高雄大學

土木與環境工程學系碩士班

95~97 學年度

工程數學考古題

國立高雄大學九十五學年度研究所碩士班招生考試試題

科目：工程數學　　系所：土木與環境工程學系碩士班土木工程組　　■可 □否 使用計算機

考試時間：100 分鐘　　本科原始成績：滿分 100 分

一、選擇題(每題七分，皆為單選題)

1. 微分方程式 $x\dfrac{dy}{dx}+y=y\ln(xy)$，其通解(general solution)為

(A) $xy-\frac{1}{3}x^2=c$ (B) $x^2y-\frac{1}{3}x^3=c$ (C) $xy^2-\frac{1}{3}x^3=c$ (D) $xy-\frac{1}{3}x^3=c$ (E)以上皆非；其中 c 為常數。

2. 聯立微分方程式 $\dfrac{dx}{dt}=-2x+y$，$\dfrac{dy}{dt}=-5x+4y$，$x(0)=1$，$y(0)=3$，其解為

(A) $x(t)=e^{3t}+e^{-t}$；$y(t)=5e^{3t}+\frac{1}{2}e^{-t}$； (B) $x(t)=\frac{1}{2}e^{3t}+\frac{1}{2}e^{-t}$；$y(t)=\frac{5}{2}e^{3t}+\frac{1}{2}e^{-t}$

(C) $x(t)=e^{2t}+e^{-t}$；$y(t)=5e^{2t}+\frac{1}{2}e^{-t}$； (D) $x(t)=\frac{1}{2}e^{2t}+\frac{1}{2}e^{-t}$；$y(t)=5e^{2t}+\frac{1}{2}e^{-t}$

(E) 以上皆非

3. 對函數 $f(t)=te^{t}\sin t$，其拉普拉斯轉換(Laplace transform) $F(s)=\int_0^\infty f(t)e^{-st}dt$ 為

(A) $\dfrac{-2(s-1)}{((s-1)^2+1)}$ (B) $\dfrac{-(s-1)}{((s-1)^2+1)^2}$ (C) $\dfrac{2(s-1)}{((s-1)^2+1)^2}$ (D) $\dfrac{-2(s-1)}{(s^2+1)^2}$ (E)以上皆非

4. 微分方程式 $y''+3y'+2y=\delta(t-a)$；$y(0)=0$，$y'(0)=0$ 的解為 $y(t)$，$y(t)$ 之 Laplace transform 為 $Y(s)$；其中 a 為常數，$\delta(t-a)$ 為 Dirac delta function；則下列何者正確

(A) $Y(s)=\left[\dfrac{1}{s+1}+\dfrac{1}{s+2}\right]e^{-as}$ (B) $Y(s)=\left[\dfrac{1}{s+1}-\dfrac{2}{s+2}\right]e^{-as}$

(C) $y(t)=\left[e^{-2(t-a)}-e^{-(t-a)}\right]H(t-a)$ (D) $y(t)=\left[e^{-(t-a)}-e^{-2(t-a)}\right]H(t-a)$

(E)以上皆非　【註：H(t-a)爲 Heaviside Unit Step function】

國立高雄大學九十五學年度研究所碩士班招生考試試題

科目：工程數學　　系所：土木與環境工程學系碩士班土木工程組　■可 □否 使用計算機

考試時間：100 分鐘　本科原始成績：滿分 100 分

5. $x = r\cos\theta$，$y = r\sin\theta$，則 θ 之梯度(gradient)為

(A) $\frac{1}{r}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$ (B) $\frac{1}{r}(\sin\theta\,\hat{i} - \cos\theta\,\hat{j})$ (C) $(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

(D) $(\sin\theta\,\hat{i} - \cos\theta\,\hat{j})$ (E)以上皆非

6. 面積分 $I = \iint_S x^3 dydz + x^2 ydzdx + x^2 zdxdy$，$S: x^2 + y^2 = a^2, 0 \le z \le b$，$a, b$ 為常數；則 I 之值為

(A) $\frac{5}{4}\pi a^4 b$ (B) $\frac{5}{4}\pi a^3 b^2$ (C) $\frac{1}{4}\pi a^4 b$ (D) $\frac{1}{4}\pi a^3 b^2$ (E)以上皆非

[提示:可應用高斯散度定理

$\iiint_R \nabla\cdot\vec{F}\,dV = \iint_S \vec{F}\cdot\hat{n}\,dA = \iint_S F_1 dydz + F_2 dzdx + F_3 dxdy$]

