t)=t$ if $t\ge 3$.【90 彰師機械 20%】

177. Solve $y''+5y'+4y=r(t)$, $r(t)=1$ if $0<t<1$ and 0 if $t>1$; $y(0)=0$, $y'(0)=1$.【90 中興材料 20%】

178. Find the Laplace transform of the function $f(t)=te^{-2t}\cos(3t)$.【90 雲科電機 10%】

179. 求 $L^{-1}\left[\cot^{-1}\dfrac{s}{\pi}\right]$。【89 高科環安 10%】

180. Given $L(f(t))=F(s)$, use the convolution theorem to show that

$$L^{-1}\left(\frac{1}{s^2}F(s)\right)=\int_0^t\int_0^r f(a)\,dadr$$

where L is Laplace transform and L^{-1} is the inverse Laplace transform.【88 台科電機 10%】

181. Find integral of $z(t)=\int_0^t e^{-m(t-a)}\sin(t-\alpha)d\alpha$.【89 中興土木 20%】

182. Find Laplace transform of $f(t)=\tan\omega t$.【中興材料 6%】

183. 若$L[f(t)]=F(s)$，則$\lim\limits_{t\to\infty}f(t)=\lim\limits_{s\to 0}sF(s)$。【88 台科電子 10%】

184. Using the Laplace transform to solve Bessel's equation of order zero.

$$ty''+y'+ty=0,\quad y(0)=1.$$

【91 交大機械 15%，89 成大電機 10%】

185. Find the step and impulse response for the equation.

$$2x''(t)+4x'(t)+10x(t)=f(t),$$

where $f(t)=\delta(t)$ and $u(t)$, respectively. The initial conditions at $t=0$ are $x(0)=x'(0)=x''(0)=0$.【90 交大電信 20%】

186. Find convolution of $u(t-\pi)*\cos t$.【89 雲科電機 10%】

187. Solve the following equation by Laplace Transform method, and state the advantage of this method.

$$y'+3y+2\int_0^t y\,dt = 2u(t-1) \quad \text{where} \quad y(0)=1 \quad \text{and} \quad u(t)=\begin{cases} 0 & t<0 \\ 1 & t\geq 0 \end{cases}.$$

【88 台科控制 20%】

188. Find $L^{-1}\left[\ln\dfrac{s^2+1}{s^2+s}\right]$.【89 雲科電機 10%】

189. $L\left\{\dfrac{\sin kt - kt\cos kt}{2k^3}\right\}=?$

(1) $\dfrac{1}{s^2+k^2}$ (2) $\dfrac{1}{(s^2+k^2)^2}$ (3) $\dfrac{1}{(s^2+k^2)^3}$ (4) $\dfrac{1}{s^2-k^2}$ (5) $\dfrac{1}{(s^2-k^2)^2}$

【87 台大電機 5%】

190. Solve $y''+9y=f(x)$, $y(0)=2$, $y'(0)=1$.【90 中興環工 10%】

191. 求 $L\left[t^2\sin\omega t\right]$ 。【88 交大機械 5%】

192. Solve $y''+4y=\sin t\,u(t-2\pi)$, $y(0)=1$, $y'(0)=0$.【89 中山電機 15%】

193. Find $L\left[\int_0^t \dfrac{\sin au}{u}du\right]$.【87 清大物理 5%】

194. Find $\displaystyle\int_0^\infty \frac{e^{-t/\sqrt{3}}\sin t}{t}dt$.【87 台科電子 15%】

195. Find: (1) $L\left[\dfrac{e^t n}{n!}\dfrac{d^n\left(t^n e^{-t}\right)}{dt^n}\right]$ (2) $L[c^t]$ (3) $L^{-1}\left[\dfrac{s}{s^3+1}\right]$.【87 中原電機 20%】

196. Find $L^{-1}\left[\dfrac{1}{(s^2+a^2)^2}\right]$.【90 中央土木 25%】

197. Find $\cos at * \frac{1}{a}\sin at$.【89 交大機械 16%】

198. Find the inverse Laplace transform $F(s)=\frac{2se^{-2s}}{\left(s^2+4\right)^2}$.【88 雲科電機 10%】

199. (1) Using the convolution formula to find the inverse of the following Laplace transform $H(s)=\frac{1}{s^2\left(s^2+1\right)}$.

