台北科技大學

土木與防災研究所

91~97 學年度
工程數學考古題

九十一學年度土木與防災研究入學考試

工程數學(甲、乙、丁組)試題

填 准 考 證 號 碼　　　　　　　　　　　　　　　　第一頁　共一頁

注意事項：
1. 本試題共【4】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。

Problem 1: (25 分)

Let $u(\rho,\phi)$ denote the steady temperatures in a long solid cylinder $a \le \rho \le b$, $-\infty \langle z \langle \infty$ when the temperature of the inner surface $\rho = a$ is a given function $f(\phi) = A + B\sin\phi$ where A and B are constants; and temperature of the outer surface $\rho = b$ is zero. Then the governing equation can be written as follows in a cylindrical coordinate.

$$\rho^2 \frac{\partial^2 u(\rho,\phi)}{\partial^2 \rho} + \rho \frac{\partial u(\rho,\phi)}{\partial \rho} + \frac{\partial^2 u(\rho,\phi)}{\partial^2 \phi} = 0 \quad (\rho \le 1, -\pi < \phi \le \pi).$$

Please calculate $u(\rho,\phi)$.

Problem 2: (25 分)

A fixed end beam subjected a concentrated load as shown in Fig. 1. The governing equation and boundary conditions of the problem are as follows.

$$\frac{d^4 y(x)}{d^4 x} = \frac{p}{EI}\delta(x - \frac{l}{3}) \quad (0 \le x \le l)$$

$$y(0) = 0, \quad y'(0) = 0, \quad y(l) = 0, \text{ and } \quad y'(l) = 0,$$

where y is the deflection, p is the concentrated load, E is the Young's modulus, and I is the moment of interia of section. Please use the **Laplace transform** to solve the deflection y.

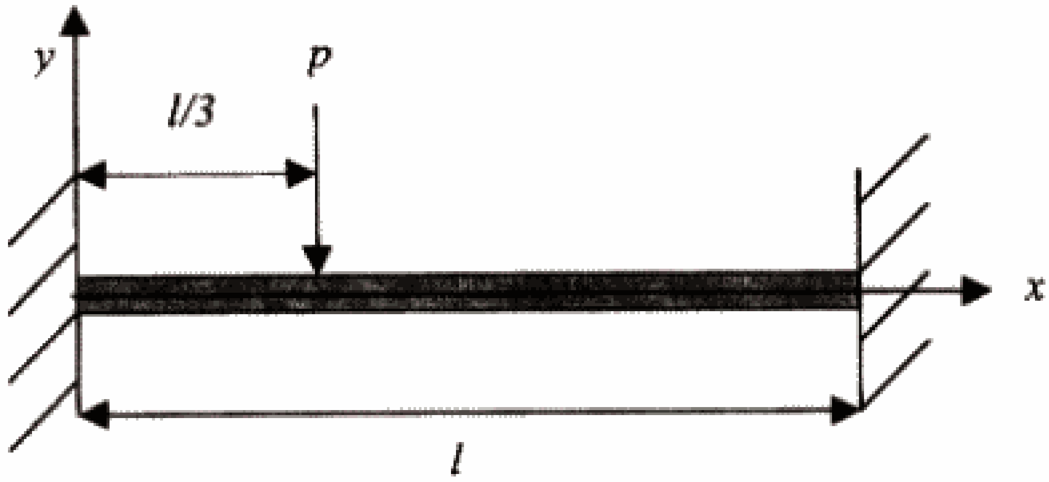

Fig. 1

Problem 3: (25 分)

Please solve the following system of linear differential equations.

$$x_1' = 3x_1 - 4x_2 + 2$$
$$x_2' = 2x_1 - 3x_2 + 4t$$
$$x_3' = x_2 - 2x_3 + 14$$
$$x_1(0) = -5, x_2(0) = -1, x_3(0) = 2$$

Problem 4: (25 分)

Please evaluate the following integral.

$\oint_c \left[\left(\frac{-y}{x^2+y^2} + x^2\right)\mathbf{i} + \left(\frac{x}{x^2+y^2} - 2y\right)\mathbf{j}\right] \cdot d\mathbf{R}$ over any simple closed path in the x-y plane that does not pass through the origin.

國立臺北科技大學

九十二學年度土木與防災研究所入學考試

工程數學試題

填准考證號碼

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Problem 1: (25 分)

The following is a 2-dimensional Laplace Equation problem.

1. Please derive the Laplace equation from the rectangular coordinate system to the polar coordinate system. (10 分)
2. Please solve the following boundary value problem in the polar coordinate system (i.e., find the steady-state temperature $u(r,\theta)$ in the semicircular plate shown in Fig. 1). (15 分)

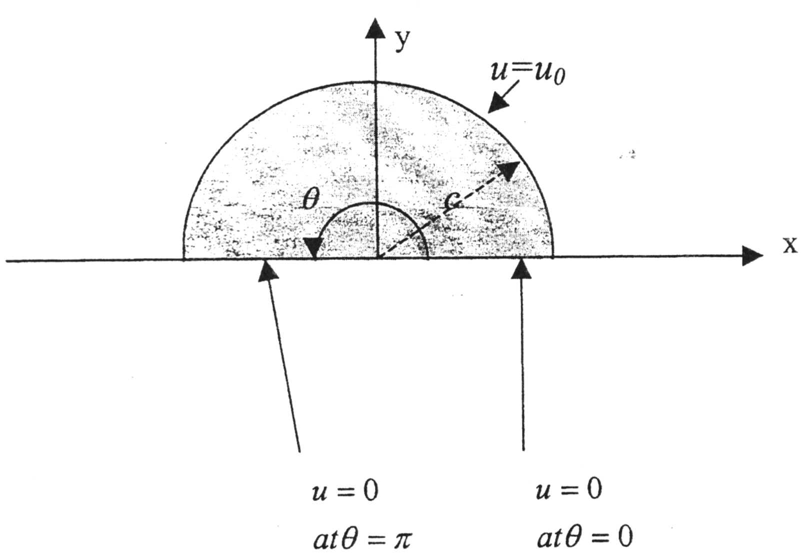

Fig. 1

Problem 2: (25 分)

