台灣師範大學

工業教育學系

91~97 學年度
工程數學考古題

國立臺灣師範大學九十四學年度碩士班考試入學招生試題

工程數學 科試題（工業教育學系電機電子組用，本試題共 1 頁）

注意：1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

1. (10 分) Find the general solution of the non-homogeneous linear system:

$$y_1' = -3y_1 + y_2 - 6e^{-2t}$$
$$y_2' = y_1 - 3y_2 + 2e^{-2t}$$

2. (15 分) Find the inverse matrix A^{-1} for the matrix A.

$$A = \begin{bmatrix} -2 & 1 & -5 \\ 1 & 1 & 4 \\ 0 & 3 & 3 \end{bmatrix}$$

3. (15 分) Use the residue theorem to evaluate the integral:

$\oint_\Gamma \frac{z-i}{2z+1}dz$; Γ is the circle of radius 1 about the original.

4. (15 分) Find the Fourier transform of $f(x) = x^2 e^{-3|x|}$

5. (15 分) Let $\mathcal{L}[f]=F(s)$ be the Laplace transform of a function $f(t)$.

Prove $\mathcal{L}^{-1}\left\{\int_s^\infty F(\tilde{s})d\tilde{s}\right\} = \frac{f(t)}{t}$

6. (15 分) Let $A[a_{jk}]$ be an $m \times n$ matrix . Prove A has rank $r \geq 1$ if and only if A has an $r \times r$ sub-matrix with nonzero determinant.

7. (15 分) Let $(f * g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau$ be the convolution of function f and g, prove the convolution theorem of Fourier transform in the form $\mathcal{F}[f*g] = \sqrt{2\pi}\,\mathcal{F}[f]\,\mathcal{F}[g]$

國立臺灣師範大學九十五學年度碩士班考試入學招生試題

工程數學 科試題 (工業教育學系用，本試題共 2 頁)

注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。

1. Solve the initial value problem using the method of Laplace transformations:

$$\begin{cases} \dfrac{dx}{dt} = 2x - 3y \\ \dfrac{dy}{dt} = y - 2x \end{cases} \quad \text{for} \quad x(0) = 8, y(0) = 3 \qquad (10 分)$$

2. Please find the value of determinant $\begin{vmatrix} 1 & 1 & 1 & 1 \\ \alpha & \beta & \gamma & \delta \\ \alpha^2 & \beta^2 & \gamma^2 & \delta^2 \\ \alpha^3 & \beta^3 & \gamma^3 & \delta^3 \end{vmatrix}$ without expanding. (15 分)

3. Consider a quadratic form of $5{x_1}^2 - 2x_1x_2 + 5{x_2}^2 = 12$, please transform into principle axis. (15 分)

4. Find the integral $\int_C (y + yz)dx + (x + 3z^3 + xz)dy + (9yz^2 + xy - 1)dz$, where C is the curve $x^2 = z$ on the plane $y = 1$ from point (1, 1, 1) to (2, 1, 4). (10 分)

請翻頁繼續作答

595

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5.Show the details to transform the quadratic form

$Q(x) = 4x_1x_2 + 4x_2x_3 + 4x_1x_3$ into its principal axes. (10 分)

6.Solve $x(x) = \sin 2t + \int_0^t x(t-\tau)\sin 2t d\tau$. (13 分)

7. Calculate $\oint_c \frac{e^z}{z^2+1} dz$, where c: $|z+i|=1$. (13 分)

8. Solve initial value problem $y'' + 4y = r(t)$, $y(0)=1, y'(0)=0$

for $y(t)$ where $r(t) = \begin{cases} 1, & \text{for } 0 < t < 1 \\ 0, & \text{for } t > 1 \end{cases}$. (14 分)

596

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國立臺灣師範大學九十六學年度碩士班考試入學招生試題

工程數學-電機電子組 科試題（工業教育學系用，本試題共 2 頁）

注意：1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

1. (10 分) Solve the differential equation $x^3y''' + xy' - y = x$

2. (10 分) Solve the differential equation $y'' + 2y' + y = 4e^{-x}\ln x$

3. (15 分) Solve the equation $y(t) = \sin t + \int_0^t y(\tau)\sin(t-\tau)d\tau$

4. (15 分) Find the Fourier series of the periodic function $f(t)$, where

$$f(t) = \begin{cases} 0 & -1 < t < 0 \\ t & 0 < t < 1 \end{cases}, \quad period = 2$$

5. (10 分)Let A be an $n \times n$ symmetric matrix. Show that $4A^2 - 5A + 3I$ is symmetric.

6. (10 分)Evaluate $\oint_c \frac{2z+1}{z^3+4z^2+3z}dz$, where c: $|z| = 5$.

請翻頁繼續作答

7. (15 分)For which real values of α do the following vectors form a linearly dependent set of R^3 ?

$$v_1 = (-1, \alpha, -1),\quad v_2 = (\alpha, -1, -1), v_3 = (-1, -1, \alpha)$$

8. (15 分)Let $\{u_1, u_2, u_3\}$ be a basis for a vector space *U*. Prove that $\{v_1, v_2, v_3\}$ is also a basis, where $v_1 = u_1 + u_2 + u_3$, $v_2 = u_1 + u_2$, and $v_3 = u_1$.

國立臺灣師範大學九十七學年度碩士班考試入學招生試題

工程數學 科試題（工業教育學系電機電子組用，本試題共 2 頁）

注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

1. Integrate $\int_c \frac{1.2\cos z}{z^2(z-1)}dz$, over

(1) C: $|z|=\frac{1}{3}$. (5 分)

(2) C: $|z-1|=\frac{1}{3}$. (5 分)

(3) C: $|z|=2$. (5 分)

2. Let $T: R^2 -> R^2$ be the linear operator given by the formula

$$T(x,y)=(2x-y,-8x+4y)$$

(1) Which of the following vectors are in the range of T?(5 分)

(a) $[1 \quad -4]^T$ (b) $[5 \quad 0]^T$

(2) Which of the following vectors are in the kernel of T?(5 分)

(a) $[3 \quad 2]^T$ (b) $[5 \quad 10]^T$

3. Let $P: R^m -> W$ be the orthogonal projection of R^m onto a subspace W of R^m.

(1) Prove that $[P]^2=[P]$. (5 分)

(2) Show that $[P]$ is symmetric. (10 分)

599

4. Let $T: R^3 -> R^3$ be the linear operator given by the formula

$$T(x_1, x_2, x_3) = (x_1 - x_2, x_2 - x_1, x_1 - x_3).$$

(1) Find the matrix for T with respect to the basis $z = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$\mathbf{v}_1 = [1 \quad 0 \quad 1]^T$, $\mathbf{v}_2 = [0 \quad 1 \quad 1]^T$, $\mathbf{v}_3 = [1 \quad 1 \quad 0]^T$. (5 分)

