高雄第一應用科技大學

光電工程研究所

93~97 學年度
工程數學考古題

國立高雄第一科技大學九十三學年度 碩士班 招生考試 試題紙

系 所 別：光電工程研究所　　　　組　別：不分組
考科代碼：1101　　　　　　　　　考　科：工程數學

注意事項：
1、本科目可使用本校提供之電子計算機。
2、請於答案卷上規定之範圍作答，違者該題不予計分。

每大題十分

(1) Solve (a) $xy^2dx + (x^2y^2 + x^2y)dy = 0$

(b) $\sin y \dfrac{dy}{dx} = (\cos x)(2\cos y - \sin^2 x)$

(2) Solve $x^3y''' + x^2y'' - 2xy' + 2y = x\ln x$ for $x > 0$

(3) Find the Laplace transform of $f(t) = t$ by using definition.

(4) Solve $\begin{cases} z'' + y' = \cos x \\ y'' - z = \sin x \end{cases}$; $z(0) = -1,\ z'(0) = -1,\ y(0) = 1,$

$y'(0) = 0$

(5) Evaluate $\oint_c \dfrac{e^z}{(z^2 + \pi^2)^2} dz$ where c is the circle $|z| = 4$.

(6) Find a basis of eigenvectors and diagonalize the matrix

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}.$$

(7) Assume that the velocity vector is $\mathbf{F} = y\mathbf{i} + \mathbf{j} - x^2 z\mathbf{k}$, speed being measured in meters/sec. Compute the flux of water through the surface S: $y = x^2,\ \ 0 \le x \le 2,\ \ 0 \le z \le 5$.

(8) Evaluate $\iint_S (xz^2\mathbf{i} + x^2y\mathbf{j} + y^2z\mathbf{k}) \cdot \mathbf{n}\, dA$, where S is the surface closed by $x^2 + y^2 + z^2 = 1$, and $\mathbf{n}$ is the outer unit normal vector of S.

(9) Evaluate $\iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{n}\, dA$, where $\mathbf{F} = y^2 e^{xz}\mathbf{i} + x^3 \cos z\mathbf{j} - e^{-xyz}\mathbf{k}$; S is the portion of $x^2 + y^2 + (z-3)^2 = 10$ that allocates above the $z = 0$ plane; and $\mathbf{n}$ is the outer unit normal vector of S.

(10) Find the Fourier cosine and sine transform of the function e^{-x}.

國立高雄第一科技大學 94 學年度 碩士班 招生考試 試題紙

系 所 別：光電工程研究所　　　　組 別：不分組

考科代碼：1501　　　　　　　　考 科：工程數學

注意事項：

1、本科目可使用本校提供之電子計算機。

2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. (10%) Solve the differential equation $y' = x^3(y-x)^2 + x^{-1}y$

2. (a) (5%) Find the Laplace transform of the function

$$f(t) = \begin{cases} 1 & if\ 0 < t < \pi \\ 0 & if\ \pi < t < 2\pi \\ \cos t & if\ t > 2\pi \end{cases}$$

(b) (5%) Find the inverse Laplace transform f(t) of

$$F(s) = \frac{2(1 - e^{-\pi s})}{(s^2 + 4)}$$

3. (10%) $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1}\, dt$, then find $\dfrac{\Gamma(\frac{2}{3})}{\Gamma(\frac{11}{3})}$

4. (10%) Evaluate $\oint_c \dfrac{e^z}{(z^2+1)^2}$, where c is the circle $|z - i| = 1$

5. (10%) Evaluate the surface integral $\iint_S (xy^2 i + y^3 j + y^2 z k) \bullet \mathrm{n}\, dA$, where S is the closed surface consisting of cylinder $x^2 + y^2 = a^2$ $(0 \le z \le b)$ and two circular disks z=0 and z=b ($x^2 + y^2 \le a^2$), and **n** is the outer unit normal vector of S.

6. (10%) Verify the Stokes's Theorem, if a vector F = $yi + zj + xk$, S is the surface by z = f(x, y) = 4 - (x^2+y^2), z $\ge$ 0.

7. (10%) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -6 & 6 & -9 \\ 6 & 3 & -18 \\ -3 & -6 & 0 \end{bmatrix}$$

8. (10%) Find the convergence interval of the series

$$\sum_{m=0}^{\infty} (\frac{2}{3})^m \ (x-3)^{2m+1}$$

9. (a) (5%) Find the Fourier series of the function

$$f(x) = \frac{x^2}{2} \quad , -\pi < x < \pi$$

(b) (5%) Using the result to show that

$$\frac{\pi^2}{6} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + ...$$

10. (10%) Evaluate the integral

$$f(x) = \int_1^3 x^{-3} J_4(x)\,dx$$, where $J_n(x)$ is the Bessel function of the first kind of order n and $J_0(1)$=0.7652, $J_0(3)$= -0.2601, $J_1(1)$= 0.4401 and $J_1(3)$= 0.3391

國立高雄第一科技大學 95 學年度 碩士班 招生考試 試題紙

系 所 別：光電工程研究所　　　　組　別：不分組

考科代碼：2201　　　　考　科：工程數學

注意事項：

1、本科目可使用本校提供之電子計算機。

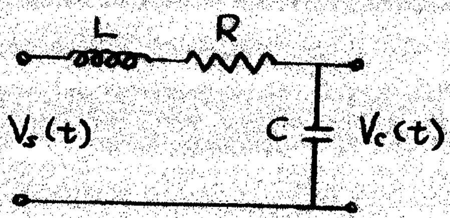
2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. (10%) Find the general solution of the differential equation $x^2y'' + 5xy' - 12y = x^2 + 1$

2. (12%) Use the Fourier transform to solve $y'' + 5y' + 6y = \delta(t-2)$

3. (10%)The Gamma function $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$, and $\Gamma\left(\frac{3}{2}\right) = 0.88622$, please find $\Gamma\left(-\frac{3}{2}\right)$

4. (12%) Please use Laplace transform to find the capacitor voltage $V_c(t)$ for t > 0 in the RLC circuit below by applying the Kirchhoff's law if the voltage source is $V_s = u(t) - u(t-a)$, where $u(t)$ is the step function and a > 0.



5. (10%) Find the total Electric Flux Φ of the vector Electric field E = xz^2i + yx^2 j+ zy^2k passing through the closed surface S in free space, where S is bounded by the surface of the cylinder $a \le x^2 + y^2 \le b$ and two planes z=0, z=c.

6. (12%) For the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, please find a matrix P and a diagonal matrix D such that $P^{-1}AP = D$

7. (12%) Write the Taylor series for $\frac{1}{1+x}$ about x_0, then find the range of x where this Taylor series converge?

8. (12%)Solve $\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2} + 3x$, $0 < x < 3$, $t > 0$; $y(0,t) = y(3,t) = 0$ for $t > 0$; $y(x,0) = \frac{\partial y}{\partial t}(x,0) = 0$ for $0 < x < 3$

9. (10%) A particle move once counterclockwise about the square with vertices (1,1), (3,1), (3,3) and (1,3) under the influence of the force F = [-cosh (x) + xy] i + [cos(y) + x] j , please calculate the work done.