Consider the spring/mass system of Fig. 2. Let $x_1=x_2=0$ at the equilibrium position, where the weight are at rest. Choose the direction to the right as positive and suppose the weights are at positions $x_1(t)$ and $x_2(t)$ at time t. The equations of motion of the system are as shown as follows.

$$m_1 \ddot{x}_1 = -(k_1 + k_2)x_1 + k_2 x_2 + f_1(t)$$

$$m_2 \ddot{x}_2 = k_2 x_1 - (k_2 + k_3)x_2 + f_2(t)$$

These equations assume that damping is negligible but allow for forcing functions acting on each mass. Suppose $m_1=m_2=1$ and $k_1=k_3=4$ while $k_2=2.5$; and suppose $f_2(t)=0$ and $f_1(t)=2[1-H(t-3)]$. **Please use the Laplace transform to solve this system.**

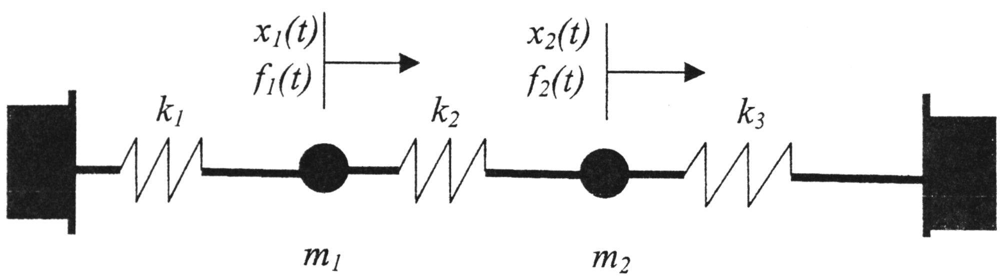

Fig. 2

Problem 3: (25 分)

$$\frac{dX(t)}{dt} = AX(t) + G(t) = \begin{bmatrix} 3 & 3 \\ 1 & 5 \end{bmatrix} X(t) + \begin{pmatrix} 8 \\ 4e^{3t} \end{pmatrix},$$ where $X(t)$ is a 2x1 matrix, and A is 2x2 matrix.

(a) Find the solution of the above equation. **(15 分)**

(b) Find the matrix A^{10}. **(10 分)**

注意：背面尚有試題

Problem 2: (25 分)

Consider the spring/mass system of Fig. 2. Let $x_1=x_2=0$ at the equilibrium position, where the weight are at rest. Choose the direction to the right as positive and suppose the weights are at positions $x_1(t)$ and $x_2(t)$ at time t. The equations of motion of the system are as shown as follows.

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These equations assume that damping is negligible but allow for forcing functions acting on each mass. Suppose $m_1=m_2=1$ and $k_1=k_3=4$ while $k_2=2.5$; and suppose $f_2(t)=0$ and $f_1(t)=2[1-H(t-3)]$. **Please use the Laplace transform to solve this system.**

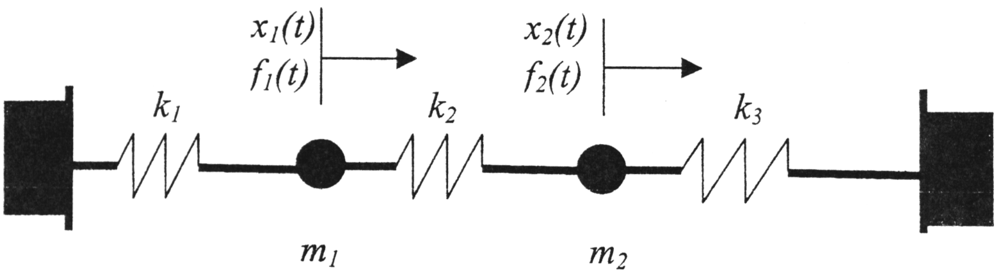

Fig. 2

Problem 3: (25 分)

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(a) Find the solution of the above equation. **(15 分)**

(b) Find the matrix A^{10}. **(10 分)**

注意：背面尚有試題

Problem 4: (25 分)

The system of equations is shown as follow.

$$AX = B$$

$$A = \begin{pmatrix} 2 & 0 & 1 \\ -5 & 5 & 6 \\ -2 & 3 & 4 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

a. Please use Gauss-Jordan elimination to solve the above equations. **(15 分)**

b. Please use Cramer's rule to solve the above equations. **(10 分)**

ce1-1

國立臺北科技大學

九十三學年度土木與防災研究所入學考試

工程數學試題

填准考證號碼 第一頁 共一頁

注意事項：

1. 本試題共六題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Find the angle between surface $X^2+Y^2+Z^2=6$ and $X/2+Y^2/2-Z=0$ at point(2,1,1). (10分)

2. Find eigenvalues and associated nonzero eigenvectors of the matrix. (15分)

$$[A] = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

3. Find out what type of conic section is represented by the given quadratic form. Transform it to principle axes. Express $X^T=[X_1, X_2]$ in term of the new coordinate vector. $Y^T=[Y_1, Y_2]$; $3X_1^2+8X_1X_2-3X_2^2=5$ (15分)

4. Find the general solution of following ordinary differential equation.

   (a) $4x(y')^2+2xy'-y=0$ (10分)

   (b) $x(dy/dx)^3-2y(dy/dx)=16x^2$ (10分)

5.Solve X_1 and X_2 of the linear systems with the following governing equations. (20 分)

$$X_1' = 5X_1 + 8X_2 + 1$$
$$X_2' = -6X_1 - 9X_2 + t$$

6.Solve the following boundary value problem. (20 分)

$$\frac{\partial u}{\partial t} - 2\frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < \pi, \quad t > 0$$