(2) Find the Laplace transform of $g(t)=\sin(\omega t+v)$, where ω and v are constants.【88 中央電機 10%】

200. Solve $y''-2y'+2y=0$, $y(0)=-3$, $y\left(\frac{1}{2}\pi\right)=0$ by Laplace transform method.【90 中正電機 10%】

201. Solve $ty''+(t-1)y'+y=0$, $y(0)=0$.【90 台科高分子 3%】

202. Solve the following problem using the Laplace transform.

$$ty''+(t-3)y'+2y=0,\quad y(0)=0.$$

【89 台科電子 10%】

203. Solve $y''+ty'-y=0$, $y(0^+)=0$, $y'(0^+)=1$.【90 中興機械 15%】

204. Let $\delta(t)$ denote the Dirac delta function, what are values of $y(0^+)$ and $y'(0^+)$ for the following initial value problem?

$$y''+4ty'-4y=3\delta(t);\quad y(0)=0,\quad y'=-7.$$

【87 台大機械 10%】

205. Using the Laplace transform only to solve the differential equation $y''+16y=\cos(4t)$ with the initial conditions $y(0)=0$ and $y'(0)=1$.【90 交大物理 15%】

206. 強迫振子(forced oscillator)的牛頓運動方程式為：

$$m\ddot{y}+ky=F_0\cos\omega t\,,\left(F_0>0,\omega>0\right)$$

初始條件為 $y(0)=0$ ， $\dot{y}(0)=0$ 。

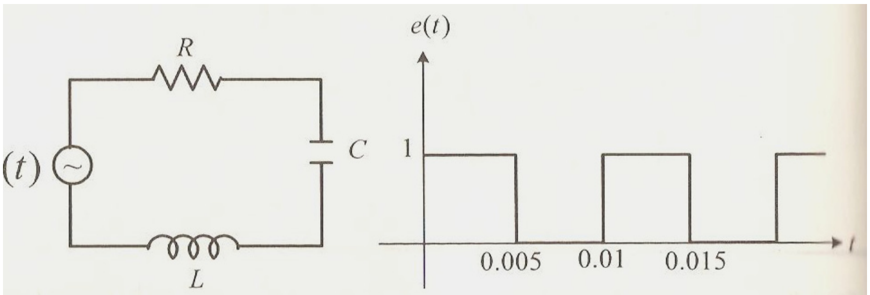
(1) 求解 $y(t)=?$

(2) 當 $\omega\to\omega_0\left(\equiv\sqrt{\dfrac{k}{m}}\right)$，繪 $y(t)$ 之橢圓。【90 交大電務 20%】

207. Using the Laplace transform method to solve the following ordinary differential equation. $ty''+(4t-2)y'-4y=0$, $y(0)=1$.【88 交大機械 10%】

208. 試證明：若 $f(t+T)=f(t)$，則 $L\left[f(t)\right]=\dfrac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt$ 。【87 台科控制 10%】

209. Find steady state current of the following circuit.



where $R=250$, $L=0.02$, $C=2\times10^{-6}$.【90 中興精密 20%】

210. (1) Find the Laplace transform of the function $f(t)$ as shown in Fig 1.

(2) What is the solution of the equation? If $y(0)=1$ and if $f(t)$ is given as in Fig. 1 with $k=1$?