國立高雄第一科技大學 96 學年度 碩士班 招生考試 試題紙

系 所 別：光電所　　　　組　別：不分組

考科代碼：2211　　　　考　科：工程數學

注意事項：

1、本科目可使用本校提供之電子計算機。

2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. (10%) Solve the differential equation $(y^2+1)\ dx = y\sec^2 2x\ dy$

2. (11%) Solve $x^2y'' - 2xy' + 2y = x^3 + 4\ln(x),\quad y(1)=5.5,\ y'(1)=6.5$

3. (12%)Use Laplace transform to find the solution for the following linear system

$$y'' - 3y' + x' + 2y - x = 0$$
$$y' + x' - 2y + x = 0$$
$$y(0) = y'(0) = 0,\ x'(0) = 1$$

4. (10%)Evaluate the integral $\int_{-\infty}^{\infty} \frac{\sin(\gamma x)}{(x^2+\alpha^2)(x^2+\beta^2)}dx$, where γ, α and β are positive constants and $\alpha \neq \beta$.

5. (10%)Evaluate $\oint_c \frac{z-4i}{z^3+4z}dz$, where c is the circle $|z|=4$.

6. (12%)Solve a boundary value problem for the motion of an infinite string, stretched the entire x-axis and released from rest with below initial position

$$\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}\quad (-\infty < x < \infty, t > 0)$$

$$y(x,0) = \begin{cases} 1+x & for -1 \le x \le 0 \\ 1-x & for\ 0 \le x \le 1 \\ 0 & for |x| > 1 \end{cases}$$

7. (10%)Explain Green's theorem (support your description with formula)

8. (10%)Explain Stokes's theorem (support your description with formula)

9. (15%)The Bessel function of the first kind of order ν

$J_\nu(x) = \sum_{n=0}^{\infty}\frac{(-1)^n}{2^{2n+\nu}n!\Gamma(n+\nu+1)}x^{2n+\nu}$, please show $J_0(x) = \frac{2}{\pi}\int_0^{\pi/2}\cos[x\sin(\theta)]\,d\theta$

Hint: $e^{ix\sin(\theta)} = \cos(x\sin(\theta)) + i\sin(x\sin(\theta))$

國立高雄第一科技大學 97 學年度 碩士班 招生考試　試題紙

系所別：光電工程研究所　　　組　別：甲、乙組

考科代碼：2211、2221　　　考　科：工程數學

注意事項：

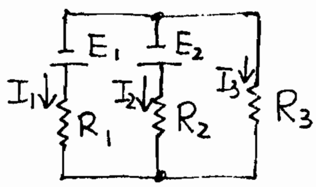
1、本科目可使用本校提供之電子計算器。

2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. (10%) Use Residue Theorem to evaluate $\int_0^{2\pi} \frac{1}{(2+Sin\theta)^2} d\theta$

2. (12%) Given the flow $f(z)=(1-i)Z$, please compute the circulation around, and the net flux across the circle $C:|z|=2$

3. (10%)Please use the Cramer's rule to find out the Current I_3 in the circuit



4. (12%) Expand the function $f(x)=x^2$, $0 < x < 1$, in a Fourier-Bessel series

$$f(x)=\sum_{i=1}^{\infty} c_i J_2(\alpha_i x), \text{ with } J_2(\alpha)=0.$$

5. (10%) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 2 & -1 & 0 \\ 5 & 2 & 4 \\ 0 & 1 & 2 \end{pmatrix}$

6. (12%) Given $f(x)=e^{-x}\cos x$, $x > 0$, find its Fourier cosine and sine integral representation.

7. (12%) $f(z)=\dfrac{1}{2+z}$, please expand the given function in the Taylor series centered at $z_0=-1$ and $z_0=i$, also find the region within which both series converge.

8. (10%) Evaluate the line integral $\oint_c \frac{-y^3}{(x^2+y^2)^2}dx + \frac{xy^2}{(x^2+y^2)^2}dy$, where C is the square with vertices (2,2), (-2,2), (-2,-2), (2,-2) in a counterclockwise direction.

9. (12%) A point charge q located at the origin create the electric field at the point (x,y,z) $E = q\frac{r}{\|r\|^3}$, where r = xi + yj + zk. Please find the outward flux of electric field E through a cubic formed by $x = y = z = \pm a$ six planes.

高雄第一應用科技大學

機械與自動化工程系

91~96 學年度
工程數學考古題

國立高雄第一科技大學 91 學年度 碩士班招生考試 試題紙

系別：機械與自动化工程系　　科目 工程數學

科目代碼 1401

一、試解: $(1+x)^3\dfrac{d^3y}{dx^3}+(1+x)^2\dfrac{d^2y}{dx^2}+3(1+x)\dfrac{dy}{dx}-8y=\dfrac{x}{\sqrt{1+x}}$ (20%)

二、試解: $£^{-1}\{\dfrac{s^2-a^2}{(s^2+a^2)^2}\}$ 並詳述計算步驟，

其中 $£^{-1}$ 表逆拉普拉斯轉換 (15%)

三、試解: $\dfrac{\partial^2 V}{\partial x^2}+\dfrac{\partial^2 V}{\partial y^2}=0 \quad (0<x<a,\quad 0<y<b)$

B.C. $x=0,\ \dfrac{\partial V}{\partial x}=f(y);\quad x=a;\ V=0$

$y=0,\ V=0;\quad y=b;\ \dfrac{\partial V}{\partial y}=0$ (15%)

四 Consider the force field $\vec{F}=y^2\,\vec{i}+2(xy+z)\,\vec{j}+2y\,\vec{k}$ (a) Determine the potential function (10%). (b) Evaluate $\int_{(111)}^{(222)}\vec{F}\cdot d\vec{r}$ (10%).

五 Find the minimal polynomial of $A=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$ (10%).

六 Find the Fourier series representation of the function $f(t)=t-t^2, 0<t<1$, (a) by half range cosine expansion (10%), (b) by half range sine expansion (10%).

國立高雄第一科技大學九十二學年度 碩士班 招生考試 試題紙

系所別：機械與自動化工程系　　　　　科　　目：工程數學（一）

組　別：　　　　　　　　　　　　　　科目代碼：1401

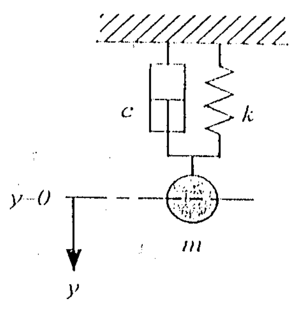
※本科目可使用本校提供之電子計算機。

共十題，每題十分

1. $xy' + y = x^3 + 2x$ Solve for $y(x)$.

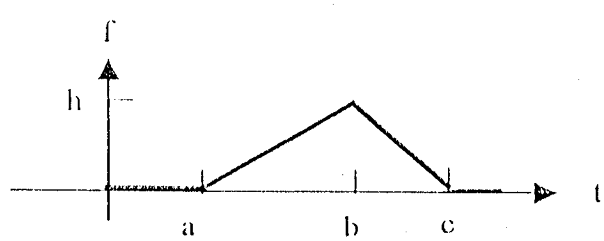
2. $3x^2 + 2e^{2x}\sin(y) + [e^{2x}\cos(y) + 3]\cdot y' = 0$, y(1)=0. Solve for $y(x)$.

