

淡江大學

土木工程學系

91~97 學年度

工程數學考古題

淡江大學九十一學年度碩士班招生考試試題

系別：土木工程學系

科目：工程數學

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本試題共 / 頁

1. (25%)

Given a matrix $A = \begin{bmatrix} 1-p & -1 & 0 \\ 0 & 2-p & 0 \\ 0 & -3 & 3-p \end{bmatrix}$, where p is a constant.

- (a) (9%) Determine the relationship between the rank of A and the value of p .
- (b) (8%) Let $p=0$, determine the eigenvalues and eigenvectors.
- (c) (8%) Let $p=4$, determine the inverse, A^{-1} .

2. (25%)

Solve the following linear system of differential equations by the Laplace transform.

$$\begin{cases} y_1'' - y_2' = 2 \cos 2t \\ y_2'' + y_1' = -2 \sin 2t \end{cases}, \text{ where } y_1' = \frac{dy_1}{dt}, y_2' = \frac{dy_2}{dt}, y_1'' = \frac{d^2 y_1}{dt^2}, y_2'' = \frac{d^2 y_2}{dt^2}$$

$$y_1(0) = -1, y_2(0) = 1, y_1'(0) = 1, y_2'(0) = 2$$

3. (25%)

Given the equation $a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = r(x)$, where a , b , and c are constants.

- (a) (8%) If $r(x)=0$ and $b^2=4ac$, find the solution $y(x)$.
- (b) (8%) If $r(x)=0$ and $b^2=3ac$ (where $ac>0$), find the solution $y(x)$.
- (c) (9%) If $r(x)=de^{fx}$ (where d and f are constants), and $b^2=4ac$, $f = -\frac{b}{2a}$, find the solution $y(x)$.

4. (25%)

Given the line integral $\int_{(1,1,5)}^{(2,0,5)} (kx^2 \sin \pi y dx + \pi x^3 \cos \pi y dy)$, where k is a constant.

- (a) (10%) Determine the value of k that the line integral is independent of path.
- (b) (15%) Evaluate the path independent integral.

淡江大學九十二學年度碩士班招生考試試題

系別：土木工程學系

科目：工 程 數 學

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本試題共 / 頁

1. Solve the following equation: (20%)

1) $1 + (3x - e^{-2x})y' = 0$

2) $y' + \frac{1}{x}y = 3x^2y^3$

3) $\cos x (e^{2y} - y_0) \frac{dy}{dx} = e^y \sin 2x, y(0) = 0$

4) $y'' - 4y' + 4y = 0, y(0) = 3, y'(0) = 1$

2. Solve $y' + y = f(t)$ by Laplace Transform (20%)

$$\text{where } y(0) = 5, f(t) = \begin{cases} 0 & 0 \leq t < \pi \\ 3 \sin(t) & t > \pi \end{cases}$$

3. Find the eigenvalues of the matrix B and, for each eigenvalue, a corresponding eigenvector. Also check that eigenvectors associated with distinct eigenvalues are orthogonal. (20%)

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

4. Use the Green's theorem to calculate $\oint_C \mathbf{F} \cdot d\mathbf{R}$. The curve is oriented counterclockwise.Where $\mathbf{F} = 2y \mathbf{i} - x \mathbf{j}$, and C the circle of radius 4 about (1, 3). (20%)5. Evaluate $\iint_{\Sigma} z d\sigma$, with Σ the part of the plane $x + y + z = 4$ lying above the rectangle $0 \leq x \leq 2, 0 \leq y \leq 1$. (20%)

淡江大學九十三年學年度碩士班招生考試試題

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本試題共 / 頁

1. Use "Integrating Factor" to solve the first-order differential equation

$$(xe^x + x^2e^x)y' = -xye^x - x^2e^x \quad (25\%)$$

2. Solve the initial value problem: $x^3y''' - 3x^2y'' + 6xy' - 6y = 2x^6$,

$$y(1) = 0, y'(1) = 1, y''(1) = -1 \quad (25\%)$$

3. Use Laplace Transform to solve the initial value problem:

$$y'' + 10y' + 25y = e^{-3t} + 2\delta(t-1), \quad y(0) = 1, y'(0) = 0 \quad (25\%)$$

4. (a) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 3 \\ 3 & -2 \end{bmatrix}$. (10%)