(2) Is T one-to-one? If so, find the matrix of T^{-1}. Please explain conclusion (5 分)

5. Solve the differential equation (10 分)

$$x^2 y' + 2 - 2xy + x^2 y^2 = 0$$

6. Solve the differential equation (15 分)

$$x^2 y'' - xy' + 4y = \cos(\ln x) + x \sin(\ln x)$$

7. Solve the differential equation (15 分)

$$y'' - 4y = g(t), \quad g(t) = \begin{cases} t, 0 \le t \le 2 \\ 0, \quad t > 2 \end{cases}, y(0) = y'(0) = 0$$

8. Please show (10 分)

$$\int_0^\infty \frac{\cos \omega x + \omega \sin \omega x}{1 + \omega^2} d\omega = \begin{cases} 0 & , \quad x < 0 \\ \pi/2 & , \quad x = 0 \\ \pi e^{-x} & , \quad x > 0 \end{cases}$$

600

國立臺灣師範大學九十四學年度碩士班考試入學招生試題

工程數學 科試題 (工業教育學系機械組用，本試題共 2 頁)

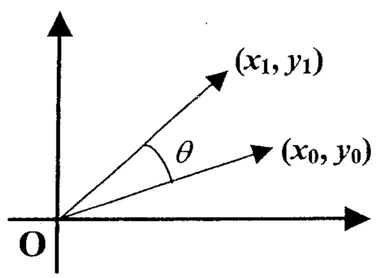
注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案紙上，否則不予計分。

【試題 1】(15 分)

Solve for the differential equation $xy'=y+\sqrt{x^2+y^2}$.

【試題 2】(15 分)

Assume that a point (x_1, y_1) is obtained by rotating point (x_0, y_0) counterclockwise about the origin with an angle θ, and $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = A \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$, please obtain A and A^n.



【試題 3】(15 分)

Solve $y''+2y'+5y=e^{-x}\cdot\cos x$, $y(0)=y'(0)=2$ using the Laplace Transform.

【試題 4】(15 分)

Let $f(x, y, z)=x^2$, Σ is the part of the paraboloid $z=4-x^2-y^2$ lying above the xy-plane. Find the surface integral $\iint_{\Sigma} x^2 dA$ over Σ.

請翻頁繼續作答

【試題 5】(15 分)

Solve the system of linear differential equations using a matrix method.

$x'_1 = -2x_1 + x_2$

$x'_2 = -4x_1 + 3x_2 + 10\cos t$

【試題 6】(10 分)

Find the integral $\int_C \vec{F} \cdot d\vec{R}$ with C a piecewise-smooth curve from $(1, 1, 1)$ to $(-2, 1, 3)$, and $\vec{F}(x, y, z) = (yz^2 - 1)\,\vec{i} + (xz^2 + e^y)\,\vec{j} + (2xyz + 1)\,\vec{k}$.

【試題 7】(15 分)

Find the residue of $f(z) = \dfrac{1+z}{2(1-\cos z)}$ at z = 0.

國立臺灣師範大學九十五學年度碩士班考試入學招生試題

工程數學 科試題 (工業教育學系用，本試題共 2 頁)

注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。

1. Solve the initial value problem using the method of Laplace transformations:

$$\begin{cases} \dfrac{dx}{dt} = 2x - 3y \\ \dfrac{dy}{dt} = y - 2x \end{cases} \quad \text{for} \quad x(0) = 8, y(0) = 3 \qquad (10\text{ 分})$$

2. Please find the value of determinant $\begin{vmatrix} 1 & 1 & 1 & 1 \\ \alpha & \beta & \gamma & \delta \\ \alpha^2 & \beta^2 & \gamma^2 & \delta^2 \\ \alpha^3 & \beta^3 & \gamma^3 & \delta^3 \end{vmatrix}$ without expanding. (15 分)

3. Consider a quadratic form of $5{x_1}^2 - 2x_1x_2 + 5{x_2}^2 = 12$, please transform into principle axis. (15 分)

4. Find the integral $\int_C (y + yz)dx + (x + 3z^3 + xz)dy + (9yz^2 + xy - 1)dz$, where C is the curve $x^2 = z$ on the plane $y = 1$ from point (1, 1, 1) to (2, 1, 4). (10 分)

請翻頁繼續作答

603

5.Show the details to transform the quadratic form

$Q(x) = 4x_1x_2 + 4x_2x_3 + 4x_1x_3$ into its principal axes. (10 分)

6.Solve $x(x) = \sin 2t + \int_0^t x(t-\tau)\sin 2td\tau$. (13 分)

7. Calculate $\oint_c \frac{e^z}{z^2+1}dz$, where c: $|z+i| = 1$. (13 分)

8. Solve initial value problem $y'' + 4y = r(t)$, $y(0) = 1, y'(0) = 0$

for $y(t)$ where $r(t) = \begin{cases} 1, & \text{for } 0 < t < 1 \\ 0, & \text{for } t > 1 \end{cases}$. (14 分)

國立臺灣師範大學九十六學年度碩士班考試入學招生試題

工程數學 科試題（應用電子科技學系用，本試題共 3 頁）

注意：1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

1. (10 分) Solve the differential equation $(x+1)^3 y''' + (x+1)y' - y = 3x$

2. (15 分) Solve the equation $y'' + 3y' + 2y = r(t)$, $r(t) = \begin{cases} 1 & if \quad 0 < t < t_0 \\ 0 & if \quad t > t_0 \end{cases}$

3. (10 分) Given the differential equation $y'' + ay' + by = 0$ and $(a^2 - 4b) = 0$, show the corresponding general solution is $y = (c_1 + c_2 x)e^{-ax/2}$, where c_1 and c_2 are any constants.

4. (15 分) Given a nonexact equation $P(x,y)dx + Q(x,y)dy = 0$,

Show that $F(x)P(x,y)dx + F(x)Q(x,y)dy = 0$ is exact, where

$$F(x) = \exp \int (\frac{1}{Q}(P_y - Q_x))dx$$

(Note: $P_y \equiv \dfrac{\partial P(x,y)}{\partial y}$, $Q_x \equiv \dfrac{\partial Q(x,y)}{\partial x}$)

【試題 4】(10 分)

Using the convolution theorem, solve

$y''+3y'+2y=r(t)$, $r(t)=1$ if $1<t<2$ and 0 otherwise, $y(0)=y'(0)=0$

【試題 5】(15 分)

To evaluate the surface integral of $F=[3z^2, 6, 6xz]$ across the parabolic cylinder S: $y=x^2$, $0\le x\le 2$, $0\le z\le 3$.