3. An object of mass m=2 (kg) is suspended from a spring having a spring constant k=10 (N/m). Attached to the object is a shock absorber, which introduces a drag force of $8v$ (v is the velocity of the object in m/sec, c=2). Set y=0 when the object is at static equilibrium. The shock absorber is set in motion by lowering the object 0.5 (m) (i.e. $y(t=0) = 0.5$) and then striking it hard enough to impact an upward velocity of 5 (m/sec) (i.e. $y'(t=0) = -2$). Find the displacement function $y(t)$ of the mass.



4. $x^2y'' - 4xy' + 6y = 4x + 6\cdot ln(x)$, $x>0$. Solve for $y(x)$.

5. Write the function whose graph is shown below in terms of the Heaviside function, and find its Laplace transform.

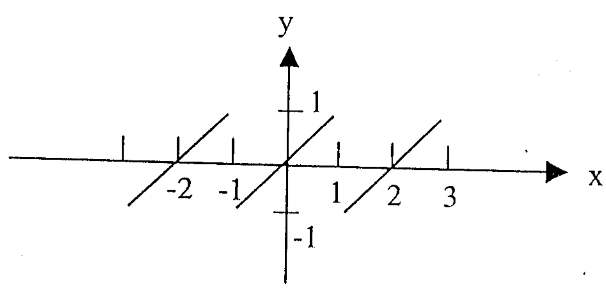


6. Use Laplace Transform to solve the initial value problem.

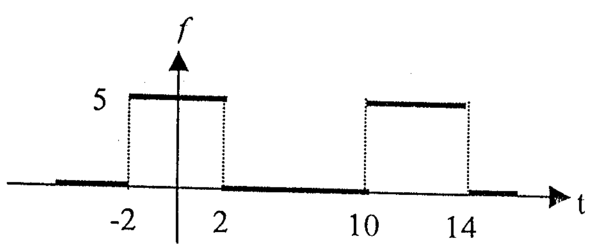
$$y'' + 2y' + 2y = f(t) \;,\quad y(0) = y'(0) = 0$$

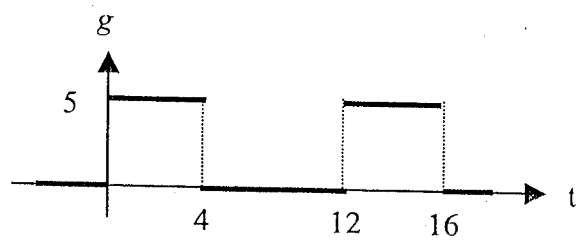
$$f(t) = \begin{cases} 0 & \text{if } t < 3 \\ 2 & \text{if } t \geq 3 \end{cases}$$

7. Part of the graph is given below. Find the phase angle form of the Fourier series of the function.



8. The graphs in the figures define two periodic functions f and g, respectively. Calculate the complex Fourier series of each function. Determine a relationship between the frequency spectra of the functions and also between their phase spectra.





9. Find the inverse Fourier transform of the function: $\dfrac{10\sin(3\omega)}{\omega + \pi}$

10. Assume that f is periodic with period p. Show that for any real number a,

$$\int_a^{a+p} f(x)dx = \int_0^p f(x)dx = \int_{-p/2}^{p/2} f(x)dx$$

國立高雄第一科技大學九十二學年度 碩士班 招生考試 試題紙

系所別：機械與自動化工程系　　　科　目：工程數學（二）

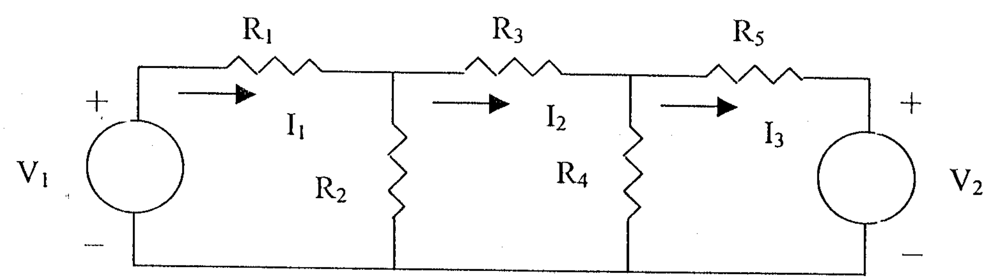
組　別：　　　　　　　　　　　　科目代碼：1402

※本科目可使用本校提供之電子計算機。

☆☆請在答案紙上作答☆☆

1. (15%) Solve the following circuit with two voltages system.

   (1) Form this system into augmented matrix. (2) Show the procedures for reduced row echelon form operation. (3) Compute the solutions; where $V_1=V_2=5$ volts, $R_1=R_2=R_3=R_4=R_5=1$ ohm.



2. (15%) If $F(x) = x^3-2x^2+x-2$, evaluate f(A), where the matrix $A=\begin{bmatrix}1 & 2\\ -3 & -4\end{bmatrix}$.

   (1) Find the eigenvalues and corresponding eigenvectors. (2) Evaluate f(A).

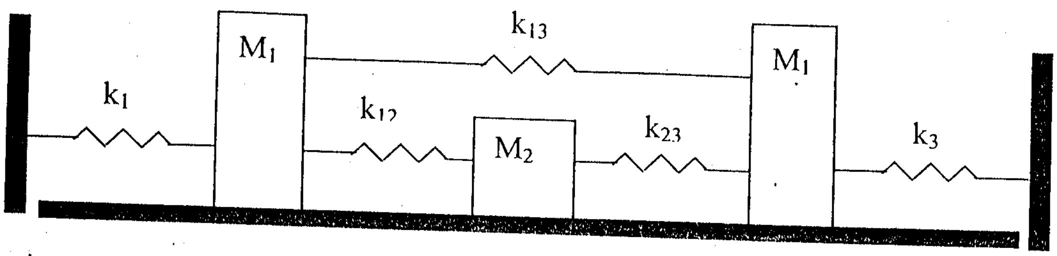
3. (15%) An external force, $\vec{F} = y\vec{i}+2x\vec{j}$, exerts on an object. Find the works from (0, 1) to (1, 0). (1) Along a straight line. (2) Along a circular arc. (3) Explain the difference.

4. (10%) Construct a set of orthonormal vectors from the given vectors：

$$\vec{V}_1=(1\quad 2\quad 2)，\vec{V}_2=(1\quad 4\quad 0)，\vec{V}_3=(2\quad 0\quad 1)$$

5. (10%) (1) Find the status of intersection for the two vectors by the four given points: $P_1 = (0,0,0)$, $P_2 = (0,3,0)$, and $V_1 = (3,3,0)$, $V_2 = (3,0,0)$. (2) If intersected, what is the intersection point?

6. (10%) The spring-mass system can be described as F = KX, where K is known as stiffness matrix; K^{-1} is known as elasticity matrix. Find the stiffness matrix and elasticity matrix for the system shown below; assume the values of k and M by yourself.