(b) Please explain the meanings of "A is Positive-Definite" and "A is Negative-Definite". (5%)

(c) Find the diagonal matrix $\hat{A}$ which is "similar" to A. (10%)

淡江大學九十四學年度碩士班招生考試試題 (2-1)

系別：土木工程學系

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本試題共 1 頁

1. Solve the following equations:

(1) $y''' - 5y'' + 3y' + 9y = 0$ (10%)

(2) $\frac{dv}{dt} = g - \frac{c}{m}v$, $v(0) = 0$, g, c, m are constants (10%)

2. Solve the boundary value problem: (25%)

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial z^2}, \quad 0 \leq z \leq 2H, \quad u(0, t) = 0 \quad \text{and} \quad u(2H, t) = 0$$

3. Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{pmatrix}$ (25%)

4. Use the Laplace transform to solve the following initial-value problem.

(1) $y' + 6y = e^{4t}$, $y(0) = 2$ (15%)

(2) $y''' - y' = \sin t$, $y(0) = 2$, $y'(0) = 0$, $y''(0) = 1$ (15%)

淡江大學 96 學年度碩士班招生考試試題

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本試題共 1 頁

1. Solve the following Differential Equation ? (20%)

a) $1 + (3x - e^{-2y}) y' = 0$

b) $x^2 y'' - 5x y' + 10y = 0$; $y(1) = 4$, $y'(1) = 6$

c) $y'' - 4y' + 4y = e^{3x} - 1$

2. Find the first five nonzero terms of Power Series Solution of the initial value problem, about the point where the initial condition is given. (20%)

$$y'' + 2xy' + (x-1)y = 0$$
 ; $y(0) = 1$, $y'(0) = 2$

3. Use the laplace transform to solve the folling I.V.P. (20%)

a) $y'' + 4y = f(t)$; $y(0) = y'(0) = 0$, $f(t) = \begin{cases} 0, & t < 3 \\ t, & t \geq 3 \end{cases}$

b) $y' + 2y = e^{-t}$; $y(0) = 1$

4. Find the general Solution of the system. (20%)

$$4x_1 - 2x_2 + 3x_3 + 10x_4 = 1$$

$$x_1 - 3x_4 = 8$$

$$2x_1 - 3x_2 + x_4 = 16$$

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本試題共 / 頁， 大題

1.

Solve the following Differential Equation ?

a) $xy dx + (2x^2 + 3y^2 - 20) dy = 0$

b) $y' = 2x^2 - \frac{y}{x} \quad y(1) = 5$

c) $x^2 y'' + 3xy' + 10y = 0$

d) $y'' - 6y' + 9y = 5e^{3x}$

2. Use Laplace transform to solve the initial value problem

$$y'' + y = 1 \quad y(0) = 6, \quad y'(0) = 0$$

3.

$$A = \begin{bmatrix} -3 & -3 \\ -2 & 4 \end{bmatrix} \quad \text{find } A^{16} = ?$$

4. let $f(x) = x$ for $-\pi \leq x \leq \pi$, Write the Fourier Series of f on $[-\pi, \pi]$

淡江大學

水資源及環境工程學系

91~97 學年度

工程數學考古題

淡江大學九十一學年度碩士班招生考試試題

系別：水資源及環境工程學系

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本試題共 / 頁

1. $y' - \left(\frac{y^2}{x} + \frac{y}{x} - \frac{2}{x} \right) = 0$ (20%)

Calculate y

2. $y'' - \left(\frac{3}{x} \right) y' + \left(\frac{4}{x^2} \right) y = 0$ (20%)

Calculate y

3. Evaluate the line integral of $\vec{G}(x, y, z) = x\vec{i} - y\vec{j} + z\vec{k}$ over the straight line segment from $(1, 1, 1)$ to $(-2, 1, 4)$. (20%)

4. A position vector $\vec{P}(t)$ is given,

$$\vec{P}(t) = [\cos t + t \sin t] \vec{i} + [\sin t - t \cos t] \vec{j} + t^2 \vec{k}, \quad t > 0$$

determine the normal component of the acceleration, the curvature, and the unit normal vector. (20%)

5. $f(t) = \begin{cases} 3 & \text{if } 0 < t < \pi \\ 0 & \text{if } \pi < t < 2\pi \\ \sin t & \text{if } t > 2\pi \end{cases}$ (20%)