【試題 6】(10 分)

Evaluate the line integral with

$\mathbf{F}(\mathbf{r})=[5z, xy, x^2z]=5z\,\mathbf{i}+xy\,\mathbf{j}+x^2z\,\mathbf{k}$

Along two different paths with the same initial point A: (0, 0, 0) and the same terminal point B: (1, 1, 1), namely (Fig. 2)

(a) C_1: the straight-line segment $\mathbf{r}_1(t)=t\,\mathbf{i}+t\,\mathbf{j}+t\,\mathbf{k}$, $0\le t\le 1$, and

(b) C_2: the parabolic arc $\mathbf{r}_2(t)=t\,\mathbf{i}+t\,\mathbf{j}+t^2\,\mathbf{k}$, $0\le t\le 1$.

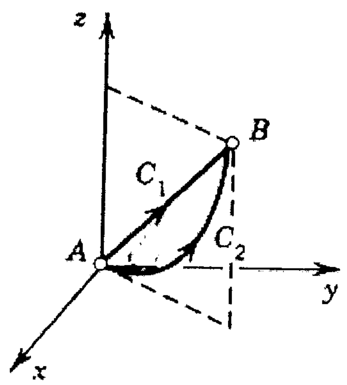


Figure 2.

606

8. (10 分) Let $\mathbf{A}$ be a unitary matrix ($\overline{\mathbf{A}}^{\mathrm{T}} = \mathbf{A}^{-1}$). Show that the determinant of $\mathbf{A}$ has absolute value one, that is $|\det \mathbf{A}| = 1$.

9. (10 分) An $n \times n$ matrix $\widehat{\mathbf{A}}$ is similar to an $n \times n$ matrix $\mathbf{A}$ if $\widehat{\mathbf{A}} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ for some nonsingular $n \times n$ matrix $\mathbf{P}$. Show $\widehat{\mathbf{A}}$ has the same eigenvalues as $\mathbf{A}$.

國立臺灣師範大學九十七學年度碩士班考試入學招生試題

工程數學 科試題（**工業教育學系機械組**用，本試題共 2 頁）

注意：
1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。

【試題 1】(10 分)

Let $f = zy + yx$, $\vec{V} = [y, z, 4z - x]$, $\vec{W} = [y^2, z^2, x^2]$, find

(a) grad f and f grad f at (3, 4, 0)

(b) div $\vec{V}$ and curl $\vec{W}$

(c) div $(\vec{V} \times \vec{W})$

【試題 2】(15 分)

There are four column vectors as $v_1 = [3, -6, 21]^T$, $v_2 = [0, 42, -21]^T$, $v_3 = [2, 24, 0]^T$, and $v_4 = [2, 54, -15]^T$.

(a) Please show that v_1 and v_2 are base vectors. (5 分)

(b) Please find that $v_3 = \alpha v_1 + \beta v_2$ and $v_4 = \gamma v_1 + \omega v_2$ for $\alpha, \beta, \gamma, \omega$ values. (10 分)

【試題 3】(10 分)

Solve the initial value problem using the method of Laplace transformations:

$$\begin{cases} \dfrac{dx}{dt} = 2x - 3y \\ \dfrac{dy}{dt} = y - 2x \end{cases}, x(0) = 8,\ y(0) = 3.$$

【試題 4】(15 分)

(a) Write the equation for the Green's theorem in the plane; **(5 分)**

(b) Prove it using $\vec{F} = (y^2 - 7y)\vec{i} + (2xy + 2x)\vec{j}$ and C: the circle $x^2 + y^2 = 1$. **(10 分)**

【試題 5】(14 分)

Solve initial value problem $xy'' - xy - y = 0; y(0) = 0; y'(0) = 1$ for $y(x)$.

【試題 6】(10 分)

Diagonalize and find eigenvector and eigenvalue of $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

【試題 7】(13 分)

Find Fourier series of $f(x) = \begin{cases} 0 & \text{for } -\pi < x < 0 \\ c & \text{for } \quad 0 < x < \pi \end{cases}$, and $f(x + 2\pi) = f(x)$

【試題 8】(13 分)

Calculate $\oint_{|z|=2} \frac{\sin z}{z^2 + 1} dz$.

台灣師範大學

光電科技研究所

91~97 學年度
工程數學考古題

國立臺灣科技大學
九十一學年度碩士班招生考試試題
系所組別： 電機工程系甲組、電機工程系乙二組
科　目： 工程數學

（共六題；滿分 100 分）

1. Let $\mathbf{F} = (yze^{xyz} - 4x)\hat{a}_x + (xze^{xyz} + z)\hat{a}_y + (xye^{xyz} + y)\hat{a}_z$ for all x, y and z.
   (a) Verify that **F** is conservative. (5%)
   (b) Find a potential function for **F**. (10%)

2. Let g be a periodic function defined by
   $g(t) = t^2$ for $0 < t < 3$ and $g(t+3) = g(t)$ for all t.
   (a) Draw the graph of g for $-6<t<6$. (5%)
   (b) Compute the Fourier series of g. (10%)
   (c) Draw the amplitude spectrum of g for the three lowest-frequency components. (5%)

3. Evaluate $\oint_C 1/(1+z^2)dz$ if C is any piecewise-smooth simple closed curve in the complex plane. Consider all possible cases, which do not pass through i or $-i$. (15%)

4. Find the general solution $y(x)$ to

$$y'' - 8y' + 16y = 8\sin(2x) + 3e^{4x}.$$ (15%)

5. Solve the initial value problem for $y(t)$ with Laplace transform:

$$y'' + 2ty' - 4y = 1; \quad y(0) = y'(0) = 0.$$ (10%)

6. Use the matrix exponential to solve the following initial value problems:

$$\frac{d}{dt}\mathrm{Y}(t) = \mathrm{A}\mathrm{Y}(t), \quad \mathrm{Y}(0) = \mathrm{Y}_0.$$

(1) $A = \begin{pmatrix} 3 & 4 \\ 3 & 2 \end{pmatrix}$, $\mathrm{Y}_0 = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$, and $\mathrm{Y}(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$ (15%)

(2) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$, $\mathrm{Y}_0 = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$, and $\mathrm{Y}(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{pmatrix}$ (10%)

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國立臺灣科技大學

九十二學年度碩士班招生考試試題

系所組別：電機工程系碩士班乙一組

科　目：工程數學

(共九題；滿分一百分)

1. Consider a differential equation as $\frac{dP}{dt} = P(t)(c_1 - c_2 P(t))$, where c_1 and c_2 are constants. Find the solution for the differential equation given $P(0)=P_0$. (10 points)