7. (25%)

(1) (5%) Find the <u>D'Alembert solution</u> of one dimensional homogeneous wave equation:

PDE: $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}, a > 0, -\infty < x < \infty, t > 0$

ICS: (i) $y(x,0) = \phi(x), -\infty < x < \infty$

(ii) $\frac{\partial y(x,0)}{\partial t} = \theta(x), -\infty < x < \infty$

(2) (5%) Find $y(1,1)$ and $y(\frac{1}{2},\frac{1}{8})$ for $a = 2$, $\phi(x) = \begin{cases} \frac{x}{2}, & 0 \le x \le \frac{1}{2} \\ \frac{1}{2}(1-x), & \frac{1}{2} \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$, and

$\theta(x) = 0, -\infty < x < \infty$.

(3) (5%) In (1), if the interval is finite, $0 < x < l$, and the additional boundary conditions are $y(0,t) = 0$, and $y(l,t) = 0$ for $t \ge 0$, find the D'Alembert solution.

(4) (5%) In (3), if the ICS: $y(x,0) = \sin\frac{\pi x}{l}$ and $\frac{\partial y(x,0)}{\partial t} = 0$, and the others are unchanged, find the solution $y(x,t)$.

(5) (5%) By using the method of separation of variables to find the solution of (4).

國立高雄第一科技大學九十三學年度 **碩士班** 招生考試 **試題紙**

系 所 別：機械與自動化工程系

考科代碼：1501

組 別：不分組

考 科：**工程數學（一）**

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注意事項：

1、本科目可使用本校提供之電子計算機。

2、請於答案卷上規定之範圍作答，違者該題不予計分。

（共六題）

1. $y' + \frac{1}{x} y = \frac{1}{x^4} y^{-3/4}$, $y(1)=1$. Solve for $y(x)$. (20 %)

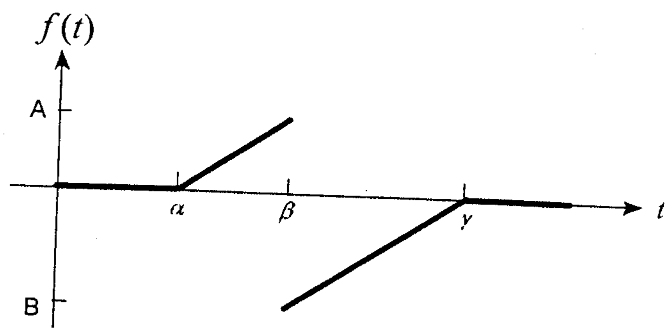
2. Solve for $y(x)$ of *the* following ODE. (15 %)

$$x^3 \frac{d^3 y}{dx^3} + 6x^2 \frac{d^2 y}{dx^2} + 7x \frac{dy}{dx} + y = 6 + \frac{2}{x}$$

3. Use Laplace Transform to solve the initial value problem. (20 %)

$$y^{(4)}(t) + y''(t) = e^{\alpha t},$$
$$y'(0) = y(0) = 0, \quad y''(0) = 1, \quad y'''(0) = -1.$$

4. Write the function $f(t)$ whose graph is shown in the following figure in terms of the Heaviside function, and find its Laplace transform. (15 %)



5. Calculate the Fourier series of the following function. (15 %)

$$f(x) = \frac{l}{4} - x, \quad \text{when } (x < 0 < \frac{l}{2})$$
$$= x - \frac{3}{4}l, \quad \text{when } (\frac{l}{2} < x < l).$$

6. Find the inverse Fourier transform of $\dfrac{4\sin(5\omega)}{2\omega + i\omega^2}$. (15 %)

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國立高雄第一科技大學九十三學年度 碩士班 招生考試 試題紙

系 所 別：機械與自動化工程系　　組　別：不分組

考科代碼：1502　　考　科：工程數學（二）

注意事項：

1、本科目可使用本校提供之電子計算機。

2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. A factory makes four different products (P_1, P_2, P_3, P_4), each product requires three raw materials (R_1, R_2, R_3). The relation is shown as follows:

	P_1	P_2	P_3	P_4
R_1	2	1	3	4
R_2	4	2	2	1
R_3	3	3	1	2

(10%) (a) Establish the production model of this factory into a matrix form. [Hint: R=AP]

(5%) (b) Estimate the required materials for the order of P1=10, P2=30, P3=20, P4=40.

(10%) 2. Find a linear transformation T: $R^3 \to R^2$, such that

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)=\begin{bmatrix}1\\-1\end{bmatrix}, T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)=\begin{bmatrix}3\\2\end{bmatrix}, T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)=\begin{bmatrix}5\\-7\end{bmatrix}$$

(15%) 3. Show the steps to diagonalize this matrix $\begin{pmatrix}3 & -2 & 0\\ -2 & 3 & 0\\ 0 & 0 & 5\end{pmatrix}$.

(10%)4. Construct a set of orthonormal vectors from the given vectors：

$$\vec{V}_1=(1 \quad 2 \quad 2),\ \vec{V}_2=(1 \quad 4 \quad 0),\ \vec{V}_3=(2 \quad 0 \quad 1)$$

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(15%)5. Calculate the surface integral of the vector function $\vec{F} = x\vec{i} + y\vec{j}$ over the portion of the surface of the unit sphere $S: x^2 + y^2 + z^2 = 1$ above the xy plane, $z \geq 0$.

(15%)6. Determine the potential function the force field

$$\vec{F} = y^2\vec{i} + 2(xy + z)\vec{j} + 2y\vec{k}.$$

(20%)7. Find the solution of the non-homogeneous wave equation:

PDE: $\dfrac{\partial^2 y}{\partial t^2} = \dfrac{\partial^2 y}{\partial x^2} + a\sin x,\ a > 0,\ 0 < x < \pi,\ t > 0$

BCS: $y(0,t) = y(\pi,t) = 0$

ICS: $y(x,0) = 0,\quad \dfrac{\partial y(x,0)}{\partial t} = 0,\ 0 < x < \pi$

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國立高雄第一科技大學 94 學年度 碩士班 招生考試 試題紙

系 所 別：機械與自動化工程系　　　　組　別：不分組

考科代碼：**1301**　　　　考　科：**工程數學（一）**

注意事項： 1、本科目可使用本校提供之電子計算機。 2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. Find the **inverse** Laplace transform of the function $\ln\left(1+\frac{\omega^2}{s^2}\right)$? (10%)

2. Use the **Laplace transform** theory to **determine and draw** the **response** of the damped mass-spring system governed by

$$y'' + 3y' + 2y = r(t), \quad y(0) = 0, \quad y'(0) = 0$$

where

(a) r(t) is the square wave, r(t)=u(t-1)-u(t-2), (20%)

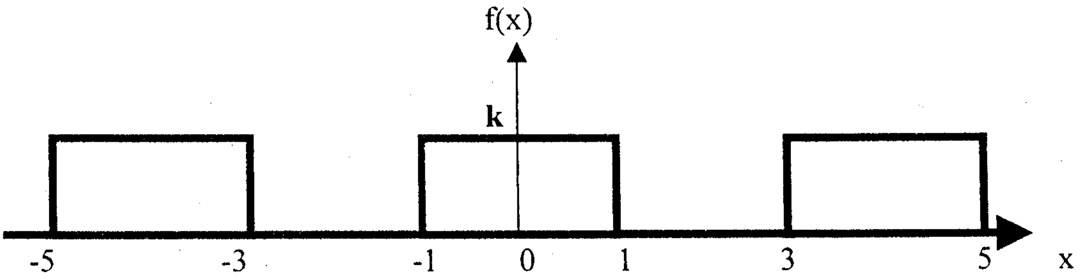
(b) r(t) is the unit impulse at time t=1, r(t)=δ(t-1). (20%)