$t > 0$: $u(0,t) = 0$, $u(\pi,t) = 0$,

$u(x,0) = \pi \quad 0 < x < \pi/2$; $u(x,0) = \pi - x \quad \pi/2 < x < \pi$

國立臺北科技大學

九十四學年度土木與防災研究所入學考試

工程數學試題

填准考證號碼　　　　　　　　　　　　　　　第一頁　共一頁

注意事項：

1. 本試題共六題，配分共 100 分。
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1. Solve the initial value problem

$$y''-4y'+4y=xe^{2x}+\sin 2x$$

$$y(0)=0\ ;\ y'(0)=1 \qquad (15\%)$$

2. Use the Laplace transform to solve the initial value problem

$$\frac{dy}{dt}+3y=13\sin 2t \quad ; \quad y(0)=6 \qquad (15\%)$$

3. Solve the following partial differential equation

$$c^2\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial t^2}=0 \qquad (0<x<a \quad , \quad t>0)$$

with the boundary conditions

$$u(0,t)=u(a,t)=0$$

and initial conditions

$$u(x,0)=f(x)$$

$$\frac{\partial u}{\partial t}(x,0)=g(x)$$

where a and c are coefficients. (20%)

4. If $A=\begin{bmatrix}-2 & 4\\ -1 & 3\end{bmatrix}$, find A^6. (15%)

5. Expand $f(x)=x^2$, $(0<x<L)$, in a Fourier series. (15%)

6. Solve $X'=\begin{bmatrix}4 & 2\\ 2 & 1\end{bmatrix}X+\begin{pmatrix}3e^t\\ e^t\end{pmatrix}$ by diagonalization. (20%)

cel-1

國立臺北科技大學九十五學年度碩士班招生考試

系所組別：3110、3120、3150 土木與防災研究所甲乙戊組

第二節 工程數學 試題

填准考證號碼

第一頁 共一頁

注意事項：

1. 本試題共五題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Given the matrix A as shown.
   (a) Determine the rank of A. (5)
   (b) Is the A matrix singular? (5)
   (c) Determine the eigenvalues of A. (5)
   (d) Determine an eigenvector of A. (5)

$$A=\begin{bmatrix}2 & -3 & 4\\ 1 & 2 & -3\\ 4 & 1 & -2\end{bmatrix}$$

2. Solve the following system of differential equations with $u(0)=v(0)=w(0)=0$ (20)

$$\begin{bmatrix}\frac{du}{dx}\\ \frac{dv}{dx}\\ \frac{dw}{dx}\end{bmatrix}+\begin{bmatrix}1 & -1 & 0\\ 1 & 2 & 1\\ -2 & 1 & -1\end{bmatrix}\begin{bmatrix}u\\ v\\ w\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}e^{2x}$$

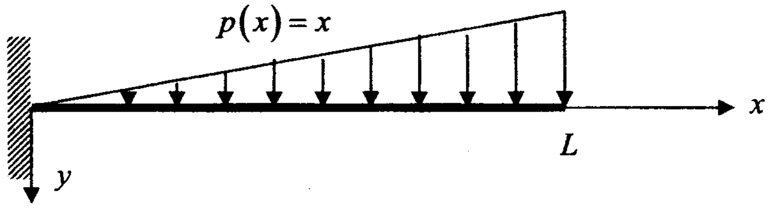
3. Solve the following partial differential equation with the boundary conditions of $u(x,b)=1$ and $u_x(0,y)=u_x(a,y)=u(x,0)=0$. $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0$ for $0\le x\le a$ and $0\le y\le b$ (20) (Note that you might also need to particularly take care of the case of a zero eigenvalue)

4. (a) Determine the solution of the differential equation $y'+y\tan x=\sin(2x)$ subject to the initial condition of $y(0)=1$. (Note that $\int\tan x\,dx=-\ln\cos x$) (10)
   (b) Find the general solution of $y''-4y'+4y=0$. (5)
   (c) Find the general solution of $y'''-6y''+11y'-6y=0$. (5)

5. A cantilever beam of length L is subjected to a distributed load $p(x)=x$ as shown in the figure. Using the Euler beam theory, the governing equation is found to be $EIy''''=x$. Apparently, the boundary conditions are $y(0)=y'(0)=y''(L)=y'''(L)=0$. Find the deflection $y(x)$ by Laplace transform. But there is no harm in extending the beam from L to ∞, provided that $p(x)=0$ is defined for $x>L$. Consequently, the problem is to

solve $EIy'''' = x\left[1-u_L(x)\right]$ with the boundary conditions of $y(0)=y'(0)=y''(L)=y'''(L)=0$.

Note that $u_L(x)=\begin{cases}0 & when \quad x<L\\ 1 & when \quad x>L\end{cases}$ where $L\geq 0$, and $\bar{f}(s)=\int_0^\infty f(x)e^{-sx}dx$ where $\bar{f}(s)=\dfrac{e^{-Ls}}{s}$ for $f(x)=u_L(x)$. (20)



台北科技大學

機電整合研究所

91~97 學年度
工程數學考古題

國立臺北科技大學

九十一學年度機電整合研究所入學考試

工程數學(微機電與控制組、機電整合設計組)試題

填准考證號碼

注意事項：

1. 本試題共【玖】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。

Problem 1. (10%)

Newton's law of cooling states that the time rate of change in temperature of an object varies as the difference in temperature between object and surroundings. If an object cools from 80°C to 60°C in 20 minutes, find the temperature in 40 minutes if the surrounding temperature is 20°C.