2. If both $\mu_1(x,y) = xy$ and $\mu_2(x,y) = (x^2+y^2)^{-1}$ are integrating factors for the differential equation $y' = f(x,y)$, then what is $f(x,y)$? (10 points)

3. Let $\Phi(x)$ and $\Psi(x)$ be linearly independent solutions of $y'' + p(x)y' + q(x)y = 0$ on an open interval *I*. Assume that $p(x)$ and $q(x)$ are continuous on *I*. Then prove that between two consecutive zeros of $\Phi(x)$, there always exists exact one zero for $\Psi(x)$. (15 points)

4. Solve $-t(1+t)y'' + 2y' + 2y = 6(t+1)$; $y(-1) = y(1) = 0$. (15 points)

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國立臺灣科技大學
九十二學年度碩士班招生考試試題
系所組別：電機工程系碩士班乙一組
科　目：　工程數學

5. Describe all solutions of $Ax = 0$ in a parametric vector form, where A is the following matrix. (10%)

$$A = \begin{bmatrix} 1 & -5 & 0 & 2 & 0 & -4 \\ 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

6. Find the inverse matrix of the following matrix, if it exists. (10%)

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

7. Given a matrix with its row equivalent matrix shown below, decide bases for $Col\ A$ and $Nul\ A$. (10%)

$$A = \begin{bmatrix} 1 & -3 & 2 & 5 \\ -2 & 6 & 0 & -3 \\ 4 & -12 & -4 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 2 & 5 \\ 0 & 0 & 4 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

8. Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3\}$ be bases for the vector space V, and suppose that $a_1 = 4b_1 - b_2$, $a_2 = -b_1 + b_2 + b_3$, and $a_3 = b_2 - 2b_3$.

(a) Find the change-of-coordinate matrix from A to B. (5%)

(b) Find $[x]_B$ for $x = 3a_1 + 4a_2 + a_3$. (5%)

9. Define $T: \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x) = Ax$, where $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a base B for $\mathbb{R}^2$ with the property that the B-matrix of T is a diagonal matrix. (10%)

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國立臺灣科技大學
九十三學年度碩士班招生考試試題

系所組別：電機工程系乙一組
科　　目：工程數學

總分 100 分

1. (15%) Solve the following systems

$$x'' - 2x' + 3y' + 2y = 4$$
$$2y' - x' + 3y = 0$$
$$x(0) = x'(0) = y(0) = 0$$

2. (15%) Find the general solution of

$$y'' - 3y' + 2y = 2x + 8\sin(2x)$$

3. (10%) For the following equation, write out the first six nonzero terms of a series solution about 0.

$$y'' - 2y' + x^3 y = 0$$

4. (10%) Solve the following equation

$$y' = -\frac{1}{x}y^2 + \frac{2}{x}y; \qquad y(1) = 4$$

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國立臺灣科技大學
九十三學年度碩士班招生考試試題

系所組別：電機工程系乙一組
科　　目：工程數學

5. (10%) Let $T:\mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that

$$T(x_1, x_2) = (x_1 + x_2, -x_1 - 3x_2, -3x_1 - 2x_2)$$

Find $x \in \mathbb{R}^2$ such that $T(x) = (-4, 7, 0)$.

6. (10% with 5% each) Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that rotates each point in $\mathbb{R}^2$ about the origin through an angle φ, with counterclockwise rotation for a positive angle.
   (a) Find the standard matrix A of this rotation.
   (b) Express the matrix $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, where a and b are both real numbers, in terms of a rotation transformation.

7. (10%) The set $B = \{1+t^2, t+t^2, 1+2t+t^2\}$ is a basis for the vector space P_2 of polynomials up to the second order. Find the coordinate vector of $P(t) = 1 + 4t + 7t^2$ relative to B.

8. (20%, with 10% each.) Find the invertible matrix P and matrix C of the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ for the matrix

$$A = \begin{pmatrix} 1 & 5 \\ -2 & 3 \end{pmatrix}$$

such that the given matrix has the form of $A = PCP^{-1}$.
   (a) What is the matrix P?
   (b) What is the matrix C?

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國立臺灣科技大學

九十四學年度碩士班招生考試試題

系所組別：電機工程系碩士班甲組、電機工程系碩士班乙二組

科　　目：工程數學

題目共 2 頁， 8 題，總分 100 分，各題分數如示。

(1) Find the general solution for the following equation:

$y^{(7)} + 18y^{(5)} + 81y''' = 0$ (15%)

(2) Find the Fourier transform for the following function:

$h(t) = \int_{-\infty}^{t} g(x)dx$ (10%)

(3) Let $u(t)$ denote the unit step function, find the Laplace transform for the following function:

$$f(x) = \sin\left[3\left(4t - \frac{\pi}{6}\right)\right]u(4t - 6\pi)$$ (10%)

(4) Consider the symmetric matrix $A = \begin{bmatrix} 5 & -4 & -2 \\ -4 & 5 & -2 \\ -3 & -2 & 8 \end{bmatrix}$, find its orthogonal diagonalizing matrix Q. (15%)

116

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國立臺灣科技大學
九十四學年度碩士班招生考試試題
系所組別：電機工程系碩士班甲組、電機工程系碩士班乙二組
科　　目：工程數學

5. Calculate the complex variable integral $\oint_C \frac{\sin 2z}{(z+3)(z+2)^2}dz$, where C is a clockwise rectangular contour with vertices at 3+i, -2.5+i, -2.5-i, 3-i. (10%)

6. Solve the complex quadratic equation $z^2-(4+i)z+(8+i)=0$. (10%)

7. Verify the Stokes's theorem by the vector function $\vec{F}=y\vec{i}+z\vec{j}+x\vec{k}$, where $\vec{i}$, $\vec{j}$, and $\vec{k}$ are the mutual orthogonal unit vectors in the x-y-z coordinate system, by the unit circle $x^2+y^2=1$ in the x-y plane. (15%)

8. Let $f(x,y,z)=2x+yz-3y^2$ and $\vec{F}$ is the gradient of f. Calculate the line integral $\int_C \vec{F}\cdot d\vec{\ell}$, where C is the quarter circle from A to B as show in Figure P8. (15%)

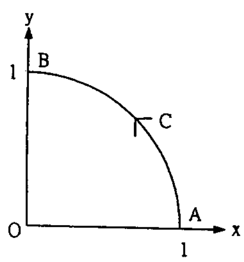

Figure P8

117

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國立台灣科技大學九十五學年度碩士班招生試題

系所組別：電機工程系碩士班甲組、乙二組

科　　目：工程數學

總分100分

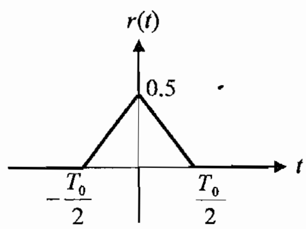
(1) Solve the following differential equation:

$$y'' - 2y' + y = e^{x} + x \qquad y(0)=1, \quad y'(0)=0 \qquad (15\%)$$

(2) Solve the initial-value problem:

$$\mathbf{x}' = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \qquad (15\%)$$

(3) (a) Find the Fourier Transform for the following function: (10%)



(b) Let $F(s) = \dfrac{1}{s^2(s^2+\omega^2)}$, find the inverse Laplace trnasform $f(t)$.