Calculate $\mathcal{L}(f(t))$

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本試題共 / 頁

(一) 請求解 Sturm-Liouville Problem 之 Eigenvalues 及其 Eigenfunctions,

並直接驗證其正交性(orthogonality); (25%)

$$(xy')' + \lambda x^{-1}y = 0, \quad y(1) = 0, \quad y'(e) = 0, \quad \text{Hint: set } x = e^t$$

(二) 請求解 A 矩陣 Eigenvectors 之基底向量，再求其主對角化之矩陣；

(25%)

$$A = \begin{bmatrix} 5 & 10 & -10 \\ 10 & 5 & -20 \\ 5 & -5 & -10 \end{bmatrix}$$

(三) 請求解 Quasi-linear Partial Differential Equation. $z = z(x, y) = ?$; (25%)

$$(y+z)\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} - (x-y) = 0,$$

Initial condition 為：

$$\text{當 } y=1, \quad |x| < \infty \text{ 時, } z=1+x$$

(四) 請求出 Even 及 Odd half-range expansions; (25%)

$$f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2} < x < L \end{cases}$$

淡江大學九十三年學年度碩士班招生考試試題

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本試題共 / 頁

1. Solve $\frac{\partial u}{\partial t} + 3 \frac{\partial u}{\partial x} + 5u = 0$,
 $0 < t < \infty$, $-\infty < x < \infty$
 with IC $u(x, 0) = \cos x$ (20%)

2. Evaluate

$$I = \iiint_S (x^3 dy dz + x^2 y dz dx + x^2 z dx dy)$$

 where S is the closed surface consisting of cylinder $x^2 + y^2 = a^2$
 $(0 \leq z \leq b)$ and the circular disks $z=0$ and $z=b$ ($x^2 + y^2 \leq a^2$) (20%)

3. Solve $y_1'' + 10y_1 - 4y_2 = 0$
 $4y_1 + y_2'' + 4y_2 = 0$ (20%)
 Subject to $y_1(0) = 0$, $y_1'(0) = 1$
 $y_2(0) = 0$, $y_2'(0) = -1$

4. Solve $x^2 y'' - 3xy' + 3y = 2x^4 e^x$ (20%)

5. Solve $f(t) = 3t^2 - e^{-t} - \int_0^t f(\tau) e^{t-\tau} d\tau$ for $f(t)$

Hint: $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$

$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

(20%)

淡江大學九十四學年度碩士班招生考試試題 ⁷⁰⁻¹

系別：水資源及環境工程學系 科目：工 程 數 學

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本試題共 / 頁	

1. 求曲面 $z^2 = 4(x^2 + y^2)$

(a) 在點 $P(1,0,2)$ 的單位正交向量 (Unit Normal Vector $\vec{n}$) (10分)

(b) 在點 $P(1,0,2)$ 的Divergence值 (Div f) (10分)

2. 求解方程式 $y'' - 2y' + y = e^x + x$ (20分)

Initial Conditions: $y(0) = 1, y'(0) = 0$

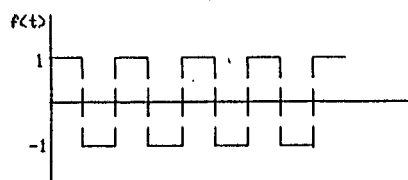
2. 求解方程式 $2xyy' - y^2 + x^2 = 0$, (20分)

Hint: Set $u = (\frac{y}{x})$

4. 求下列函數 $f(t)$ 之 Laplace Transform $F(s)$: (20分)

(1) $f(t) = t^2, (0 < t < 3)$

(2)



5. 求解聯立 Ordinary Equations 之近似解 $y_1(t), y_2(t)$, For $t = 0 \sim 2$ (20分)

$$\frac{dy_1}{dt} = -0.5y_1$$

$$\frac{dy_2}{dt} = 4 - 0.3y_2 - 0.1y_1$$

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本試題共 / 頁

1. Solving $2(e^x \cos y + 1)y' - 4x + 2e^x \sin y = 0$
(20%)

2. Determining $\mathcal{L}[f]$ if $f(t) = 0$ for $0 \leq t < 2$
and $f(t) = 1 + t^2$ for $t \geq 2$
(20%)

3. Finding the function $y(x)$ that passes through the points $(0, 0)$ and $(1, 1)$ and minimizes $J[y] = \int_0^1 [y^2 + y'^2] dx$
(20%)