Problem 2. (10%)

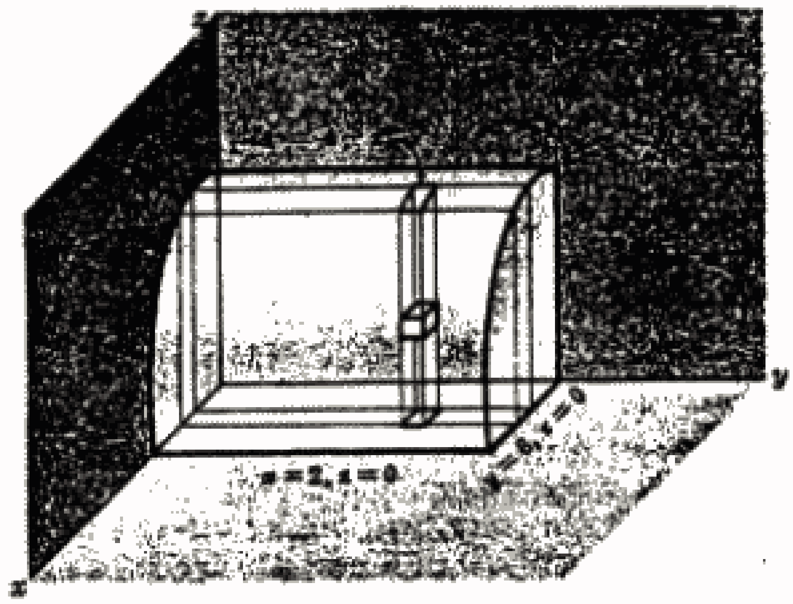
We have $F(s)=\int_0^\infty e^{-su}f(u)\,du$, $G(s)=\int_0^\infty e^{-su}g(u)\,du$,

If $\mathcal{L}\{f(t)\}=F(s)$, $\mathcal{L}\{g(t)\}=G(s)$, then

Prove $\mathcal{L}\{\int_0^t f(t)g(t-u)du\}=F(s)G(s)$

Problem 3. (15%)

Find the (a) volume and (b) centroid of the region R bounded by the parabolic cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 6, z = 0$ assuming the density to be a constant. The region R is shown in the Figure as follow.



Problem 4. (15%)

The gamma function denoted by $\Gamma(n)$ is defined by $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$

which is convergent for $n > 0$. The beta function, denoted by $B(m,n)$ is defined by

$B(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$, Which is convergent for $m > 0$, $n > 0$.

Prove that $B(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

Problem 5. (10%)

Find the (a) eigenvalues and (b) eigenvectors of $A = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 5 \end{pmatrix}$

Problem 6. (10%)

Solve $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ $0 < x < 3$, $t > 0$, given that $u(0, t) = u(3, t) = 0$,

$u(x, 0) = 5 \sin 4\pi x - 3 \sin 8\pi x + 2 \sin 10\pi x$, $|u(x,t)| < M$

Problem 7. ((a) 5%, (b) 5%)

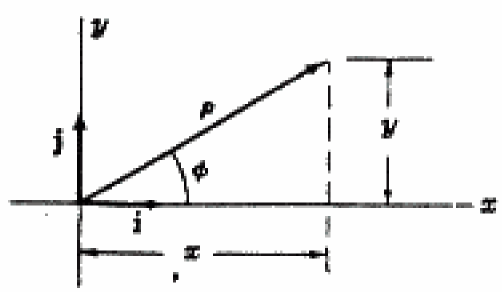
Evaluate $\oint_C \frac{e^z\,dz}{(z-1)(z+3)^2}$, where C is given by (a) $|z| = 3/2$, (b) $|z| = 10$.

Problem 8. (10%)

Solve $x^3 y''' - 3x^2 y'' + 6xy' - 6y = x^4 \ln x$

Problem 9. (10 %)

A particle of mass m moves in the xy plane under the influence of a force F of attraction to the origin 0 of magnitude $F(\rho) > 0$, where ρ is the distance of the mass from. Set up the equations describing the motion.



國立臺北科技大學

九十二學年度電機工程系碩士班入學考試

工程數學（甲、乙、丙組）試題

填准考證號碼　　　　　　　　　　　　第一頁　共一頁

注意事項：

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1. Let $y_1 = e^{-3x}$ be one of the solutions to the differential equation $y'' + 6y' + 9y = 0$.

   (a) Derive the other linearly independent solution y_2 from y_1. (10%)

   (b) Show they are indeed linearly independent. (5%)

   (c) Find the dimension of the solution space. (5%)

2. Solve $y'' + 2y' + 2y = \delta(t - \pi)$; $y(0) = y'(0) = 0$. (15%)

3. Consider the periodic function $f(t) = \sum_{n=-\infty}^{\infty} \frac{2}{n^2\pi} e^{jnt}$, $n \neq 0$.

   (a) Find the forced response of $y'' + 0.02y' + 25y = f(t)$ in the form of complex Fourier series. (10%)

   (b) Express the derived response in trigonometric form. (5%)

4. Suppose that S consists of all vectors $(x, y, -y, -x)$ in R^4.

   (a) Show that S is a subspace. (5%)

   (b) Determine a basis for the subspace S of R^4. (5%)

   (c) Determine the dimension of the subspace. (5%)

5. Let $A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$ and $f(x) = \sin(x)$.

   (a) Find the eigenvalues of $f(A)$. (5%)

   (b) Find $f(A)$. (10%)

6. Consider the quadratic form $x_1^2 + 2x_2^2 + 2\sqrt{2}x_1x_3$. Find the matrix Q that transforms the quadratic form into the standard form (i.e. $\lambda_1 y_1^2 + \lambda_2 y_2^2 + \lambda_3 y_3^2$). (20%)

國立臺北科技大學

九十三學年度電機工程系碩士班入學考試

工程數學試題(甲組、乙組與丙組)

填准考證號碼　　　　　　　　　　　　　第一頁　共一頁

注意事項：

1. 本試題共 5 題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Given $A^2 > 4B$, "A" and "B" are constants
   Show that $y(t) = e^{\alpha t}(c_1 \cosh(\beta t) + c_2 \sinh(\beta t))$ is the general solution of $y'' + Ay' + By = 0$ for appreciate choices of α and β; C_1 and C_2. (20%)

2. Given $f(t) = |E\sin(\omega t)|$, "E" and "ω" are positive numbers.

   Find the Laplace Transform of *f(t)*. (15%)