(10%)

4. Evaluate the complex integral $\oint_C \tan z\,dz$ for the contour C in the circle $|z|=3$. (15%)

5. Evaluate $\int_C (x-1)yz\,dx + \cos(yz)\,dy + x(z-1)\,dz$, where C is straight-line segment from $(1,1,1)$ to $(-2,1,3)$. (15%)

6. Let V describe the region bounded by the hemisphere $x^2+y^2+(z-2)^2=9$, $2 \le z \le 5$, and the plane $z=2$. Please verify the divergence theorem if $\vec{F} = x\vec{i} + y\vec{j} + (z-2)\vec{k}$. (20%)

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109

國立台灣科技大學九十七學年度碩士班招生試題

系所組別： 電機工程系碩士班己組

科　　目： 工程數學

總分 100 分

(1) Find a unit normal vector **n** on the plane $4x^2 + y^2 = z$ at the point $(1, -2, 8)$. (16%)

(2) Evaluate the integral $\oint_C \frac{1}{z^2(z-2i)} dz$ where C is (a) $|z-1|=1$, (b) $|z-1|=2$, (c) $|z-1|=3$. (18%)

(3) Find the probability of $P(x>V)$ for a Rayleigh distribution $p(x) = \frac{x}{\psi} e^{-x^2/2\psi}, x \geq 0$. (16%)

(4) Given $A = \begin{pmatrix} 2 & 1 & 0 & -5 \\ -1 & 0 & 1 & 2 \end{pmatrix}$

(a) Find a basis for the nullspace of A. (8%)

(b) Given that $\{(2,1,0,-5)^T, (-1,2,5,0)^T\}$ is an orthogonal basis for the column space of A^T, find the vector in the column space of A^T that is closest to $(-1,0,0,1)^T$. (12%)

(5) Find the inverse Laplace transform of $Y(s) = \frac{2}{s^3(s+2)^2}$. (15%)

(6) Given the Fourier transform pair: $x(t) \leftrightarrow X(\omega)$, derive the Fourier transform of $x(at)$. Also find $X(\omega)$ when $x(t) = e^{-c|t|}$ where $c > 0$. (15%)

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台灣師範大學

機電科技研究所

91~97 學年度

工程數學考古題

國立臺灣師範大學九十四學年度碩士班考試入學招生試題

工程數學 科試題（**機電科技學系 光電與系統組**用，本試題共 2 頁）

注意：1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

1. Solve the following initial value problem: (14 分)

$$y'' + y = r(t), \quad r(t) = t \;\; if \; 1 < t < 2 \; and \; 0 \; if \; t > 2;$$
$$y(0) = 1, \; y'(0) = -2$$

2. Solve the following initial value problem: (14 分)

$$y''' + 3y'' + 3y' + y = 8\sin(x), \; y(0) = -1, \; y'(0) = -3, \; y''(0) = 5$$

3. Show that x, sin(x) and cos(x) are linearly independent. (14)

4. Determine the stability of the critical point and find a real general solution for the following problem. (15 分)

$$y_1' = -2y_1 - 6y_2,$$
$$y_2' = -8y_1 - 4y_2$$

5.Find the current I(t) in the Fig. 1 with L=1 henry, C=1 farad, zero initial current and charge on the capacitor, and V(t)=t if 0< t<1 and v(t)=1 if t >1. (15 分)

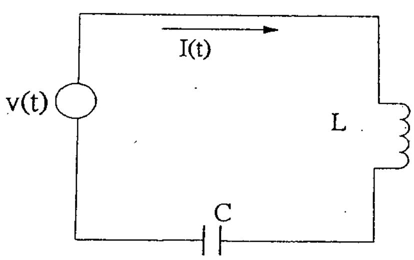


Fig. 1

6. Find $\nabla^2 f$,the Laplacian of f(x,y,z)=c/r, where c is a constant and

r=$\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$ (14 分)

請翻頁繼續作答 621

7. Find a basis of eigenvectors and diagonalization for the following matrix A, (14 分)

$$A = \begin{bmatrix} -2.5 & -3 & 3 \\ -4.5 & -4 & 6 \\ -6 & -6 & 8 \end{bmatrix}$$

國立臺灣師範大學九十五學年度碩士班考試入學招生試題

工程數學 科試題（機電科技學系機電光整合組用，本試題共 2 頁）

注意：1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

【試題 1】

(a) If $\overline{X}$, $\overline{Y}$, $\overline{Z}$ are three vectors which are not parallel to the same plane, show that any vector $\overline{F}$ can be expressed as a linear combination of $\overline{X}$, $\overline{Y}$, $\overline{Z}$ as
$\overline{F} = \frac{[\overline{F}\ \overline{Y}\ \overline{Z}]}{[\overline{X}\ \overline{Y}\ \overline{Z}]}\overline{X} + \frac{[\overline{X}\ \overline{F}\ \overline{Z}]}{[\overline{X}\ \overline{Y}\ \overline{Z}]}\overline{Y} + \frac{[\overline{X}\ \overline{Y}\ \overline{F}]}{[\overline{X}\ \overline{Y}\ \overline{Z}]}\overline{Z}$ **(10 分)**

(b) If the vectors $\overline{X} = (1,\ 2,\ 3)$, $\overline{Y} = (2,\ 4,\ 2)$, $\overline{Z} = (2,\ 1,\ 3)$ and $\overline{F} = (11,\ 13,\ 16)$, please find the $\overline{F} = \alpha\overline{X} + \beta\overline{Y} + \gamma\overline{Z}$ for α, β, γ values. **(5 分)**

【試題 2】(10 分)

Solve the system of linear differential equations using a matrix method.

$$y_1' = -14y_1 + 10y_2$$
$$y_2' = -5y_1 + y_2$$
$$y_1(0) = -1,\ y_2(0) = 1$$

【試題 3】(10 分)