3. Solve $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$? (10%)

4. Find the **complex form of the Fourier series** of $f(x)=e^x$, if $-\pi < x < \pi$ and $f(x+2\pi)=f(x)$ and obtain from it the **usual Fourier series.** (20%)

5. Find the **Fourier series** of the periodic function？ (20%)

$$f(x) = \begin{cases} 0 & \text{if } -2 < x < -1 \\ k & \text{if } -1 < x < 1 \\ 0 & \text{if } 1 < x < 2 \end{cases}, \quad \text{one period} = 4$$



國立高雄第一科技大學 94 學年度 碩士班 招生考試 試題紙

系 所 別：機械與自動化工程系　　組 別：不分組

考科代碼：1302　　考 科：工程數學（二）

注意事項：

1、本科目可使用本校提供之電子計算機。

2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. Assume that $\mathbf{v} = [v_1 \quad v_2 \quad v_3]^T$ represents all vectors in R^3 such that $3v_1 - 2v_2 + v_3 = 0$, $4v_1 + 5v_2 = 0$. Is the given set of vectors a vector space? If your answer is yes, determine the dimension and find a basis for this vector space. (15%)

2. A model of a vehicle suspension system is shown in Fig. 1.

(a) Derive the equations of motion in matrix form. Since this system can be formulated as an eigenvalue problem, please find the resulting eigenvalues for $k_1 = 10^3$ (N/m), $k_2 = 10^4$ (N/m), $m_2 = 50$ (kg), and $m_1 = 2000$ (kg).

(b) Continue Part (a). Suppose that the tire hits a bump that corresponds to an initial condition of

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.01 \end{bmatrix}, \text{ and } \begin{bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Calculate the response of the car $x_1(t)$.

(20%)

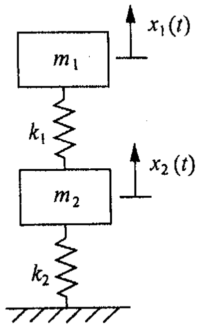


Figure 1 Two-degree-of-freedom model of a vehicle suspension system, where m_1: car mass, m_2: tire mass, k_1: spring constant of the car, k_2: spring constant of the tire, $x_1(t)$: car's vibration displacement at time t, and $x_2(t)$: tire's vibration displacement at time t.

3. Find the length of the circular-helix curve:

   $\mathbf{r}(t) = a\cos t\,\mathbf{i} + a\sin t\,\mathbf{j} + ct\,\mathbf{k}$, from $(a,0,0)$ to $(a,0,2\pi c)$

   (15%)

4. The flow of heat in the temperature field, $T(x,y) = e^{x^2-y^2}\sin 2xy$, takes place in the direction of maximum decrease of temperature T. Find this direction at the given point $(x,y)=(1,1)$.

   (15%)

5. Evaluate the line integral $\oint_C \mathbf{F}(\mathbf{r})\cdot d\mathbf{r}$ counterclockwise around the boundary C of the region R, where $\mathbf{F} = [x\cosh 2y,\ 2x^2\sinh 2y]$, and R: $x^2 \le y \le x$.

   (15%)

6. Solve the following one-dimensional heat equation for a long thin bar of length L oriented along the x-axis:

$$\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$$

   with the boundary conditions,

$$u(0,t)=0,\quad u(L,t)=0 \quad \text{for all } t,$$

   and the initial condition,

$$u(x,0)=f(x)$$

   where $f(x) = \begin{cases} x, & \text{if } 0 < x < \frac{L}{2} \\ L-x, & \text{if } \frac{L}{2} < x < L \end{cases}$

   (20%)

國立高雄第一科技大學 95 學年度 碩士班 招生考試 試題紙

系 所 別：機械與自動化工程系　　組 別：不分組

考科代碼：1301　　考 科：工程數學（一）

==

注意事項：

1、本科目可使用本校提供之電子計算機。

2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. Solve the differential equation: $x\tan\frac{y}{x} - y + x\frac{dy}{dx} = 0$. (20%)

2. Find the eigenvalues and eigenvectors of the matrix (20%)

$$\mathbf{A} = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

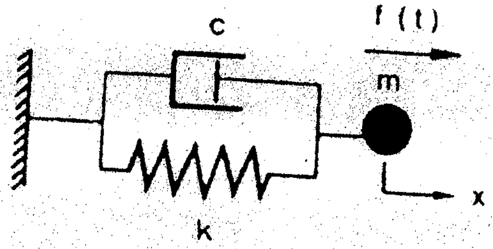
3. Use Laplace Transform to solve for the following initial value problem. (20%)

$$y'' + 4y' + 4y = f(t)\ ;\ y(0) = 1,\ y'(0) = 2 \quad \text{with } f(t) = \begin{cases} 1 & \text{for } 0 \le t < 2 \\ 0 & \text{for } t \ge 2 \end{cases}$$

4. Solve the inverse Laplace transform of the function: (20%)

$$F(s) = \frac{10}{s^3 + 5s^2 + 17s + 13}$$

5. A mass-spring-damper mechanical vibration system is as shown below.



(a) Construct a differential equation for the system. (5%)

(b) Let $m = 1$ (kg), $c = 3$ ($N \cdot s \cdot m^{-1}$), $k = 2$ (N/m), and $f(t) = 20 \cos 2t$ (N), find the steady-state oscillation of the system based on your detail calculation. (15%)

國立高雄第一科技大學 95 學年度 碩士班 招生考試 試題紙

系 所 別：機械與自動化工程系　　組　別：不分組

考科代碼：1302　　考　科：工程數學（二）

注意事項：

1、本科目可使用本校提供之電子計算機。

2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. Please find out the loop currents, i_1 and i_2, in the circuit of Fig 1. Assume that all currents and charges are zero until the switch is closed at time zero. (Note: voltage across the inductor = $L\,i'$, voltage across the capacitor = $(1/C)\int i\,dt$)

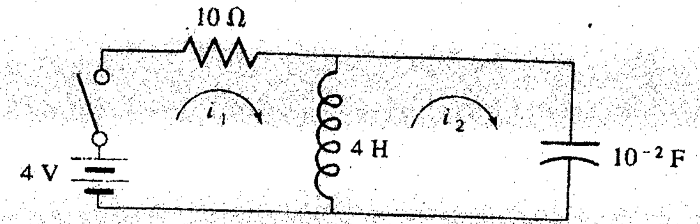


Fig 1　(20%)

2. Suppose a particle is moving along a curve, which is described by parametric equations given below:

$$x = \cos(t) + t\sin(t),\quad y = \sin(t) - t\cos(t),\quad z = t^2,\quad \text{for } t > 0$$

Please determined the following: (1) unit tangent vector and unit normal vector, (2) the curvature of the moving path, (3) the tangential and normal components of the acceleration of the particle. (20%)

3. Consider a vector field: $\vec{F} = 2x\cos(2y)\vec{i} - 2x^2\sin(2y)\vec{j}$. Please evaluate the line integral of this vector field over the following path: (1) a line segment from (0, 0) to $(1, \pi/8)$, (2) a circle from (0,0) to (0,0) (20%)