3. Given $f(t) = \begin{cases} 0 & \forall t < 5 \\ t^2 + 2t + 1 & \forall t \geq 5 \end{cases}$

   Find the Laplace Transform of *f(t)*. (15%)

4. Given $x - 2y + 3z = 1$, $2x + ky + 6z = 6$, and $-x + 3y + (k+3)z = 0$ (20%)
   (1). Find "k" such that the equations are ***inconsistent***
   (2). Find "k" such that there exists a ***unique*** solution for the equations

5. Select the ***wrong*** statement(s) **and** give the ***reason(s) (30%)***

   (1). Given $F = G + H$ *where* $F \in R^3$, $G \in R^3, H \in R^3$ then $F \bullet (G \times H) = 0$

4. Please prove that:
   (a) A signal $g(t)$ and its Hilbert transform $\hat{g}(t)$ have the same magnitude spectrum.(5%)
   (b) If $\hat{g}(t)$ is the Hilbert transform of $g(t)$, then the Hilbert transform of $\hat{g}(t)$ is $-g(t)$.(5%)

5. A FM signal is defined as $s(t) = A_c \cos(2\pi f_c t + 2\pi \int_0^t m(\tau)d\tau)$. The modulating signal $m(t)$ is defined as below:

   $m(t) = A_m \sin^2(2\pi f_m t)$

   Please find the approximate form of a narrowband($\frac{A_m}{f_m}$ is small compared to one radian) FM signal.(10%)

6. The AM signal is defined as below: (10%)
   $s(t) = A_c[1 + 0.8\sin(2\pi f_m t)]\cos(2\pi f_c t)$
   Please find the value of $\frac{(SNR)_O}{(SNR)_C}$.

7. According to the Nyquist's theorem, to avoid the aliasing effect, please find the minimal sample rate(Hz) for the following signal. (10%)
   (a) $g(t) = \sin c(100\pi t)$
   (b) $g(t) = \sin c^2(100\pi t)$
   (c) $g(t) = \sin c(100\pi t) + \sin c^2(100\pi t)$
   (d) $g(t) = \sin c(100\pi t) * \sin c^2(100\pi t)$

8. The filter input $x(t)$ consists of a pulse signal $g(t)$ corrupted by additive white Gaussion noise $w(t)$, as shown by
   $x(t) = g(t) + w(t), 0 \le t \le T$.

   Please derive the optimal match filter $h_{opt}(t)$.(10%)

9. (a) Please sketch the waveforms of the in-phase and quadrature components of the MSK signal in response to the input binary sequence 10101100011. (5%)
   (b) Please sketch the MSK waveform itself for the binary sequence specified in part (a). (5%)

國立臺北科技大學

九十四學年度電機工程系碩士班入學考試

工程數學（甲乙丙組）試題

填准考證號碼　　　　第一頁　共一頁

注意事項：

1. 本試題共七題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. (10%) Find $L^{-1}\{\ln\left(\frac{s+2}{s+1}\right)\}$

2. (10%) Find the general solution：$y'' - 4y = \sum_{n=1}^{20} \frac{1}{n}\cos(nx)$

3. (15%) Find the general solution：$(x-2)^2 y'' + 3(x-2)y' + y = x$

4. (15%) Solve the initial value problem：

$$(x^2-2x)y' + (x^2-5x+4)y = (x^4-2x^3)e^{-x} \quad ; \quad y(3)=18e^{-3}$$

5. (15%) Let $B = \begin{bmatrix} 6 & -2 & -4 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}$, find $B^m = ?$ (m is a positive integer)

6. (20%) Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 0 & 1 \\ 2 & -4 & 2 \\ 4 & 0 & 0 \end{bmatrix}$, find an orthonormal basis for the column space of A.

7. (15%) Let $E = [u_1, u_2, u_3]$ and $F = [b_1, b_2, b_3, b_4]$, where

$u_1 = (1, 2, 1)^T$, $u_2 = (-1, 1, 1)^T$, $u_3 = (1, 0, -2)^T$

and $b_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 & 0 \\ -1 & 0 \end{bmatrix}$, $b_3 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, $b_4 = \begin{bmatrix} 0 & 3 \\ 0 & -1 \end{bmatrix}$.

Find the matrix representing L with respect to the ordered bases E and F, where $L(\mathbf{x}) = \begin{bmatrix} x_1 + x_2 & x_1 - x_3 \\ x_2 - x_3 & x_2 + x_3 \end{bmatrix}$

國立臺北科技大學九十五學年度碩士班招生考試

系所組別：1610、1620、1630 電機工程系碩士班甲乙丙組

第二節 工程數學 試題

填准考證號碼

注意事項：
1. 本試題共十題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。
4. 第一～五題僅需寫答案，不需作答過程。其它題目必須有作答過程。

一、Let A is an $m\times n$ matrix, and B is an $n\times n$ matrix.

1. (2%) Write the condition of rank(B) if rank(AB) < rank(A).
2. (2%) Write the condition of rank(B) if rank(AB) = rank(A).
3. (2%) Write the condition of rank(B) if rank(AB) > rank(A).

二、(6%) Find the transition matrix A which is the linear transformation from $\begin{bmatrix}0\\0\\2\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}$ to $\begin{bmatrix}0\\2\\2\\3\end{bmatrix}, \begin{bmatrix}1\\1\\2\\4\end{bmatrix}, \begin{bmatrix}4\\2\\3\\1\end{bmatrix}$.

三、(6%) Find all possible matrix X for which $AX = \begin{bmatrix}0\\0\\0\end{bmatrix}$,

where $A = \begin{bmatrix}-2 & 3 & -4\\3 & -5 & 6\\3 & -7 & 6\end{bmatrix}$.

四、(6%) Let $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ 0 \\ 1 \end{bmatrix}$ be one of basis of nullspace of matrix

$\begin{bmatrix} 3 & 1 & 0 & 2 & 1 \\ 1 & 0 & 1 & 2 & 1 \\ 2 & 1 & 1 & 4 & 1 \\ 3 & 0 & 1 & 2 & 2 \end{bmatrix}$, compute the value of a_1, a_2, a_3, b_1, b_2, b_3.