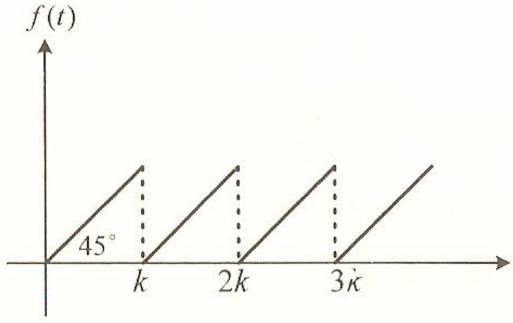


Fig. 1

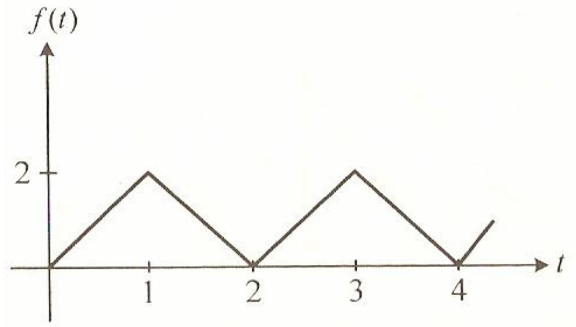
【90 中興精密 20%】

211. Solve the differential equation by means of the Laplace transformation.

$$y'' + 2y' + 10y = r(t)$$

where $r(t) = \begin{cases} 1 \ (0 < t < \pi) \\ -1 \ (\pi < t < 2\pi) \end{cases}$, $r(t + 2\pi) = r(t)$.【90 中央電機 15%】

212. Find the Laplace transform for the following periodic function.
(Note: In this problem, you should assign: $y = f(t)$, $x = t$ and $t \geq 0$.)



【90 中原醫工 20%】

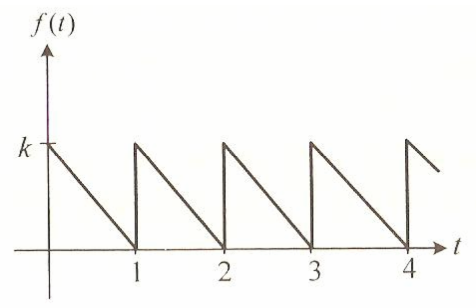
Find $L^{-1}\left[\frac{k}{ps^2} - \frac{ke^{-ps}}{s(1-e^{-ps})}\right]$, both p and k are constants, and plot the curve.

【89 中央電機 5%】

213. Consider the Integral-differential equation,

$$y'(t) + 2\int_0^t y(\tau)\cdot\cos(t-\tau)\,d\tau = f(t), \quad y(0) = 0,$$

where $f(t)$ is shown as below.



(1) Find the Laplace transform of $f(t)$.

(2) Take the Laplace transform of the equation and find the associated

$Y(s)=L[y(t)]$.

(3) $y(t)$ maybe written as $y(t)=\sum_{n=0}^{\infty}z(t-n)$, find $z(t)$.【86 台大電機 30%】

214. A spring-mass system is excited by the function of

$$f(t)=\begin{cases} 20 & ,0<t<1 \\ -20 & ,1<t<2 \end{cases}$$

with $f(1+2)=f(t)$. Find the solution of transient response together with steady state response with $mass=1$, dashpot $c=3$ and spring rate $k=2$.【90 彰師機械 20%】

215. Find the Laplace transform of the function $f(t)=|\sin \alpha t|$, $\alpha>0$.【87 中山機械 10%，88 中山電機 15%】

216. Let $L\left[f(t)\right]=\int_0^{\infty}e^{-st}f(t)\,dt$.

(1) For what value of s is $L\left[1-e^{-2t}\right]$ convergent.

(2) Given $L[f(t)]=\dfrac{1}{(s^2+1)\cdot(1-e^{-\pi s})}$, is $f(t)$ a periodic function of period $T=\pi$? What is $\int_0^{T}e^{-st}f(t)\,dt$?【86 台大機械 13%】

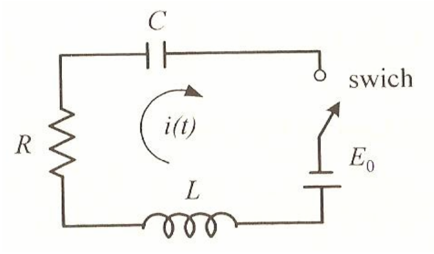
217. $y''+4y=f(t)$, $y(0)=0$, $y'(0)=0$, $f(t)=\begin{cases} 1 & ,0\le t\le \pi \\ -1 & ,\pi\le t<2\pi \end{cases}$, $f(t+2\pi)=f(t)$.