7. 矩陣 $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$ 之特徵向量(eigenvector)為 $\vec{x}_1$ 及 $\vec{x}_2$，則下列何者正確

(A) $\vec{x}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\vec{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (B) $\vec{x}_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ $\vec{x}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ (C) $\vec{x}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ $\vec{x}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(D) $\vec{x}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\vec{x}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (E)以上皆非

8. 矩陣 $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$; $f(x) = \frac{x}{x+1}$，$f(A)$ 之特徵值(eigenvalue)為

(A) 1/2, 1/4 (B) 1, -1 (C) 1/2, 2/3 (D) 1, 2/3 (E)以上皆非

國立高雄大學九十五學年度研究所碩士班招生考試試題

科目：工程數學　　系所：土木與環境工程學系碩士班土木工程組　■可 □否 使用計算機

考試時間：100 分鐘　本科原始成績：滿分 100 分

9. 積分 $I = \int_0^{2\pi} \frac{\sin^2\theta}{5+4\cos\theta} d\theta$ 之積分值為

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{8}$ (D) $\frac{\pi}{16}$ (E)以上皆非

10. $f(x) = 1 + x, -1 < x < 1$，$f(x)$ 為週期 T=2 之週期函數，函數 $f(x)$ 之傅立葉(Fourier)級數為

(A) $1+\sum_{n=1}^{\infty} 2\frac{(-1)^{n+1}}{n\pi}\cos(n\pi x)$ (B) $1-\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\pi}\cos(n\pi x)$

(C) $1-\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\pi}\sin(n\pi x)$ (D) $1+\sum_{n=1}^{\infty} 2\frac{(-1)^{n+1}}{n\pi}\sin(n\pi x)$ (E)以上皆非

二、計算題（每題十五分）

1. 有一組聯立微分方程式如下：

$$u\frac{dL}{dx} = -k_1 L$$

$$u\frac{dC}{dx} = -k_1 L + k_2(C_s - C)$$

已知初始條件為 $L(0) = L_o$，$C(0) = C_o$，其中，u、C_s、k_l、k_2 為常數。求 $C(x) = ?$

2. 一偏微分方程式 $\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} - D\frac{\partial^2 C}{\partial x^2} = 0$

已知邊界條件為 $C(x, t) \to 0$ 當 $x \to \pm\infty$，

初始條件為 $C(x,0) = M\delta(x)$。其中，u、D、M 為常數，$\delta(x)$為 Dirac delta function，求解 $C(x, t)$？

國立高雄大學九十六學年度研究所碩士班招生考試試題

科目：工程數學　　系所：土木與環境工程學系土木工程組　　是否使用計算機：是
考試時間：100 分鐘　　土木與環境工程學系環境工程組
本科原始成績：100 分

1. 求解下列微分方程式 (每一小題佔 10%)

(a). $y'' + 4y = x\cos(2x)$； $y' = dy/dx$

(b). $x^2 y'' - 2xy' + 2y = x^3 \sin x$

(c). $x^2 y'' + xy' + (x^2 - 1/4)y = x^{3/2}\cos x$ (Hint: Set $y = ux^{-1/2}$)

2. 對常係數微分方程 $y'' + ay' + by = 0$，其中 $y' = dy/dx$，其求解過程為令 $y = e^{mx}$ 代入微分方程後得特徵方程式 $m^2 + am + b = 0$，求得特徵方程式之根 $m = m_1$，m_2 後即可得通解 $y_h = c_1 e^{m_1 x} + c_2 e^{m_2 x}$，其中 c_1 與 c_2 為任意常數。

(a) 解釋為何要令 $y = e^{mx}$ 來求解?　　(5%)

(b) 當特徵方程式之根為重根時 $m_1 = m_2$，證明 $y_2 = \left.\dfrac{\partial y(x,m)}{\partial m}\right|_{m=m_1}$ 為微分方程之另一組解 (10%)

3. 面積分 $I = \iint_S x^3 dydz + x^2 y dzdx + x^2 z dxdy$， $S: x^2 + y^2 = a^2, -b \le z \le b$，a, b 為常數，求 I 之值。 (10%)

4. 對聯立微分方程式 $Z' = AZ$ ，其中 $Z = [x(t), y(t)]^T$ ， $Z' = [dx/dt, dy/dt]^T$ ，

$$A = \begin{bmatrix} 1 & 3 \\ -3 & 7 \end{bmatrix}$$

(a) 求矩陣 A 之特徵值(eigenvalue)與對應之特徵向量　　(10%)