五、(6%) Find the interval of convergence of the following series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2 3^n}(x-2)^n$$

六、(10%) Let A be a nonsingular $n\times n$ matrix with a nonzero cofactor A_{nn} and set $c = \dfrac{\det(A)}{A_{nn}}$, show that if we subtract c from a_{nn}, then the resulting matrix will be singular.

七、Let $A = \begin{bmatrix} 4 & 3 \\ 5 & 6 \end{bmatrix}$,

1. (4%) Find the eigenvalues and eigenvectors of A.
2. (12%) Solve the matrix equation $X^2 = A$.

八、(15%) Solve $y'' + 9y = -4x\cos(3x)$

九、(15%) Use the Laplace transform to solve

$$t\,y'' + (4t+2)\,y' - 4\,y = 0; \qquad y(0) = 2$$

十、For $x^2 y'' + x y' + (x^2-1)\,y = 0$,find

1. (4%) indicial equation
2. (10%) two linearly independent solutions.

國 立 臺 北 科 技 大 學

九十一學年度機電整合研究所入學考試

工程數學(微機電與控制組、機電整合設計組)試題

填 准 考 證 號 碼

注意事項：

1. 本試題共【玖】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在試卷答案欄內，否則不予計分。

Problem 1. (10%)

Newton's law of cooling states that the time rate of change in temperature of an object varies as the difference in temperature between object and surroundings. If an object cools from 80°C to 60°C in 20 minutes, find the temperature in 40 minutes if the surrounding temperature is 20°C.

Problem 2. (10%)

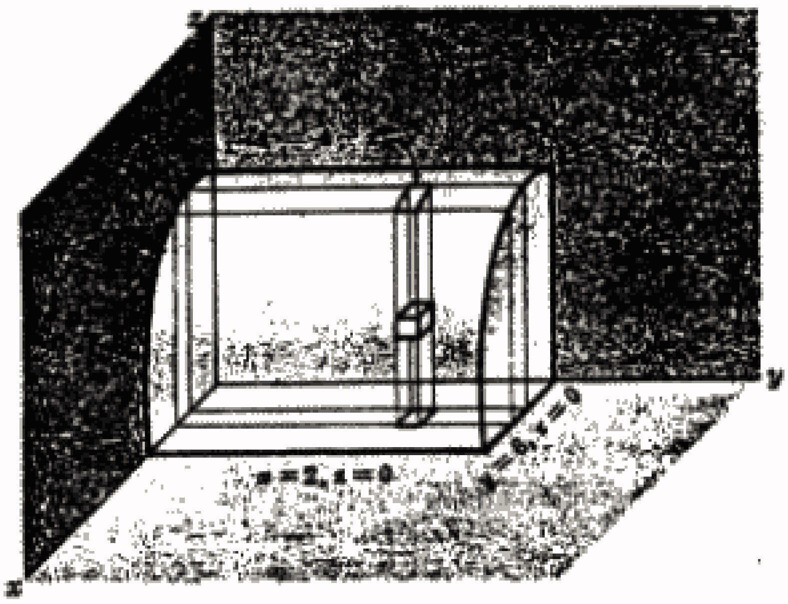
We have $F(s) = \int_0^\infty e^{-su} f(u)\,du$, $G(s) = \int_0^\infty e^{-su} g(u)\,du$,

If $\mathcal{L}\{\, f(t) \,\} = F(s)$, $\mathcal{L}\{\, g(t) \,\} = G(s)$, then

Prove $\mathcal{L}\{\, \int_0^t f(t)g(t-u)du \,\} = F(s)G(s)$

Problem 3. (15%)

Find the (a) volume and (b) centroid of the region R bounded by the parabolic cylinder $z = 4 - x^2$ and the planes $x = 0$, $y = 0$, $y = 6$, $z = 0$ assuming the density to be a constant. The region R is shown in the Figure as follow.



Problem 4. (15%)

The gamma function denoted by $\Gamma(n)$ is defined by $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\,dx$

which is convergent for $n > 0$. The beta function, denoted by $B(m,n)$ is defined by

$B(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1}\,dx$, Which is convergent for $m > 0$, $n > 0$.

Prove that $B(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

Problem 5. (10%)

Find the (a) eigenvalues and (b) eigenvectors of $A = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 5 \end{pmatrix}$

Problem 6. (10%)

Solve $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ $0 < x < 3$, $t > 0$, given that $u(0, t) = u(3, t) = 0$,

$u(x, 0) = 5 \sin 4\pi x - 3 \sin 8\pi x + 2 \sin 10\pi x$, $|u(x,t)| < M$

Problem 7. ((a) 5%, (b) 5%)

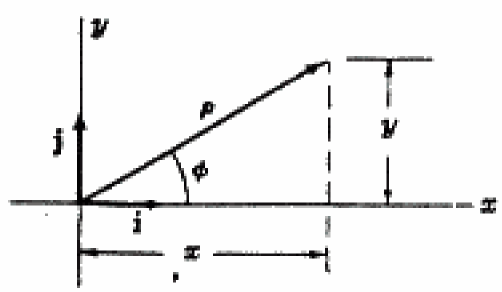
Evaluate $\oint_C \frac{e^z\,dz}{(z-1)(z+3)^2}$, where C is given by (a) $|z| = 3/2$, (b) $|z| = 10$.

Problem 8. (10%)

Solve $x^3 y''' - 3x^2 y'' + 6xy' - 6y = x^4 \ln x$

Problem 9. (10 %)

A particle of mass m moves in the xy plane under the influence of a force F of attraction to the origin 0 of magnitude $F(\rho) > 0$, where ρ is the distance of the mass from. Set up the equations describing the motion.