4. Please determine the Fourier series of a periodic function f with fundamental period 3, in <u>phase angle form</u>, and also shows the amplitude spectrum of the indicated function. The function f is define as:

$$f(x) = x^2 \quad \text{for } 0 \leqq x < 3$$

$$\text{and } f(x+3) = f(x) \quad \text{for all } x$$ (20%)

5. Please solve for the given differential equation with Fourier Transform:

$$y' - 4y = H(t)e^{-4t}$$

where $H(t)$ is the unit step function: $H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$ (20%)

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國立高雄第一科技大學 96 學年度 **碩士班** 招生考試 **試題紙**

系所別：機械系　　　　組別：不分組
考科代碼：1301　　　　考科：**工程數學（一）**

注意事項：
1、本科目可使用本校提供之電子計算機。
2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. Definitions (每小題 2 分，共計 14 分)

(1)ordinary differential equation

(2)linear differential equation

(3)separable differential equation

(4)homogeneous differential equation

(5)exact differential equation

(6)integrating factor

(7)order and degree

2. Solve the following problems: (每小題 6 分，共計 36 分)

(1) $ydx+(x^2y^3+x)dy=0$

(2) $(1+x^2)(dy-dx)=2xydx$, y(0)=1

(3) $y''+7y'+12y=0$

(4) $y''+2y'+5=0$

(5) $y''+3y'+2y=10e^{3x}+4x^2$

(6) $y''-2y'+y=xe^x-e^x$

3. Find the expansion of the periodic function whose definition in one period is

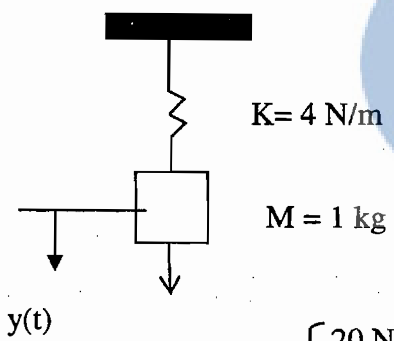
$$f(t)=\begin{cases}0, -\pi<t<0\\ \sin t, 0<t<\pi\end{cases}$$ (15 分)

4. Solve the following differential equation by using Laplace transform: (20 分)

(1) $y''' - 2y'' - y' + 2y = u(t-2),\ y_0 = y_0' = 0,\ y_0'' = 1$

(2) $y''' + 3y'' + 3y' + y = \cosh t,\ y_0 = y_0' = y_0'' = 0$

5. Find the response of the spring system as shown below: (15 分)



$$F(t) = \begin{cases}20\text{ N}, 0<t<1\\ -20\text{ N}, 1<t<2\end{cases}$$

國立高雄第一科技大學 96 學年度 碩士班 招生考試 試題紙

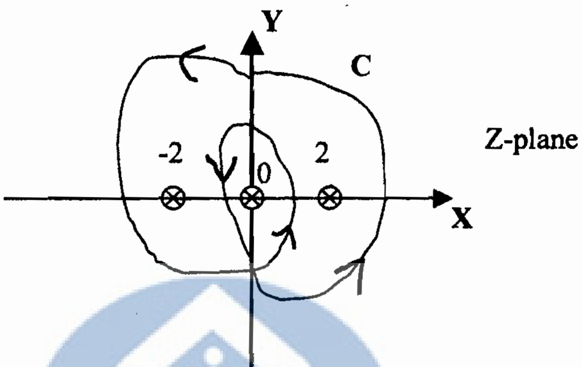
系所別：機械系　　　　組別：不分組

考科代碼：1302　　　　考科：工程數學（二）

注意事項：
1、本科目可使用本校提供之電子計算機。
2、請於答案卷上規定之範圍作答，違者該題不予計分。

Question 1 (15%)

Find $\oint_C \frac{z^2+3z+1}{z(z^2-4)}dz = ?$



Question 2 (30%)

a. Find $\oint_C \frac{1}{(z-a_1)(z-a_2)\ldots\ldots\ldots(z-a_n)}dz = ?$ C: A closed loop which includes $z = a_1, a_2, \ldots\ldots, a_n$, $n \geqq 2$. (10%)

b. Find $\oint_C \frac{1}{(z^{100}+3)(z-9)}dz = ?$ (1) C: $|z| = \infty$, (2) C: $|z| = 6$. (20%)

Question 3 (10%)

Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 0$ at the point (1,-2,1)

Question 4 (20%)

Evaluate the flux of $\vec{F} = xz\vec{i} - y\vec{k}$ across the part of the sphere $x^2 + y^2 + z^2 = 4$ lying above the plane z=1.

Question 5 (15%)

Consider the matrix, $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

a. What is the trace of A? (5%)

b. What is the determinant of A? (5%)

c. Find the inverse matrix of A? (5%)

Question 6 (10%)

Given a matrix, $A=\begin{bmatrix} 7 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 7 \end{bmatrix}$ with $P^{-1}AP = D$, where D is a diagonal matrix, try to find the matrix P?

高雄第一應用科技大學

營建工程系

91~97 學年度
工程數學考古題

國立高雄第一科技大學 學年度 碩士班招生考試 試題紙

系別：營建工程系　　科目：工程數學

科目代碼：1203

本科目分六大題，題目卷共一頁

1. 試求 $z = -10$ 及 $y = 1$ 二個平面與 $z = x^3 - 2y^2$ 所代表之曲面交會處的切平面方程式。(15%)

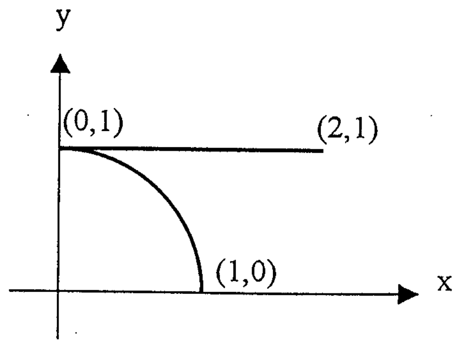
2. Assuming a curve on x-y coordinate plane expressed as $y = g(x)$, find the radius of curvature at point $(x1, y1)$ of the curve, in terms of 'g' and its derivatives. (15%)

3. 求一函數 F(x)其 Fourier 正弦延伸級數表示式；並指出該級數在 $x = 1$ 與 $x = 2$ 之收斂值各為何？(20%)

$$F(x) = \begin{cases} x & if \quad 0 \le x \le 1 \\ 1 - x & if \quad 1 < x \le 2 \end{cases}$$

4. 試證明以下有關於特徵向量線性獨立之定理：
"Let the $n \times n$ matrix A have n distinct eigenvalues. Then corresponding eigenvectors are linearly independent." (15%)

5. Let C be the curve consisting of the quarter-circle $x^2+y^2=1$ in the x-y plane from $(1,0)$ to $(0,1)$, then the horizontal line segment from $(0,1)$ to $(2,1)$. Let $\vec{\mathbf{F}}(x, y) = 4x\mathbf{i}$. Compute $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{R}}$.(15%)



6. Solve the following ODEs. (20%)
   (a) $x^2 y'' + 2xy' - 6y = 0, \quad with \quad y(1) = 9 \quad and \quad y(2) = 5$
   (b) $x^2 y'' + 2xy' - 6y = x + 1 \quad for \quad x > 0$