(b) 求聯立微分方程式之通解(general solution)　　(15%)

5. 利用分離變數法(separation of variable)求解下列偏微分方程 (20%)

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \;;\quad u = u(x,t),\ 0 < x < L,\quad t \ge 0 \;;\quad c \text{ 為常數}$$

邊界條件: $u(0,t) = 0$, $u(L,t) = 0$

初始條件: $u(x,0) = x(L - x)$

國立高雄大學九十七學年度研究所碩士班招生考試試題

科目：工程數學
考試時間：100 分鐘

系所：
土木與環境工程學系碩士班土木工程組　　是否使用計算機：是
本科原始成績：100 分

1. Solve the logistic differential equation $\dfrac{dy}{dt} = ky\left(1 - \dfrac{y}{L}\right)$, where k and L are positive constants. (20)

2. Solve the boundary value problem $\dfrac{\partial u}{\partial t} = c_v \dfrac{\partial^2 u}{\partial z^2}$ $(0 \le z \le 2H,\ t \ge 0)$ with the following boundary conditions:

$$u(z,0) = u_0$$
$$u(0,t) = 0 \text{ and } u(2H,t) = 0 \text{ if } t > 0,$$

where c_v, H and u_0 are positive constants. (30)

3. (a) Show how to find a particular solution by variation of parameters (15).

> Consider a 2nd Order linear non-homogeneous ODE in (1)
>
> $$y'' + p(x)y' + q(x)y = r(x) \tag{1}$$
>
> One may find two basis functions to form the general solution for the ODE.
>
> $$y_h = c_1 y_1 + c_2 y_2 \ (c_1, c_2 = \text{const}) \tag{2}$$
>
> And obtain the particular solution y_p of (1) in the form
>
> $$y_p(x) = -y_1 \int \frac{y_2 r}{W}\, dx + y_2 \int \frac{y_1 r}{W}\, dx \tag{3}$$
>
> where $W = y_1 y'_2 - y_2 y'_1$.

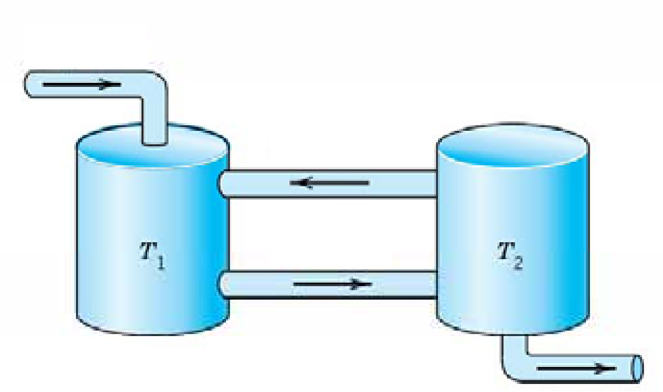
(b) Use (a) to find the complete solution to the ODE, y'' ? $2y' + 2y = 2e^x \cos x$ (15).

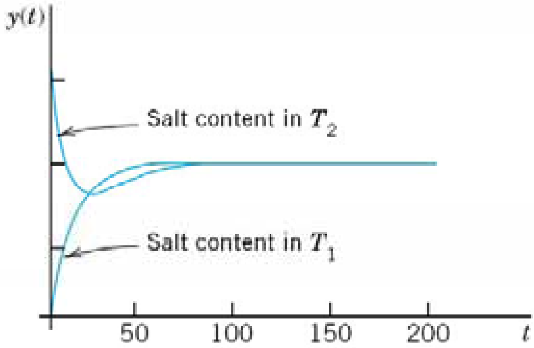
4. Consider a system of two tanks as shown below. Find the salt content for each tank if the system can be modeled as

$$y_1'' = 4y_2 - 4e^t,\ y_2'' = 3y_1 + y_2,\ y_1(0) = 1,\ y_1'(0) = 2,\ y_2(0) = 2,\ y_2'(0) = 3$$

(20)

科目：工程數學　　系所：土木與環境工程學系碩士班土木工程組　　是否使用計算機：是
考試時間：100 分鐘　　本科原始成績：100 分





高雄大學

電機工程學系碩士班

93~97 學年度
工程數學考古題

國立高雄大學九十三學年度研究所碩士班招生考試試題

系所組別：電機工程學系碩士班光電組　　　　科目：工程數學

A. 微分方程(工程數學)

不可使用計算機，需按照題目順序作答。

10% 1. $\dfrac{dy}{dx} = \dfrac{y}{e^{2x}\ln y}$, solve y.