Please find the value of determinant

$$A_n = \begin{vmatrix} x+a & a & \cdots & \cdots & a \\ a & x+a & \cdots & \cdots & a \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ a & a & \cdots & \cdots & x+a \end{vmatrix}$$

【試題 4】(10 分)

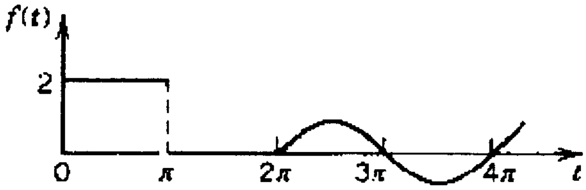
Solve the initial value problem

$$2\sin(y^2)dx + xy\cos(y^2)dy = 0,\ y(2) = \sqrt{\pi/2}$$

請翻頁繼續作答

【試題 5】

(a) Find the Laplace transform of the function defined by following figure. **(5 分)**



$$f(t)=\begin{cases}2 & \text{if } 0<t<\pi\\ 0 & \text{if } \pi<t<2\pi\\ \sin t & \text{if } t>2\pi\end{cases}$$

(b) Find the inverse Laplace transform $f(t)$ of

$$F(s)=\frac{2}{s^2}-\frac{2e^{-2s}}{s^2}-\frac{4e^{-2s}}{s}+\frac{se^{-\pi s}}{s^2+1}$$ **(5 分)**

【試題 6】(15 分)

Evaluate the line integral which are taken around the given contour C in the clockwise sense as viewed from the origin.

$$\int_C (\sin z\,dx-\cos x\,dy+\sin y\,dz)$$

C : the boundary of the rectangle

$0\le x\le \pi,\ 0\le y\le 1,\ z=3$

【試題 7】(15 分)

Please solve the following Cauchy equation

P.D.E : $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=3$

Initial Condition : $u(1\,,y)=\ln y$

【試題 8】(15 分)

Let $\vec{F}=f_1\vec{i}+f_2\vec{j}+f_3\vec{k}$, then $\iint_S \vec{F}\bullet\vec{n}\,dA=\iint_S (f_1\,dydz+f_2\,dzdx+f_3\,dxdy)$.

Using the above theorem, evaluate $I=\iint_S (2x^3\,dydz+x^2y\,dzdx+x^2z\,dxdy)$ for

$S: x^2+y^2=a^2,\ 0\le z\le b$

國立臺灣師範大學九十六學年度碩士班考試入學招生試題

工程數學 科試題（機電科技學系光電與系統組用，本試題共 2 頁）

注意：1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

1. Prove that if f(t) is a periodic function with period T, then the Laplace transform of f(t) (15 分)

$$L[\, f(t)\,] = \frac{\int_0^T f(t)e^{-st}dt}{1-e^{-Ts}}$$

2. Solve the initial value problem

$$y'' + 2y' + 5y = 4, y(0) = 0, y'(0) = 0,$$ (15 分)

3. Solve the following initial value problem: (20 分)

$$y_1' = y_1 + 4y_2 - 4t^2 - 3; \quad y_1(0) = 2$$
$$y_2' = y_1 + y_2 - t^2 + 2t - 3; \quad y_2(0) = 3$$

4. For the continuous-time signals x(t) and u(t) shown in Fig. 1, compute the convolution x(t)*u(t) for all t >=0 and plot your resulting signal. (18 分)

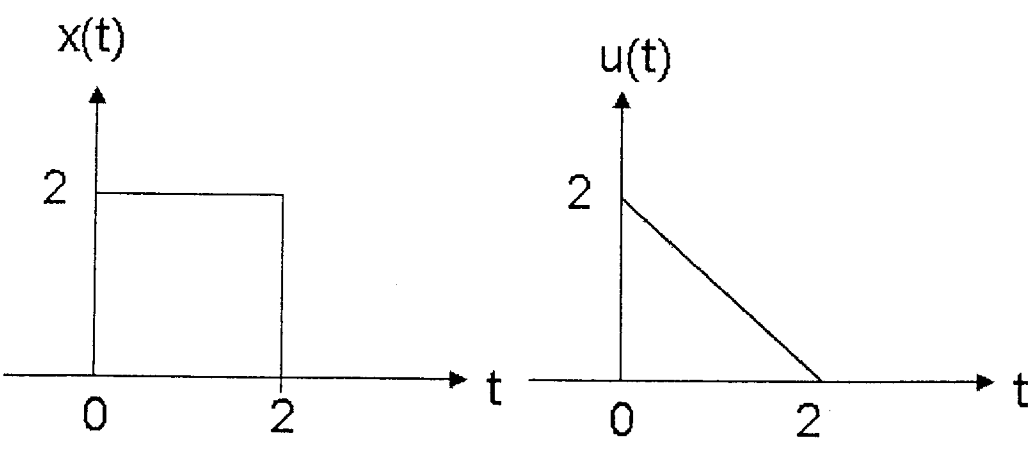


Fig. 1

625

請翻頁繼續作答

5. Calculate the total charge Q with the indicated volume: (16 分)

$0 \le \rho \le 0.1$, $0 \le \phi \le \pi$, $2 \le z \le 4$; *the volume charge density* $\rho_v = \rho^2 z^2 \sin(0.6\phi)$

6. Find all real values (ranges) of k which the matrix A has real characteristic values (16 分)

$$A = \begin{bmatrix} 1 & k & 3 \\ -k & 2 & -k \\ 1 & k & 3 \end{bmatrix}$$

國立臺灣師範大學九十七學年度碩士班考試入學招生試題

工程數學 科試題（**機電科技學系光機電系統組用**，本試題共 2 頁）

注意：1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

1.Solve the Cauchy-Euler problem $x^2y'' - 2xy' + 4y = 0$, (12 分)

2.What is the Laplace transform of the periodic function with period T in Fig.1, (16 分)