Solve the initial value problem.【86 成大土木 16%】

218. Consider an RLC circuit in series with a battery, with

$$R=60\Omega,\ C=10^{-3}\text{F},\ L=1H,\ E_0=10V.$$

Suppose the switch is alternate closed and opened at times $t=0$, 0.1π, 0.2π, 0.3π …, and $i(0)=i'(0)=0$. Derive the current $i(t)$, $(0.1)n\pi<t<0.1(n+1)\pi$.



【86 交大電子 11%】

219. $f(x)=\begin{cases}1 & for\ \ 0\le x<1\\ 0 & for\ \ 1\le x<2\end{cases}$ and $f(x+2)=f(x)$, find Laplace transform of $f(x)$ and solve $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+2y=f(x)$, $y(0)=0$, $y'(0)=0$.【88 清大電機 10%】

220. Find the steady state current in the circuit by Laplace transform $V(t)=t$, $0<t<1$, and $V(t+1)=V(t)$.【86 中山光電 10%】

221. (1) If $\dfrac{\partial^2 u}{\partial t^2}=\dfrac{\partial^2 u}{\partial x^2}$, $0\le x\le\infty$ is one sided that $v(t)=0$ for $t<0$ and $v(t)=V_0$ for $0<t<a$, and 0 if $a<t<2a$, and $v(t+2a)=v(t)$ for $t>0$. Find the Laplace transform of $v(t)$.
(2) If $y(t)$ satisfies the differential equation: $y'+y=v(t)$, and $y=0$ for $t\le 0$. Find the steady-state solution of y by Laplace transform method.【90 清大電子，電機】

222. Consider the circuit in Fig.1, the voltage-source $v_s(t)=Vu(t)$, where $V=10(V)$ and $u(t)$ is unit step function, initial current $i_1(0)=0(A)$. Derive and solve the $i_1(t)$ by differential equation.

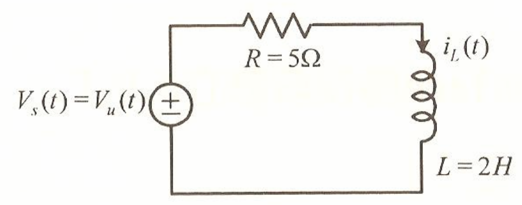


Fig. 1

【90 北科通訊 15%】

223. Given $y_1'' = -ky_1 + k(y_2 - y_1)$ ……………………………………………………...(a)

$y_2'' = k(y_2 - y_1) - ky_2$ …………………………………………………………(b)

where y_1, y_2 are functions of t and k which is a constant. The initial conditions are $y_1(0)=1$, $y_2(0)=1$, $y_1'(0)=(3k)^{1/2}$, $y_2'(0)=-(3k)^{1/2}$. Solve for Equations (a) and (b) using Laplace transformation method.【90 台大環工】

224. Solve

$$x_1'' = \frac{13}{2}x_1 + \frac{5}{2}x_2 + 2[1 - H(t-3)],$$

$$x_2'' = \frac{5}{2}x_1 - \frac{13}{2}x_2,$$

$$x_1(0) = x_2(0) = 0,\ x_1'(0) = x_2''(0) = 0.$$

【91 北科冷凍 15%，90 海洋光電 15%】

225. Using the Laplace transform to solve the system.

$x' + 2x - y' = 0$, $x' + y + x = t^2$; $x(0) = y(0) = 0$.【90 成大微電子 15%】

226. Using the Laplace transform to solve the given system of differential equations.

$$x'' + x' + y' = 0,$$
$$y'' + y' - 4x' = 0,$$

and $x(0)=1$, $x'(0)=0$, $y(0)=-1$, $y'(0)=5$.【89 台大化工 10%】

227. Solve the initial value problem using the method of Laplace transformations:

$$\begin{cases} \dfrac{dx}{dt} = 2x - 3y \\ \dfrac{dy}{dt} = y - 2x \end{cases} \text{ subject to } x(0)=8,\ y(0)=3.$$

【88 中央化工 15%，88 交大電子 11%】

228. Using the Laplace transform method to solve the simultaneous differential equation.

$$\frac{4d^2u}{dt^2} + \frac{d^2v}{dt^2} - v = 0,\ \frac{d^2u}{dt^2} - u - v = 0,$$

where $u(0)=1$ and $u'(0)=v(0)=v'(0)=0$.【88 成大土木 20%】

229. Solve

$$\frac{d^2y}{dt^2}-2\frac{dx}{dt}+3\frac{dy}{dt}+2y=4,$$

$$2\frac{dy}{dt}-\frac{dx}{dt}+3y=0,$$

$$x(0)=y(0)=\frac{dx(0)}{dt}=0.$$

【90 台科營建 30%】

230. (1) What is the general procedure for solving an ordinary differential equation with Laplace Transforms?