國立高雄第一科技大學九十二學年度 碩士班 招生考試 試題紙

系所別：營建工程系　　　　科　　目：工程數學

組　別：結構組　　　　　　科目代碼：１２０２

※ 本科目可使用本校提供之電子計算機。

※ 本科試題共分五大題，請依題號次序作答，未依題號次序作答者，該題不予計分。

1. 是非題，以下共五子題，請判斷其陳述正確與否。(每題 3 分，合計 15 分)

(1.a) 若某線性聯立方程組有無窮多組解時，採用矩陣之高斯消去法可判斷之。

(1.b) If $f(x,y)=\sqrt{1-x^2-y^2}$, then $D_{\mathbf{u}}f(0,0)=0$ for any unit vector $\mathbf{u}$. (D：derivative)

(1.c) If the function $f(x)$ is continuous on the closed interval $[\alpha,\beta]$, then there exists a number c in the closed interval $[\alpha,\beta]$ such that $\int_\alpha^\beta f(x)dx = f(c)(\beta-\alpha)$.

(1.d) If x is a real number, then $\sqrt{x^2}$ can be x or $-x$.

(1.e) Assume that $\mathbf{r}$ is a differentiable vector-valued function of t, then $\frac{d}{dt}\left(\|\mathbf{r}(t)\|\right)=\|\dot{\mathbf{r}}(t)\|$.

2. 填空題，請依題號將答案寫在答案紙上。(每個空格 5 分，合計 40 分)。

(2.a) 有一流體，其速度 V 之向量場函數為

$$\vec{\mathbf{V}}(x,y,z)=x^2\vec{i}+(-3y^2+2xy)\vec{j}+(6yz-4xz)\vec{k}$$

(x, y, z) 為位置座標，$(\vec{i},\vec{j},\vec{k})$ 為座標單位向量。則其散度等於_________，該流體屬於___________之流體。

(2.b) 某一長 L 之簡支樑，其彈性係數 E 及斷面二次矩 I 為常數。受力後變形之撓度 v 為 $v(x)=\dfrac{q}{24EI}(x^4-4Lx^3+6L^2x^2)$，$x=0\sim L$。其中 x 為沿樑軸之位置座標，q 為均佈載重強度。則此樑之彈性應變能為_____________。(註：樑彎矩與變形之曲率 (curvature) 比例值為 EI。)

(2.c) 應用梯形法(Trapezoidal method)取八段進行數值積分，則 $\int_{-1}^{1}\sqrt{1-x^2}\,dx$ 約為___________________(計至百分位)。

(2.d) 已知矩陣 **K** 及 **M** 如下：

$$\mathbf{K}=\begin{bmatrix}20 & -20 & 0\\ -20 & 40 & -20\\ 0 & -20 & 40\end{bmatrix}\quad;\quad \mathbf{M}=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$$

則滿足 $(\mathbf{K}-\lambda\mathbf{M})\mathbf{x}=\mathbf{0}$ 之特徵方程式為_____________(提示：λ 之三次多項式方程式)，且其最小特徵值為_____________(提示：可用牛頓法計算其近似值到十分位)。

(2.e) $\int(x\sqrt{x^2+1})dx=$___________。

(2.f) $\int(\ln x)\,dx=$___________。

3. 有一質量 m 彈簧常數 k 之單自由度振動系統，沿 y 座標軸位移。於時間 t = 1 ~ 2 秒 之間承受一定值 50 牛頓 之衝擊力長達 1 秒，其他時間不施外力。試以拉氏變換求解無阻尼情況下，質塊之位移函數 y(t)。假設已知 t = 0 時， y = 0，且為靜止狀態。(本題滿分 15 分)

4. 求解下列常微分聯立方程式：(本題滿分 15 分)

$$\begin{cases} \dfrac{\dot{x}_1}{3} = x_1 + \dfrac{4x_2}{3} \\[2ex] \dfrac{\dot{x}_2}{2} = \dfrac{3x_1}{2} + x_2 \end{cases}$$

x_1, x_2 均為 t 之函數，且 $x_1(t=0)=6$ ， $x_2(t=0)=-1$ 。

5. 問答題，試回答下列問題：(每一小題 5 分，合計 15 分)。

(5.a) 比較傅立葉級數 (Fourier Series) 及傅立葉轉換 (Fourier Transform) 定義上之差異。

(5.b) 函數 $f(x)$ 之拉氏變換(Laplace transform)的存在條件為何？

(5.c) 你獲得一組共有十筆數據之彈簧拉力試驗記錄，每一筆數據包含彈簧伸長量以及砝碼重量。由於量測伸長量上的不確定因素，通常以最小二乘法(線性回歸)求該彈簧常數。請條列最小二乘法回歸之步驟。

國立高雄第一科技大學九十三學年度 碩士班 招生考試 試題紙

系 所 別：營建工程系　　　　組　別：結構組（一般生）

考科代碼：1302　　　　考　科：工程數學

注意事項：
1、本科目可使用本校提供之電子計算機。
2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. 計算 $f(z)=1/z$ 沿複數平面上某曲線 $C:|z-3|=1$ 逆時鐘方向作一圓周之積分

$\oint_c f(z)\,dz = ?$ (註: $z=x+yi$, $i=\sqrt{-1}$) (15)

2. 已知函數 $y=3x^3+2\sin x$ 爲 x-y 平面上一曲線，試求在點 $x=\pi$ 處，該曲線之曲率半徑爲何? (15)

3. 某彈簧懸吊 4.0 公斤質塊,會伸長 1.0 公分,則此系統之上下振動週期爲何? (10)

4. 以拉氏變換求解

$y'' + 2y' + y = e^{-x}$ ， $y(x=0)=-2$ ， $y'(x=0)=2$ (15)

″, ′ 表 y 對 x 二次,一次之微分。

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5. 計算 x-y 平面上，沿 (0,0) →(1,2) →(0,2) →(0,0) 三線段路徑移動之某力向量

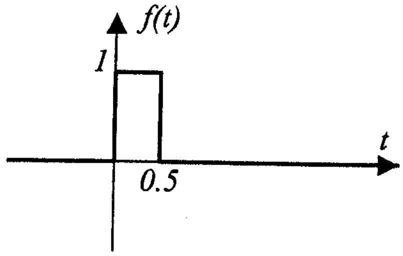
$$F = < 4x^2 y, 2y >$$ 所做之功 (註: 力與位移之積) (15)

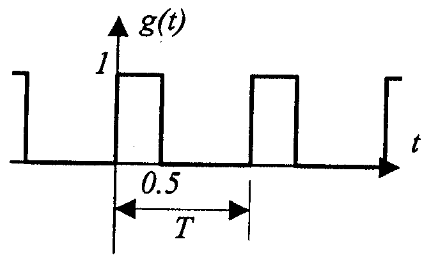
6. 一非週期性函數 $f(t)$ 定義如下，爲了以 Fourier Series 描述 $0 \le t \le 1$ 範圍內的函數值，假設一週期性函數 $g(t)$ 如下。

$$f(t)=\begin{cases}1 & 0 \le t \le 0.5 \\ 0 & otherwise\end{cases}$$

$$g(t)=\begin{cases}1 & 0 \le t < 0.5 \\ 0 & 0.5 \le t < T\end{cases}$$

$$g(t+T)=g(t)$$





已知 $T \ge 1$，試回答下列問題：

(a) $g(0.5) - f(0.5) = ?$

(b) 若 $T = 2$，請列出 $g(t)$ 之 Fourier series (15)