4. Solving partial Differential Egn
 $u_t = a^2 u_{xx}$
B.C. $\begin{cases} u_x(0, t) = 0 \\ u_x(L, t) = 0 \end{cases} \quad 0 \leq x \leq L$
I.C. $u(x, 0) = f(x)$
(20%)

5. Evaluating the line integral of $\vec{F}(x, y, z) = -x\vec{i} + y\vec{j} - z\vec{k}$ over the straight line segment from $(1, 1, 1)$ to $(-2, 1, 3)$
(20%)

淡江大學 96 學年度碩士班招生考試試題

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本試題共 1 頁

1. 求解矩陣 $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$ (20 分)

(a) 反矩陣(Inverse Matrix)

(b) 特徵值(eigenvalues)

2. 求下列函數之 Laplace 轉換(Laplace Transform) (20 分)

(a) $f(t) = t^2 e^{5t}$

(b) $f(t) = e^{-2t}(2 \cos 3t - \sin 3t)$

3. 求解 Bernoulli equation $y' + xy = xy^{-1}$ (20 分)

4. 求解方程式 $y'' + y' - 2y = 0$ (20 分)

邊界條件： $y(0) = 0, y(1) = e - \frac{1}{e^2}$

5. 求偏微分方程式的解 $u(x, y)$ (20 分)

$$u_x - y u_y = 0$$

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本試題共 / 頁，5 大題

1. Solve the initial value problem

$$(e^{x+y} + ye^y)dx + (xe^y - 1)dy = 0, \quad y(0) = 1 \quad (20 \text{ 分})$$

2. Solve the ordinary differential equation

$$(x^2 - x)y'' - xy' + y = 0 \quad (20 \text{ 分})$$

3. Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \quad (20 \text{ 分})$$

4. Evaluate $\iint_S (8x\vec{i} + 2z\vec{k}) \cdot \vec{n} dA$ over the sphere

$$x^2 + y^2 + z^2 = 9 \quad (20 \text{ 分})$$

5. Find the Fourier series of the function

$$f(x) = \begin{cases} 0 & \text{if } -2 < x < -1 \\ k & \text{if } -1 < x < 1 \\ 0 & \text{if } 1 < x < 2 \end{cases}$$

$$\text{Period } p=2L=4, \quad L=2 \quad (20 \text{ 分})$$

淡江大學

航空太空工程學系

91~97 學年度

工程數學考古題

淡江大學九十一學年度碩士班招生考試試題

系別：航空太空工程學系

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本試題共 / 頁

1. Write the complex Fourier series of the following periodic function f and determine what this series converges to.

$$f(x) = \begin{cases} x^2, & 0 \leq x < 1, \\ f(x+1) & \text{for all } x. \end{cases} \quad (20\%)$$

2. Use the Laplace transform to solve the following system

$$\begin{cases} x' + y' - x = \cos(2t), \\ x' + 2y' = 0, \end{cases} \quad x(0) = y(0) = 0. \quad (20\%)$$

3. Consider the regular Sturm-Liouville problem

$$(r(x)y'(x))' + (q(x) + \lambda p(x))y(x) = 0,$$

$$A_1 y(a) + A_2 y'(a) = 0, \quad B_1 y(b) + B_2 y'(b) = 0.$$

If λ_n and λ_m are distinct eigenvalues of this problem, with corresponding eigenfunctions φ_n and φ_m respectively, then prove

$$\int_a^b p(x) \varphi_n(x) \varphi_m(x) dx = 0. \quad (20\%)$$

4. Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{\cos(4x)}{(x^2+1)^2} dx. \quad (20\%)$$

5. Solve the system $X' = AX$, with

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -2 & -2 \\ 0 & 2 & 0 \end{bmatrix}. \quad (20\%)$$

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本試題共 / 頁

1. Solve the initial value problem

$$y'' - 6y' + 18y = 0, \quad y(0) = 0, \quad y'(0) = 6 \quad 15\%$$

2. Solve the initial value problem

$$x^2 y'' - 4xy' + 4y = 0, \quad y(1) = 4, \quad y'(1) = 13 \quad 15\%$$

3. Apply the power series method to solve the following equation

$$y' = 2y \quad 15\%$$

4. Solve the following differential equation by the method of Laplace

$$\text{transforms } y'' + 2y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 1$$