(2) Please solve the following ordinary differential equation with Laplace Transforms.

$$\frac{dx}{dt}+\frac{dy}{dt}+x=-e^{-t},$$

$$\frac{dx}{dt}+2\frac{dy}{dt}+2x+2y=0,$$

Initial conditions: $x(0)=-1$, $y(0)=1$.

【90 成大機械 10%】

231. Please solve the following integral equation:

$$y(t)=\sin t+\int_0^t y(T)\sin(t-T)\,dT$$

【90 清大通訊、電機 7%】

232. 求解積分方程式 $f(t)=3t^2-e^{-t}-\int_0^t f(s)e^{t-s}ds$ 。【90 北科化工 15%】

233. $\omega(t)=f(t)+\int_0^t f(t-u)v(u)\,du$; $v(t)=\int_0^t g(t-u)\omega(u)\,du$,

$$Q(t)=\int_0^t \omega(t)\,dt-\int_0^t v(t)\,dt;\quad f(t)=\lambda e^{-u};\quad g(t)=\mu e^{-\mu t};$$

where λ and μ are constants. Please solve $Q(t)$.【90 台大環工 20%】

234. Solve the integral equation $f(t)=\cos t+e^{-2t}\int_0^t f(\tau)e^{2\tau}d\tau$.【88 北科光電 10%】

235. Solve the following integral equation $f(t)=2t^2+\int_0^t f(t-\alpha)e^{-\alpha}d\alpha$.【88 台大機械 10%】

236. Solve the integral equation for $y(t)$.

$$y(t)=t^2-4\int_0^t (t-\tau)y(\tau)d\tau$$

【88 成大航太 14%】

237. Find $y(t)$, $y'(t)+\int_0^t e^{-\tau}y(t-\tau)d\tau=1$; $y(0)=0$.【90 台科營建 15%】

238. Solve $Y(t)=t^2+\int_0^t Y(u)\cdot\sin(t-u)du$.【91 中山材料 20%】

239. Solve the integral equation of $y(t)e^{3t}=e^t-3\int_0^t y(\tau)e^{3\tau}d\tau$.【90 崑山電機 20%】

240. Find Laplace transform of $e^{-2t}\int_0^t e^{2\alpha}\cos 3\alpha d\alpha$.【90 海洋光電 10%】

241. Find $y(t)$, $y(t)-\int_0^t y(\alpha)\sin(t-\alpha)d\alpha=e^{-2t}$.【90 中原電機 10%】

242. Find $y(t)$, $y'(t)=\cos t+\int_0^t y(\alpha)\cos(t-\alpha)d\alpha$, $y(0)=1$.【90 中原機械 15%】

243. Solve by Laplace transform $x\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial t}=xt$, $u(x,0)=0$, $u(0.t)=0$.【90 中興化工 20%，91 北科化工 15%】

244. Solve the following boundary value problem by Laplace transform.

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}\ (x>0,t>0),$$

$$u(x,0)=A(x>0),\ u(\infty.t)\ \text{is finite.}$$

Note: The Laplace transform of $erfc\left(\dfrac{a}{2\sqrt{t}}\right)$ is $\dfrac{1}{s}e^{-a\sqrt{s}}$ and $erfc(x)=1-erf(x)$.