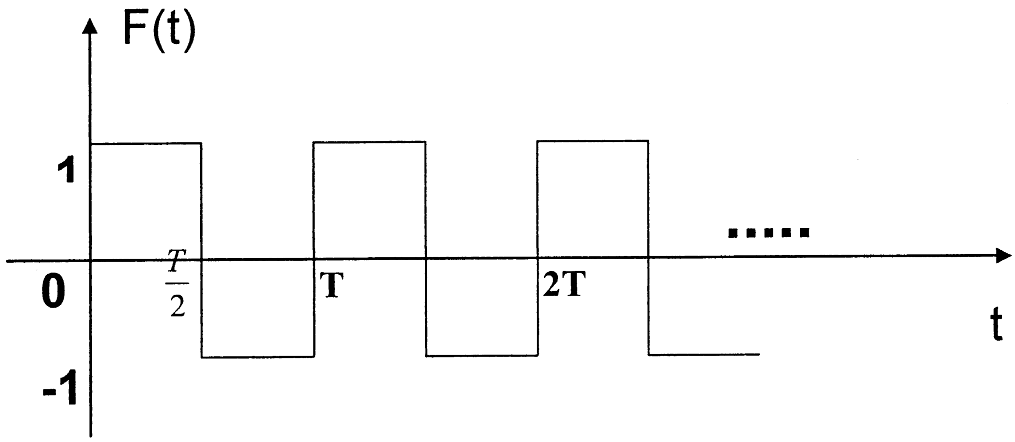


Fig. 1

3. Solve the following initial value problem: (14 分)

$$y_1' = 5y_2 + 23, \quad y_1(0) = 1, \; y_2(0) = -2,$$
$$y_2' = -5y_1 + 15t$$

4. Find and graph the result of the convolution y(t)=x(t)*h(t) in Fig. 2. (16 分)

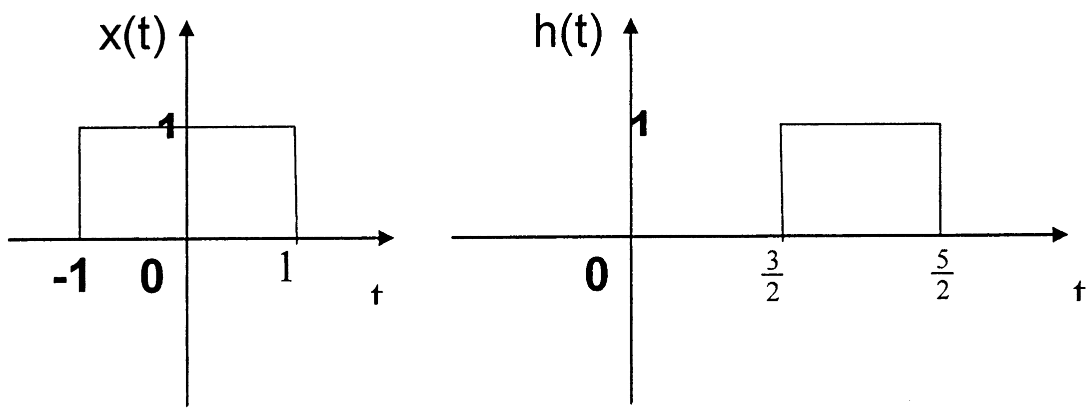


Fig. 2

627

5. Find the the inverse of matrix A, (12 分)

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & -2 \\ 1 & 0 & -3 \end{bmatrix}$$

6. Evaluate e^{At} for the given matrix A, where (16 分)

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

7. Find all eigenvalues and eigenvectors of the matrix A, (14 分)

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

628

國立臺灣師範大學九十四學年度碩士班考試入學招生試題

工程數學 科試題（機電科技學系 機電光整合組用，本試題共 1 頁）

注意：1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

1. （17分） Solve $(ax+by)dx+(rx+sy)dy=0$, where a,b,r,s are constants, if it is an exact differential equation. What is the required condition?

2. （17分） Solve $xy''-3y'+\frac{9}{x}y=0$

3. （17分） Given $A=\begin{bmatrix}-3 & 2\\ 1 & -4\end{bmatrix}$, calculate A^n, show the details.

4. （15分） Given a temperature distribution field described by $f(x,y)=ax^3-bxy^2$, find the heat flow direction at point $(1,2)$.

5. （17分） Solve $\oint_C \frac{-e^z}{\cos \pi z}dz$, where $C:|z|=1$

6. （17分） A period function $f(t)$ has period $T=\frac{2\pi}{\omega}$, and within interval $-\frac{\pi}{\omega}<t<\frac{\pi}{\omega}$ is defined by $f(t)=\begin{Bmatrix} 0 \text{ for } -\frac{\pi}{\omega}<t<0 \\ E\sin\omega t \text{ for } 0<t<\frac{\pi}{\omega}\end{Bmatrix}$, find its Fouries series.

國立臺灣師範大學九十五學年度碩士班考試入學招生試題

工程數學 科試題（機電科技學系光電與系統組用，本試題共 1 頁）

注意：1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

1. Solve the following initial value problem: (14 分)

$$xy'' - 4y' = 0, \quad y(1) = 0,\ y'(1) = 3$$

2. Find a general solution of the following equation. (14 分)

$y'' - 2iy' - y = 0,$

3. Solve the following differential system wherein a prime indicates differentiation with respect to t. (15 分)

$$x' + y' + 2y = \sin t,$$
$$x' + y' - x - y = 0$$

4. A series circuit in which Q(0)=i(0)=0 contains the elements L=2 henrys, R=4 ohms, C=0.05 farads. If a constant voltage E=100 volts is suddenly switched into the circuit, find the Q(t) and the steady state of Q(t). (15 分)

5. Show that the linear space spanned by the function e^x , e^{2x} ,…,e^{nx} Has dimension n. (14 分)

6. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$, verify that $(A^2)^{-1}=(A^{-1})^2$ (14 分)

7. Find a pair of matrices (P, Q) such that PAQ is a diagonal matrix for the following matrix A, (14 分)

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix}$$

國立臺灣師範大學九十六學年度碩士班考試入學招生試題

工程數學 科試題（機電科技學系機電光整合組用，本試題共 3 頁）

注意：1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

【試題 1】(10 分)

Solve $(e^{x+y}-y)dx+(xe^{x+y}+1)dy=0$

【試題 2】(15 分)

Find a general solution of

$$\mathbf{y}' = \mathbf{A}\,\mathbf{y}+\mathbf{g}=\begin{bmatrix}-3 & 1\\ 1 & -3\end{bmatrix}\mathbf{y}+\begin{bmatrix}-6\\ 2\end{bmatrix}e^{-2t}$$

by the method of variation of parameters

【試題 3】(20 分)

Consider a mass-spring system as shown in Fig. 1, in which y_1 and y_2 are the displacement of masses m_1 and m_2, respectively, from equilibrium positions. The spring constants are k_1 and k_2.

(a) Try to derive the governing equations for this system.

(b) If $Y=\begin{bmatrix}y_1\\ y_2\end{bmatrix}$, and $Y''=AY$, find the matrix A.

(c) Considering the eigenvalue problem and find the general solution of this system assuming $m_1=m_2=1$, $k_1=3$ and $k_2=2$.

(d) To determine the specific values for constants in the general solution using the boundary conditions of $y_1(0)=-2,\ \dot{y}_1(0)=0,\ y_2(0)=1,\ \dot{y}_2(0)=0$.