15%

5. Find the eigenvalues and eigenvectors of the following matrix

$$\begin{bmatrix} 6 & 10 & 6 \\ 0 & 8 & 12 \\ 0 & 0 & 2 \end{bmatrix} \quad 20\%$$

6. Find the temperature, $T(t, x)$, in a laterally insulated bar of length L

whose ends are kept at temperature 0.

$$\frac{\partial T}{\partial t} = c^2 \frac{\partial^2 T}{\partial x^2}, \quad T(t, 0) = 0, \quad T(t, L) = 0$$

The parameter c is a constant, and assuming that the initial

$$\text{temperature is } T(0, x) = \sin \frac{\pi}{L} x, \text{ if } 0 \leq x \leq L \quad 20\%$$

淡江大學九十三年學年度碩士班招生考試試題

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本試題共 / 頁

1. Solve $y'' - 4y' + 3y = -3\sin(x+2)$; $y(-2) = 2$, $y'(-2) = 2$. (20%)

2. Solve the following system

$$x' + 4y' - y = 0, \quad x' + 2y = e^{-t}; \quad x(0) = y(0) = 0. \quad (20\%)$$

3. Compute

(1) $\int_0^{2\pi} \frac{\sin^2(\theta)}{2 + \cos(\theta)} d\theta$. (10%)

(2) $\int_{-\infty}^{\infty} \frac{1}{x^6 + 64} dx$. (10%)

4. Let A and B be $n \times n$ matrices such that $AB = BA$. Prove that

$$e^{(A+B)t} = e^{At} e^{Bt}. \quad (15\%)$$

5. Use the Laplace transform to solve the following boundary value problem

$$c^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}, \quad (0 < x < \infty, t > 0, c \text{ is a constant}),$$

$$y(0, t) = f(t),$$

$$\frac{\partial y}{\partial t}(x, 0) = 0,$$

$$y(x, 0) = 0. \quad (25\%)$$

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系別：航空太空工程學系

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本試題共 1 頁

1. Solve $y'' + 5y' + 6y = 3\delta(t-2) - 4\delta(t-5)$; $y(0) = y'(0) = 0$. (20%)

Note: Here $\delta(\)$ is the Dirac delta function.

2. Find the family of orthogonal trajectories of the family $y^2 = Kx^3$, where K is a constant. (20%)

Note: Sometimes we encounter two families of curves in which any curve of one family is orthogonal to any curve of the other wherever two such curves intersect. Two families of curves that are in this sense are called orthogonal trajectories.

3. Solve $x^2y'' - 2xy' + 2y = 10\sin(\ln(x))$; $y(1) = 3$, $y'(1) = 0$. (20%)

4. Find the inverse of the following matrix or else show that this matrix is singular.

$$A = \begin{bmatrix} -2 & 6 & 0 & 0 \\ 1 & 4 & 4 & 11 \\ 4 & -4 & -5 & 3 \\ -3 & 1 & 2 & -6 \end{bmatrix}. \quad (10\%)$$

5. Find the Fourier series of $f(x) = x^4$ on $[-\pi, \pi]$. (15%)

6. Use the Laplace transform to solve the following boundary value problem

$$a^2 \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}, \quad (x > 0, t > 0; a \text{ is a constant.})$$

$$y(0, t) = t, \quad (t > 0)$$

$$\frac{\partial y}{\partial t}(x, 0) = B, \quad (x > 0; B \text{ is a constant.})$$

$$y(x, 0) = 0. \quad (x > 0) \quad (15\%)$$

淡江大學 95 學年度碩士班招生考試試題

81

系別：航空太空工程學系

科目：工 程 數 學

准帶項目請打「V」	
✓	簡單型計算機
本試題共 2 頁	

1. Solve $y'' + y' = te^t$; $y(0) = 0$, $y'(0) = 0$. (20%)
2. Use the Convolution theorem to find the inverse Laplace transform of each function

$$(a) \frac{s}{(s+1)(s+2)^3}, \quad (10\%)$$

$$(b) \frac{s^2 + 4s + 4}{(s^2 + 4s + 13)^2}. \quad (10\%)$$

3. Find the general solution of

$$\mathbf{x}' = \begin{bmatrix} -8 & -1 \\ 16 & 0 \end{bmatrix} \mathbf{x}. \quad (20\%)$$