(c) 若 $G(t)$ 爲 $g(t)$ 之 Fourier series，則 $G(0.5) = ?$

7. 試求不定積分：

(a) $\int [x \ln x] dx$

(b) $\int \frac{1}{\sqrt{10^{2x}-1}} dx$ (15)

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國立高雄第一科技大學 94 學年度 碩士班 招生考試 試題紙

系 所 別：營建工程系　　　　組 別：結構組

考科代碼：1102　　　　考 科：工程數學

注意事項：

1、本科目可使用本校提供之電子計算機。

2、請於答案卷上規定之範圍作答，違者該題不予計分。

1.實驗時經常得到含有誤差的數據，再加上實際物理行為與理想化的數學模式之差異，我們通常必須多記錄些實驗數據，再以最小二乘法回歸成理想化的數學模式之參數。其原理係令最佳數學模式之參數將使各筆實驗值與假設的數學模式之預測值間的差異(誤差)平方和為最小值。今假設自由落體實驗數據如下表所示，其中第一欄(t_i)為量測時間，單位為秒；第二欄(y_i)為拋射物高度，單位為公尺。試以一個二次多項式回歸此自由落體位置函數，$y(t)$。

t_i	y_i
0	0
1	45.3
2	69.2
3	77.2
4	61.5

2. 請依 Simpson's Rule 估計以下定積分值，並證實你的答案之誤差小於 0.0001。

$$\int_0^2 \cos(x^2)dx$$

3. 於座標平面 (x ,y)上， 某力向量 F= ＜ $2y^2$, x^3-y ＞ ，試求該力沿三角形路徑: (0,0) -> (1,-1) -> (1,1) -> (0,0), 移動時所作功。

4. Find the solution y(t) for

$$y''(t)+0.1y'(t)+y(t)=0. \quad \text{given}$$

$$y(t=0)=A \quad \text{and} \quad y'(t=0)=B$$

5. Find the Fourier Integral of f(x) as

$$\int_0^\infty [...]d\omega \quad ; \quad \text{given} \quad f(x)=2. \quad \text{for} \quad -1<x<1 \quad \text{and}$$

$$=0. \quad \text{for} \quad x>1, x<-1$$

6. Find eigenvalues and eigenvectors (normalized unit vectors) for matrix

$$\begin{bmatrix} 200 & 0 & 0 \\ 0 & 300 & 173 \\ 0 & 173 & 100 \end{bmatrix}$$

7. 空間上四個點分別是 A(1,-4, 1)、B(2,3,0) 、C(-3,2,4)以及原點 O。試回答下列問題：
   i. 向量外積：$\overrightarrow{OA}\times\overrightarrow{OB}=?$
   ii. 由 O、A、B、C 四點為頂點之四面體(tetrahedron)之體積為何？

(1-6 題每題 15 分; 第 7 題 10 分)

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(限用試場指定之計算機)

1. 求 $V(x,y,z)=x+3y^2+4z^3$ 在點(1/2, 1/2, 2)位置沿曲面 $z=4x^2+4y^2$ 的法線方向之方向導數. (15)

2. 一水平向懸臂梁(質量忽略)之自由端附掛質塊 4 kg 靜止時撓度為 0.1 cm。該系統自由振動時，週期與頻率為何? (10)

3. 已知位置向量 $F = 3t\ i - 2\ j + t^2\ K$，求其：速度(V), 速率(v), 切線加速度($a_T$), 法線加速度($a_N$), 單位切線向量(T), 單位法線向量(N)及 Binormal(B). (20)

4.Using Laplace Transform, find the solution for

$y'' + 4y = e^t$; given $y(t=0)=1$ and $y'(t=0)=0$ (20)

5.Find: eigen-values and matrix inverse for

$$\begin{bmatrix} 0 & 2 & 1 \\ -4 & -1 & -1 \\ 3 & 6 & -7 \end{bmatrix} \qquad (15)$$

6.Using Fourier Integral to express the function

$$\begin{aligned} f(x) &= 0, && x < -1 \\ &= -2, && -1<x<0 \\ &= 1, && 0<x<1 \\ &= 0, && x>1 \end{aligned} \qquad (20)$$

（以下空白）

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1. Given two vector functions U (x,y,z) = <u1,u2,u3> and V (x,y,z) = <v1,v2,v3>, please prove that

$$\operatorname{div}(U \times V) = V \cdot \operatorname{curl}(U) - U \cdot \operatorname{curl}(V)$$

(10)

2. 試以 x-y 平面上函數 P (x,y) 及 Q (x,y) 說明格林定理.

(10)

3. 下列公式是否爲真? (是 / 非)

矩陣之乘法 (符號 A, B, C 表矩陣， T 表轉置。)

3.1 $(A+B)^T C^T = (CA + CB)^T$

3.2 $AA^T = A^T A$

(10)

4. Find eigen-values and corresponding eigen-vectors for the 3 x 3 matrix.

$$\begin{bmatrix} -30 & 0 & 0 \\ 0 & 0 & 20 \\ 0 & 20 & 0 \end{bmatrix}$$

(20)

5. 求函數 f(x)之傅立葉積分式.

$x < 0, \quad f(x) = 0$

$0 < x < 2, \quad f(x) = -x$

$x > 2, \quad f(x) = 0$

(20)

6. 求解二階常微方程式 $y'' - 9y = e^{-2x} + 2x$ 且 $y(x=0) = 0$

及 $y'(x=0) = 0$.

註: 須有通解及特解.

(20)

7. 求出 x-y 平面上二條曲線: $y = 2 - x^2$ 及 $y = x$ 所圍成之面積.

(10)

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1. 一長度 L、撓曲剛度為 EI 之鋼柱，承受一通過形心之載重 P，假設鋼柱之一端為固定端、一端為鉸接端。試計算鋼柱可承受之挫屈載重 P_{cr}。(20)

2. 一彈簧質量系統之運動方程式為 $y'' + 4y' + 3y = e^{t}$，其中 t 為時間。請求該質量系統隨時間變化之運動軌跡，假設該系統之初始狀態為 $y(0)=0$，$y'(0) = 2$。(20)

3. 求$\nabla\cdot(\nabla \times \vec{F})$，其中$\vec{F}$為一連續之向量場。(15)

4. 假設 a1,a2,a3 為相異實數，判斷 x^{a1}, x^{a2}, x^{a3} 是否為線性獨立。(15)

5. 計算向量$\vec{F}$與$\vec{G}$間之夾角，其中$\vec{F}=5\vec{i}+3\vec{j}+4\vec{k}$，$\vec{G}=20\vec{i}+6\vec{k}$。(15)

6. 請計算 $\iint_S \vec{F}\bullet\vec{n}\,dA$，其中$\vec{F}=x\vec{i}+y\vec{j}+z\vec{k}$，且 S 為滿足球面方程式 $x^2+y^2+z^2=1$ 之曲面。(15)