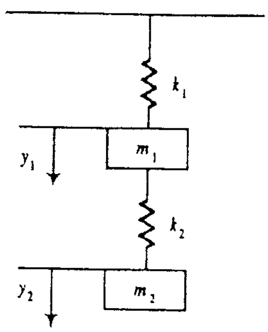


Figure 1.

【試題 4】(10 分)

Using the convolution theorem, solve

$y''+3y'+2y=r(t)$, $r(t)=1$ if $1<t<2$ and 0 otherwise, $y(0)=y'(0)=0$

【試題 5】(15 分)

To evaluate the surface integral of $F=[3z^2, 6, 6xz]$ across the parabolic cylinder S: $y=x^2$, $0\le x\le 2$, $0\le z\le 3$.

【試題 6】(10 分)

Evaluate the line integral with

$\mathbf{F}(\mathbf{r})=[5z, xy, x^2z]=5z\,\mathbf{i}+xy\,\mathbf{j}+x^2z\,\mathbf{k}$

Along two different paths with the same initial point A: (0, 0, 0) and the same terminal point B: (1, 1, 1), namely (Fig. 2)

(a) C_1: the straight-line segment $\mathbf{r}_1(t)=t\,\mathbf{i}+t\,\mathbf{j}+t\,\mathbf{k}$, $0\le t\le 1$, and

(b) C_2: the parabolic arc $\mathbf{r}_2(t)=t\,\mathbf{i}+t\,\mathbf{j}+t^2\,\mathbf{k}$, $0\le t\le 1$.

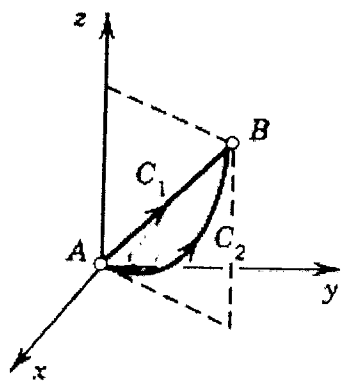


Figure 2.

【試題 7】(10 分)

Please solve the following partial differential equation

$$\text{P.D.E}:\ 2\frac{\partial u}{\partial x}-3\frac{\partial u}{\partial y}+2u=2x$$

Where the initial condition is $u(x\ ,y)=x^2$ for the line $2y+x=0$

【試題 8】(10 分)

Find the Fourier series of the following periodic function

$$f(x)=\begin{cases} x^2 & 0\le x\le 1 \\ 1 & 1\le x\le 4 \end{cases}$$

國立臺灣師範大學九十七學年度碩士班考試入學招生試題

工程數學 科試題（機電科技學系光機電系統組用，本試題共 2 頁）

注意：1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。

1.Solve the Cauchy-Euler problem $x^2y'' - 2xy' + 4y = 0$, (12 分)

2.What is the Laplace transform of the periodic function with period T in Fig.1, (16 分)

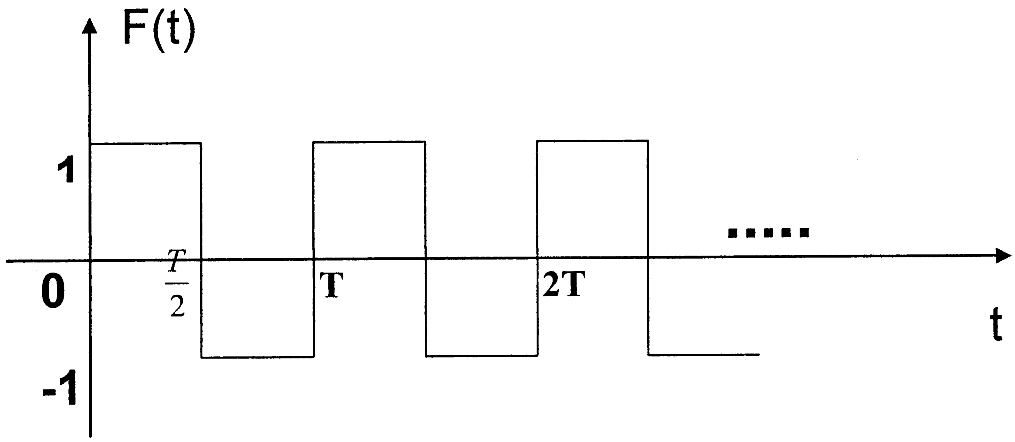


Fig. 1

3. Solve the following initial value problem: (14 分)

$$y_1' = 5y_2 + 23, \quad y_1(0) = 1,\ y_2(0) = -2,$$
$$y_2' = -5y_1 + 15t$$

4. Find and graph the result of the convolution y(t)=x(t)*h(t) in Fig. 2. (16 分)

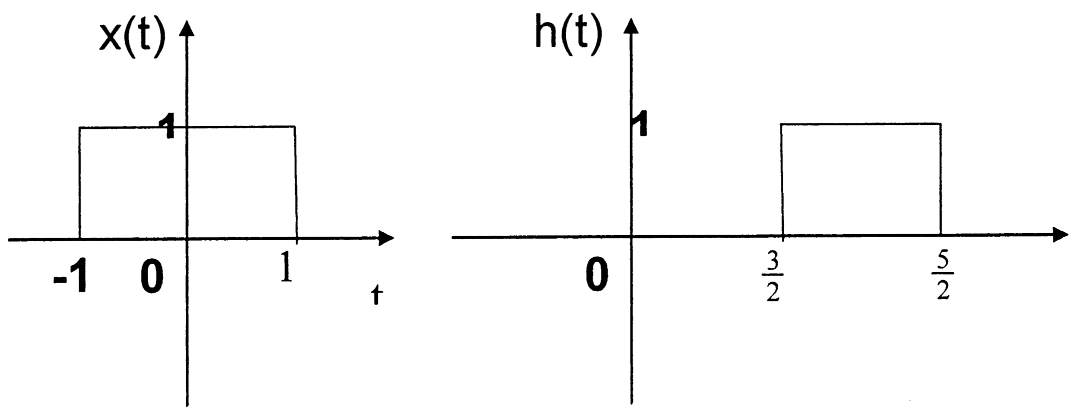


Fig. 2

634

5. Find the the inverse of matrix A, (12 分)

$$A=\begin{bmatrix}1 & 2 & 0\\ 3 & -1 & -2\\ 1 & 0 & -3\end{bmatrix}$$

6. Evaluate e^{At} for the given matrix A, where (16 分)

$$A=\begin{bmatrix}2 & 1 & 4\\ 0 & 2 & 0\\ 0 & 3 & 1\end{bmatrix}$$

7. Find all eigenvalues and eigenvectors of the matrix A, (14 分)

$$A=\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$$

635