4. Let $\mathbf{A}$ be unitary, hermitian, or skew-hermitian. Prove that

$$\overline{\mathbf{A}}\mathbf{A}^T = \overline{\mathbf{A}}\mathbf{A}. \quad (20\%)$$

5. Use the Method of Separation of variables to construct a series solution of the heat equation

$$\frac{\partial u}{\partial t} - k \frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < L, \quad t > 0, \quad k \text{ is a constant,}$$

and the following initial and boundary conditions

$$\begin{aligned} u(0, t) &= u(L, t) = 0, \\ u(x, 0) &= \sin(2\pi x / L). \end{aligned} \quad (20\%)$$

淡江大學 96 學年度碩士班招生考試試題

系別：航空太空工程學系

科目：工 程 數 學

准帶項目請打「V」	
✓	簡單型計算機

本試題共 / 頁

1. Solve the given non-homogeneous differential equation,

$$y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$$

(20%)

2. Use the Laplace transform method to solve the given initial-value problem,

$$y'' + 16y = \cos 4t, \quad y(0) = 0, \quad y'(0) = 1$$

(20%)

3. Solve the given differential equation subject to the indicated initial conditions,

$$x^2 y'' + xy' + y = 0; \quad y(1) = 1, \quad y'(1) = 2$$

(20%)

4. Use Gaussian elimination or Gauss-Jordan elimination to solve the given system of equations,

$$2x_1 + x_2 + x_3 = 3$$

$$3x_1 + x_2 + x_3 + x_4 = 4$$

$$x_1 + 2x_2 + 2x_3 + 3x_4 = 3$$

$$4x_1 + 5x_2 - 2x_3 + x_4 = 16$$

(20%)

5. Use the method of separation of variables to solve the heat equation

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0, \quad \text{subject to the given conditions}$$

$$u(0, t) = 100, \quad u(1, t) = 100, \quad u(x, 0) = 0$$

(20%)

淡江大學 97 學年度碩士班招生考試試題

79

系別：航空太空工程學系

科目：工 程 數 學

准帶項目請打「V」	
☒	簡單型計算機

本試題共 乙 頁，五 大題

1. (20 points) Find the general solutions (real-valued) of the systems below.
 - (a) $x_1' = 5x_1 + 3x_2$; $x_2' = -x_1 + x_2$.
 - (b) $x_1' = 2x_1 - x_2$, $x_2' = 8x_1 - 2x_2$.
2. (20 points) Find all solutions of $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where the matrix $\mathbf{A}$ is as given below.
 - (a) $\mathbf{A} = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}$.
 - (b) $\mathbf{A} = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}$.
3. (20 points) Consider the initial value problem:

$$\mathbf{x}' = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & -2 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$
 - (a) If one of its eigenvalues is -1 , find the other two eigenvalues.
 - (b) Obtain the associated eigenvectors.
 - (c) Obtain the solution of the initial value problem.
4. (20 points) Use Convolution Theorem to find the inverse Laplace transform of each function. Evaluate the integral to your limits.
 - (a) $\frac{s}{(s^2 + 1)^2}$.
 - (b) $\frac{\mathcal{L}[f]}{s^2 + 1}$.
5. (20 points) Use Laplace transform to solve the following initial value problem.

$$y'' - 7y' + 12y = 6e^{3t}, \quad y(0) = -1, \quad y'(0) = 2.$$

淡江大學

電機工程學系

91~97 學年度

工程數學考古題

淡江大學九十一學年度碩士班招生考試試題

系別：電機工程學系

科目：工程數學

准帶項目請打「○」否則打「×」	
計算機	字典
×	×

本試題共 1 頁

一. $xy' = \frac{y^2}{x} + y$, $y(x) = ?$ (20%)

二. $y'' - \frac{4}{x}y' + \frac{4}{x^2}y = x^2 + 1$, $y(x) = ?$ (20%)

三. Produce a matrix that diagonalizes matrix A

$$A = \begin{bmatrix} -1 & -4 \\ 3 & 2 \end{bmatrix} \quad (20\%)$$

四. $\oint_T \frac{2z+1}{z^3+3iz^2} dz = ?$ $T = |z+i| = 6$ (20%)