國立臺北科技大學

九十二學年度機電整合研究所入學考試

工程數學試題

填准考證號碼 　　　　　　　　　　　　　　第一頁　共一頁

注意事項：

1. 本試題共【10】題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

1. Solve the initial value problem $xy' - y = \dfrac{y}{\ln y - \ln x}$; $y(2) = 2$. (10%)

2. Find the general solution of differential equation $y^{(4)} + 16y = t^2 + 1$ (10%)

3. Solve the initial value problem $y'' + 4y = f(t)$, $y(0) = 1$, $y'(0) = 0$ with

$$f(t) = \begin{cases} 0 & \text{for } 0 \le t < 4 \\ 3 & \text{for } t \ge 4 \end{cases}$$ (10%)

4. Find the first four nonzero terms of the power series solution of the initial value problem, about the point where the initial conditions are given

$$y'' + y' - xy = 0 \;;\quad y(0) = -2 \;,\quad y'(0) = 0$$ (10%)

5. Show that the eigenvalues of $\begin{bmatrix} \alpha & \beta \\ \beta & \gamma \end{bmatrix}$ are real. (α, β and γ are real numbers)

(10%)

6. Find the general solution of system of linear differential equations

$$x_1' = -2x_1 + x_2$$
$$x_2' = -4x_1 + 3x_2 + 3$$ (10%)

7. Evaluate $\oint_c \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$ with C the positively oriented square with vertices (1,1), (-1,1), (-1,-1) and (1,-1) by (10%)

(a) directly integrating along the curve. (hint: $\int \frac{1}{1+x^2}dx = \tan^{-1} x$)

(b) the use of Green's theorem.

8. Consider the level surface $\varphi(x, y, z) = z - \sqrt{x^2+y^2} = 2$ (10%)

(a) show the graph of this level surface in 3-D diagram.

(b) find the equation of tangent plane to this surface at point (3,4,7).

9. Periodic function $f(t)$ has period 2π and (10%)

$$f(x) = x+\pi \qquad -\pi < x < \pi$$

(a) find the Fourier series of $f(t)$.

(b) find the the value of this series at $x = \pi$

(c) use the result above to show that $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \cdots$

10. Wave equation be expressed as boundary value problem: (10%)

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \qquad 0 < x < \pi, \quad t > 0$$

$$u(0, t)=0, \quad u(\pi, t)=0 \qquad t > 0$$

$$u(x,0)= 0, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = x$$

Show that u(x,t) can be expressed as

$$u(x, t)= \sum_{n=1}^{\infty}(A_n \cos nat + B_n \sin nat)\sin nx$$ and find A_n and B_n.

(By assuming that $u(x, t)$= X (x)T(t) and $\frac{X''}{X} = -\lambda^2$)

國立臺北科技大學

九十三學年度機電整合研究所入學考試

工程數學試題

填准考證號碼　　　　　　　　　　　　　　　　第一頁　共一頁

注意事項：

1. 本試題共四題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

Problem 1. (25%)

a) (15%) Please prove the following convolution integral result

$$L^{-1}\{F(s)\cdot G(s)\} = f(t) * g(t)$$

where L^{-1} is inverse Laplace transform, * is convolution integral

$$f(t) = L^{-1}\{F(s)\} \ , \ g(t) = L^{-1}\{G(s)\}$$

b) (10%) Please use the above result to compute the following

$$L^{-1}\{\frac{s}{(s^2+\varpi^2)^2}\} = ?$$

Problem 2. (25%)

a) (15%) Please find the solution of ordinary differential equation

$$(x^2+1)y'' - 2xy' + 2y = 6(x^2+1)^2$$

Boundary condition $y(x=0) = -1, y(x=1) = 5$

b) (10%) Please find the solution of ordinary differential equation

$$\frac{d^2y}{dx^2} = x(\frac{dy}{dx})^3$$

Problem 3. (25%)

Please find the following Principal Value Integral (P.V.)

$$P.V.\int_{-\infty}^{\infty} \frac{e^{qx}}{1-e^{x}}dx = ? \quad \text{for } 0 < q < 1$$

Problem 4. (25%)

Please Check or Interpret the following terms from a) to e)

a) (5%) "Riemann surface" for double roots, "Branch Cut" for the Plane

b) (5%) What is existence condition of f(z) by the "Conformal Mapping"?
for $z \in C$

c) (5%) What is existence condition of f(t) by the "Laplace Transform"?
for $0 < t < \infty$

d) (5%) What is existence condition of f(t) by the "Fourier Transform"?
for $-\infty < t < \infty$

e) (5%) What is condition of "Hermitian Matrix" of A?

國立臺北科技大學

九十四學年度機電整合研究所入學考試

工程數學試題

填准考證號碼

第一頁　共一頁

注意事項：

1. 本試題共七題，配分共 100 分。
2. 請按順序標明題號作答，不必抄題。
3. 全部答案均須答在答案卷之答案欄內，否則不予計分。

*** 參考積分公式 ***

$$\int \frac{du}{u\sqrt{u^2+a^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C \qquad \int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$$

$$\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C \qquad \int \frac{du}{\sqrt{u^2+a^2}} = \ln(u+\sqrt{u^2+a^2}) + C$$

$$\int e^{au}\sin bu = \frac{e^{au}(a\sin bu - b\cos bu)}{a^2+b^2} + C \qquad \int e^{au}\cos bu = \frac{e^{au}(a\cos bu + b\sin bu)}{a^2+b^2} + C$$

1. Refer to Fig. 1, if the relation between before and after the axis rotation can be represented by

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \mathbf{T} \cdot \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}.$$

Find the orthonormal transformation matrix T when a CCW rotation θ is applied to x-axis. (5%)