【90 北科光電 20%】

245. A semi-infinite string at rest along the positive axis with the left end moving in a prescribed fashion. The displacement of the string can be described as the following:

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2},$$

$$y(0,t) = \begin{cases} \sin(2\pi t), & 0 \le t \le 1 \\ 0, & t > 1 \end{cases},$$

$$y(x,0) = \frac{\partial y}{\partial t}(x,0) = 0 \quad (x > 0).$$

Please find $y(x,t)$ using the Laplace transformation.【87 台大化工 10%，88 中山機械 15%】

246. The displacement $u(x,t)$ of a semi-infinite string is governed by the following partial differential equation.

$$c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x, \quad 0 < t$$

where c is a constant with the initial conditions

$$u(x,0) = 0, \quad \frac{\partial u(x,0)}{\partial t} = 0.$$

And the excitation $f(t)$ at one end of string, that is,

$$u(0,t) = f(t)$$

Then, what is the solution of $u(x,t)$?【91 成大土木 20%】

247. Solve the partial differential equation $\frac{\partial^3 u}{\partial t^3} = \frac{\partial u}{\partial x}$, where u is a function of t and x satisfying $u(x, t = -\infty) = 0$, $u(t = 0, x) = 0$, $\left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$ and $\left.\frac{\partial^2 u}{\partial t^2}\right|_{t=0} = e^{8x}$.

【88 清大電機 13%】

248. 求解 advection equation：

$$\frac{\partial c}{\partial t} + V \frac{\partial c}{\partial x} = u_a(x) - u_b(x); \quad I.C.: C(x,0) = 0; \quad B.C: C(0,t) = 0$$

其中：$V = constant$；$u_c(x)$ 爲 step function；$b > a$。【88 雲科環安 15%】

249. Using the Laplace transform to solve the partial differential equation for a non-periodic function $u(y,t)$.

$$\frac{\partial u}{\partial t}=v\frac{\partial^2 u}{\partial y^2}$$

The initial and boundary conditions for u are

$$u(y,0)=0,\ u(0,t)=-u_B(t),\ u(\infty,t)=0.$$

Hint: (1) Using convolution theorem &

(2) $L^{-1}\left[e^{-\sqrt{\frac{s}{v}}y}\right]=L^{-1}\left[e^{-\frac{y}{\sqrt{v}}\sqrt{s}}\right]=\frac{y}{2\sqrt{v\pi}}t^{-\frac{3}{2}}e^{-\frac{y^2}{4vt}}$

【89 成大水利 12%】

250. Find the solution $p(x,t)$ of the following partial differential equation:

$$\frac{\partial^2 p}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}=\delta(x-at),\ c>a>0,\ 0\le x<\infty;\ 0\le t<\infty;$$

with $p(x,0)=0$, $\partial p(x,0)/\partial t=0$, $p(0,t)=0$, $p(x,t)<\infty$ as $x\to\infty$.

Note: δ denotes the Dirac delta function.【88 交大機械 20%】

251. Solve the partial differential equation of

$$\frac{\partial^2 \phi}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2}-\sin\omega t \quad \text{for } 0\le x\le\infty \text{ and } 0\le t<\infty.$$

Boundary condition $\phi(0,t)=0$.

Initial conditions $\phi(x,0)=0$, $\frac{\partial \phi}{\partial t}(x,0)=0$.【88 北科機電整合 20%】

252. Solve $\frac{\partial \omega}{\partial x}+x\frac{\partial \omega}{\partial t}=0$, $\omega(x,0)=0$, $\omega(0,t)=2t$, $t>0$.【88 交大土木 20%】

253. Solve $\frac{\partial \omega}{\partial x}+2x\frac{\partial \omega}{\partial t}=2x$, $\omega(x,0)=\omega(0,t)=1$.【88 台科高分子 10%】

254. Solve $\frac{\partial \omega}{\partial x}+\frac{\partial u}{\partial t}+2u=0$, $-\infty<x<\infty$, $t>0$, $u(x,0)=\sin x$.【88 成大水利 10%】

255. Using Laplace Transformation (with respect to t) to solve the partial differential

equation.

$$y_t(x,t) = a^2 y_{xx}(x,t) - g \text{, where } a \text{ and } g \text{ are constants;}$$

and $y(x,t)$ satisfies the boundary conditions $y(x,0) = y_t(x,0) = 0$, $y(0,t) = 0,\ \lim_{x\to\infty} y_x(x,t) = 0$.【89 逢甲土木 15%】