五.
$$\begin{cases} \frac{\partial^2 y}{\partial x^2} = 9 \frac{\partial^2 y}{\partial x^2} & (-\infty < x < \infty, t > 0) \\ y(x, 0) = 4e^{-5|x|} & (-\infty < x < \infty) \\ \frac{\partial y}{\partial t}(x, 0) = 0 \end{cases} \quad (20\%)$$

淡江大學九十二學年度碩士班招生考試試題

系別：電機工程學系

科目：工 程 數 學

准帶項目請打「○」否則打「×」
簡單型計算機
×

本試題共 1 頁

一. $y''(t) + y(t) = f(t)$; $y(0) = y'(0) = 0$, where

$$f(t) = \begin{cases} 0 & \text{for } t < 1 \\ t & \text{for } t \geq 1 \end{cases} \quad \text{Find } y(t). \quad (20\%)$$

二. $y_1(x) = x$ for $x > 0$ is a solution of the following differential equation $x^3 y''(x) - 4x y'(x) + 4y(x) = 0$. Please find a second nontrivial solution. (20%)

三. Given $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{pmatrix}$. (1) Find all the solutions $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ of the equation $A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$. (10%) (2) Compute the sum of all the eigenvalues of A . (5%) (3) Is it true that all the eigenvalues of A have positive real parts? (5%)

四. Evaluate $\oint_C \frac{-y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy$, where C is the simple, closed, positively oriented curve satisfying the equation $x^2 + \frac{y^2}{4} = 1$. (20%)

五. (1) Find the Fourier transform of $f(t) = e^{-a|t|}$, $t \in \mathbb{R}$, where $a > 0$. (10%) (2) Find the inverse Fourier transform of $\frac{12}{(1+w^2)(4+w^2)}$. (10%)

淡江大學九十三年學年度碩士班招生考試試題

系別：電機工程學系

科目：工 程 數 學

准帶項目請打「○」否則打「×」
簡單型計算機
×

本試題共 1 頁

一. $(y^2 - 6xy) dx + (3xy - 6x^2) dy = 0$, $y(1) = -2$ (20%)

二. $y'' + 2y' + 2y = \delta(t-3)$, $y(0) = y'(0) = 0$ (20%)

三. Find the rank and nullity of the matrix

$$\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix} \quad (20\%)$$

四. $\oint_P \frac{\sin z}{z^2(z^2+4)} dz$, $P: |z-2i|=3$ (20%)

五. Let $f(x) = e^{-b|x|}$ for all real x , with b a positive constant. Find the complex Fourier integral representation of $f(x)$. (20%)

淡江大學九十四學年度碩士班招生考試試題³³⁻¹

系別：電機工程學系

科目：工 程 數 學

准帶項目請打「V」	
☒	簡單型計算機
本試題共 1 頁	

一. Solve $(y' + y)^2 = e^x$; $y(0) = 0$. (20%) $(y' \triangleq \frac{dy}{dx})$

二. Solve $f(t) = 6t^2 + \int_0^t f(t-\tau)e^{-\tau}d\tau$ for $f(t)$, $t \geq 0$. (20%)

三. Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$.

(1) Find all the eigenvalues of A . (10%)

(2) Find the rank of the augmented matrix $\begin{bmatrix} A & A^{10} \\ A^{10} & A^{10} \end{bmatrix}$ (10%)

四. Find the tangent plane to the surface

$\sin(x^2 + y^2) = z$ at the point $(1, 1, \sin(2))$. (20%)

五. Let $f(t) = \begin{cases} z & \text{if } -1 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$. Find the Fourier transform of the following functions.

(1) $f(t) \cos(10t)$. (10%)

(2) $f(t) \cos^2(10t)$. (10%)

系別：電機工程學系

科目：工 程 數 學

准帶項目請打「V」	
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本試題共 / 頁	

1. Linear Algebra

True or False. If you think the statement is true, mark "T"; if you think the statement is false, mark "F". You don't need to write down your reasons.

- (a) (5%) Every real 3×3 matrix having $\lambda = 1 + i$ as an eigenvalue is diagonalizable over the complex numbers. (Note: $i = \sqrt{-1}$)
- (b) (5%) If matrix A is similar to matrix B , then $\text{rank}(A) = \text{rank}(B)$.
- (c) (5%) If A is an $m \times n$ matrix having orthonormal columns, then $\det(AA^T) = \det(A^T A)$.
- (d) (5%) If A is a symmetric matrix such that $A^5 = 0$, then $A = 0$.
- (e) (5%) If A and B are 2×2 matrices with the same trace and the same determinant, then A and B have the same eigenvalues.