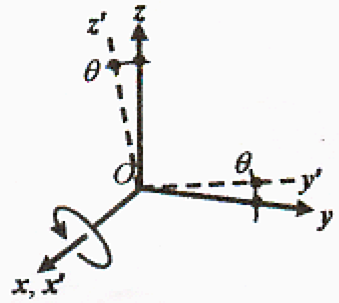


Fig. 1

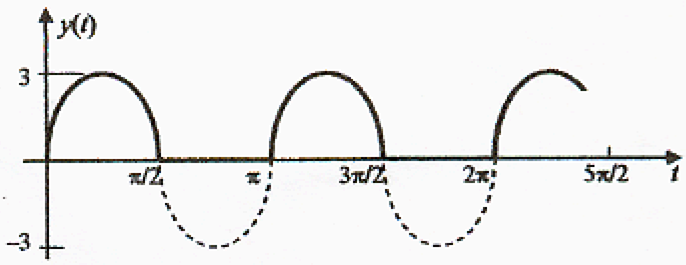


Fig. 2

2. A periodic function $y(t)$ can be represented by the rectified sine function as shown in Fig. 2. Let $u(t)$ denote the unit-step function, then
   (a) Write $y(t)$ in terms of unit step functions; (5%)
   (b) Find the Laplace transform of $y(t)$. (10%)

3. If $x(t)$ and $y(t)$ satisfy the differential equations
$$\begin{cases} x' - y' + x = 5 \\ x'' - y' + 3x - y = e^{3t} \end{cases}$$
where $'$ and $''$ denote $\frac{d}{dt}$ and $\frac{d^2}{dt^2}$, respectively. What are $x(t)$ and $y(t)$? (20%)

4. Find a solution for $f(x, y)$ that satisfies the partial differential equation
$$x\frac{\partial f}{\partial x} + 2y\frac{\partial f}{\partial y} = 0. \quad (15\%)$$

5. A vector field $\mathbf{F}$ is defined by $\mathbf{F} = 2x\,\mathbf{i} + 6y\,\mathbf{j} + 2z\,\mathbf{k}$. Refer to Fig. 3, if one moves a particle along path C starting from O to D. Let $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ be a position vector from O.
   (a) Please compute $\int_C \mathbf{F} \cdot d\mathbf{r} = ?$ (10%)
   (b) If path C is reverse, i.e., $O \to \text{D} \to \text{B} \to \text{A}$, and back to O, then $\int_C \mathbf{F} \cdot d\mathbf{r} = ?$ (5%)

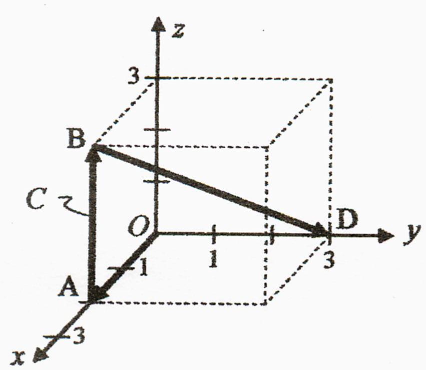


Fig. 3

6. If a matrix $\mathbf{A}$ is defined as
$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix},$$
Find all the eigenvalues of A. (15%)

7. Solve the differential equation $xy'' - 2\sqrt{1 + (y')^2} = 0$ for $y(x)$. (15%)

國立臺北科技大學九十五學年度碩士班招生考試

系所組別：1111、1112、1120 機電整合研究所甲乙組

第一節 工程數學 試題

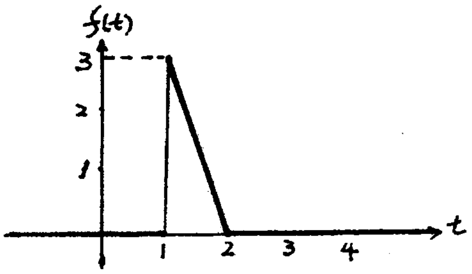
填准考證號碼

第一頁 共一頁

注意事項：

1. 本試題共 10 題，每題 10 分，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. 若向量函數 $\mathbf{F}(x,y)=(ye^{xy}+xy^2e^{xy}+2x)i+(xe^{xy}+x^2ye^{xy}-2y)j=\nabla\phi(x,y)$.
   試求位勢函數(potential function) $\phi(x,y)$.
2. 寫出微分方程式 $y''+2y'+y=e^{-t}$ 之任二個特解.
3. 函數 f(t)在 1<t<2 時,為如右圖之一直線,其他時為 0.求其拉氏轉換 Laplace transform F(s)



4. 向量函數 $\mathbf{F}(x,y,z)=xyi-\cos(yz)j+xzk.$,
   C 為從點(1,0,3)到(-2,1,3)之一直線.
   求 $\mathbf{F}(x,y,z)$沿直線 C 之線積分.
5. 求解聯立微分方程式 $X'=\begin{pmatrix}1 & 6\\ 1 & 2\end{pmatrix}X$, $X(0)=\begin{pmatrix}1\\ 0\end{pmatrix}$
6. $z=\sqrt{x^2+y^2}$ 代表一圓錐面.面上一點 P 之坐標為 x=1,y=1.求此圓錐面在 P 點之切平面方程式.
7. 流場 F=xi+yj-zk, Σ 為平面 x+y+z=1 在第一象限內部分,
   求此流場從原點方向流經 Σ 之通量 Flux.
8. 函數 f(x)= $x^4+2\cos 4x-3x+5, -2\le x\le 2$. 若以 Fourier series 表示為
   $$\frac{1}{2}a_0+\sum_{n=1}^{\infty}[a_n\cos\frac{n\pi x}{2}+b_n\sin\frac{n\pi x}{2}],\quad n=1,2,3\cdots$$
   試求 b_3 之值.

9. (1) 求解 $y''(x)+\frac{4\pi^2}{9}y(x)=0$， y(0)=y(3)=0

(2) 求解 $y''(x)+\frac{4}{9}y(x)=0$， y(0)=y(3)=0

10. 一長 L,原來均勻溫度 80℃之直棒,假設只有沿軸向(x 方向)之熱傳,若棒左端(x=0)保持絕熱狀態,右端(x=L)則和永遠保持 0℃之物體接觸.試寫出此邊界值問題之微分方程式及邊界條件(不必解出). [溫度分佈以 u(x,t)表示].