2. Linear Algebra

Suppose the singular value decomposition of a matrix A is $A = U\Sigma V^T$ where

$$U = \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad V = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

- (a) (5%) Find the eigenvalues of $A^T A$.
- (b) (5%) Find a basis for the nullspace of A .
- (c) (5%) Find a basis for the column space of A .

3. Complex Variables

- (a) (10%) Find the residue at $z = 0$ of $z^{-3} \csc(z^2)$.
- (b) (10%) Use residual theorem to compute the integral $\int_0^\infty \frac{1}{x^2 + a^2} dx$, $a > 0$.

4. Differential Equations

Consider a dynamical system described by the following set of two first-order differential equations

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} 1 & 9 \\ -1 & -5 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

- (a) (10%) Use eigenvalue/eigenvector concept to find $x_1(t)$ and $x_2(t)$ for $t \geq 0$.
- (b) (10%) Write down the equivalent second-order differential equation and the initial conditions in terms of $x_1(t)$ for the dynamical system. Use "Laplace transform" to solve the equivalent second-order differential equation and determine $x_1(t)$ and $x_2(t)$ for $t \geq 0$.

5. Fourier Series

- (a) (10%) Find the Fourier series of $|\sin(x/2)|$ on $[0, 2\pi]$.
- (b) (10%) Use the result in (a) to find $\sum_{n=1}^\infty \frac{(-1)^n}{4n^2 - 1}$.

系別：電機工程學系

科目：工 程 數 學

准帶項目請打「V」	
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本試題共 1 頁

一. Find the solution of the equation

$$[e^x \sin y - 2x] dx + [e^x \cos y + 1] dy = 0 \quad (20\%)$$

$$二. \quad x^2 y'' - 5x y' + 10y = 0 \quad y(1) = 4, \quad y'(1) = -6$$

(20%)

$$三. \quad \oint_C \frac{-y}{x^2 + y^2} dx + \frac{x}{x^2 + y^2} dy = ?$$

with C any simple, positively oriented path in the plane but not passing through the origin. (20%)

$$四. \quad \text{Find the Fourier transform of } f(t) = \begin{cases} 1 - |t|, & \text{for } |t| < 1 \\ 0, & \text{otherwise} \end{cases}$$

(20%)

$$五. \quad \text{Evaluate } \oint_P \frac{4iz - 1}{\sin(z)} dz, \text{ with } P \text{ any simple, positively oriented path in the plane enclosing the poles } 0, \pi, 2\pi \text{ and } -2\pi \text{ but no other singularities of } \frac{4iz - 1}{\sin(z)}. \quad (20\%)$$

淡江大學 97 學年度碩士班招生考試試題

79

系別：航空太空工程學系

科目：工 程 數 學

准帶項目請打「V」	
☒	簡單型計算機

本試題共 乙 頁，五 大題

1. (20 points) Find the general solutions (real-valued) of the systems below.
 - (a) $x_1' = 5x_1 + 3x_2$; $x_2' = -x_1 + x_2$.
 - (b) $x_1' = 2x_1 - x_2$, $x_2' = 8x_1 - 2x_2$.
2. (20 points) Find all solutions of $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where the matrix $\mathbf{A}$ is as given below.
 - (a) $\mathbf{A} = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}$.
 - (b) $\mathbf{A} = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}$.
3. (20 points) Consider the initial value problem:

$$\mathbf{x}' = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 0 & -2 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$
 - (a) If one of its eigenvalues is -1 , find the other two eigenvalues.
 - (b) Obtain the associated eigenvectors.
 - (c) Obtain the solution of the initial value problem.
4. (20 points) Use Convolution Theorem to find the inverse Laplace transform of each function. Evaluate the integral to your limits.
 - (a) $\frac{s}{(s^2 + 1)^2}$.
 - (b) $\frac{\mathcal{L}[f]}{s^2 + 1}$.
5. (20 points) Use Laplace transform to solve the following initial value problem.

$$y'' - 7y' + 12y = 6e^{3t}, \quad y(0) = -1, \quad y'(0) = 2.$$