10% 2. $\dfrac{dy}{dx} = -\dfrac{2xy}{x^2 + y^2}$, solve y.

10% 3. y'' + 2y' + y = 6 , y(0) = 5 and y'(0) = 1, solve y.

10% 4. y''-9y = 54 t sin (3t) , solve y.

10% 5. Solve the Laplace transformation L\{ f(t) \}(s).

a). L\{ cos(kt) \}

b). L\{e^{at} * cosh (k t) \}

國立高雄大學九十三學年度研究所碩士班招生考試試題

系所組別：電機工程學系碩士班半導體組（元件）　　科目：工程數學

B. 線性代數

1. Determine whether the solution space of the system $Ax = 0$ is a line through the origin, a plane through the origin, or the origin only. If it is a plane, find an equation for it, and if it is a line find parametric equations for it.
10%

(a) $A = \begin{bmatrix} 2 & -8 & 6 \\ -3 & 12 & -9 \\ 7 & -28 & 21 \end{bmatrix}$　　(b) $A = \begin{bmatrix} 2 & 6 & 8 \\ 3 & 3 & 15 \\ 2 & 4 & 12 \end{bmatrix}$

2. Find the QR-decomposition of A under the Euclidena inner product.
10%

(a) $A = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$　　(b) $A = \begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & -1 \\ 1 & -1 & -2 \end{bmatrix}$

3. Consider the bases $B = \{\mathbf{u}_1, \mathbf{u}_2\}$ and $B' = \{\mathbf{v}_1, \mathbf{v}_2\}$ for R^2, where
10%　$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(a) Find the transition matrix $P_{B',B}$ from B to B'. 3%

(b) Find the transition matrix $P_{B,B'}$ from B' to B. 3%

(c) Compute the coordinate matrix $[\mathbf{w}]_{B'}$ where $\mathbf{w} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$. 2%

(d) Use your answers to parts (b) and (c) to compute $[\mathbf{w}]_B$. 2%

4. Use diagonalization to compute A^{10} for $A = \begin{bmatrix} 5 & 3 & -7 \\ -1 & 1 & 1 \\ 3 & 3 & -5 \end{bmatrix}$.
10%

5. Let $T : R^3 \to R^3$ be the linear operator defined by $T(x_1, x_2, x_3) = (x_2 + x_3, x_1 + x_3, x_1 + x_2)$.
10%　(a) Find the matrix $[T]_B$, where $B = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, and $\mathbf{v}_1 = (1,1,3)$, $\mathbf{v}_2 = (1,2,0)$, $\mathbf{v}_3 = (-1,0,1)$.

(b) Use the matrix from (a) to compute $T(1,1,1)$.

國立高雄大學九十四學年度研究所碩士班招生考試試題

系所(組別)：電機工程學系碩士班
(光電組)

科目：工程數學

考試時間：100 分鐘

本科原始成績滿分 100 分

微分方程

DE-1.(10%) $\dfrac{dy}{dx} = \dfrac{4y^2 - x^2}{2xy}$

$y(1) = 1$

DE-2.(10%) $ydx + (2x - ye^y)dy = 0$

DE-3.(10%) $y'' - y' = -3x - 4x^2 e^{2x}$

$y(0) = -\dfrac{7}{2}, y'(0) = 0$

DE-4.(10%) $y'''' + 6y''' + 18y'' + 30y' + 25y = e^{-x}\cos(2x) + e^{-2x}\sin(x)$

DE-5.(10%) $\begin{array}{l} f(x) = e^x, 1 \le x < 1 \\ f(x+2) = f(x) \end{array}$, find the Fourier series

國立高雄大學九十四學年度研究所碩士班招生考試試題

系所(組別)：電機工程學系碩士班
(通訊組)

科目：工程數學

考試時間：100 分鐘

本科原始成績滿分 100 分

線性代數

1. Apply the Gram-Schmidt orthonormalization process to the basis $B = \{1, x, x^2\}$ in P_2, using the inner product

10%

$$\langle \mathbf{p}, \mathbf{q} \rangle = \int_{-1}^{1} p(x)q(x)\,dx.$$

2. Find the projection of the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ onto the subspace S of R^3 spanned by the

10%

vectors $\mathbf{w}_1 = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$ and $\mathbf{w}_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$.

3. Let $T\colon R^5 \to R^4$ be defined by $T(\mathbf{x}) = A\mathbf{x}$, where $\mathbf{x}$ is in R^5 and

10%

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 1 & 3 & 1 & 0 \\ -1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 8 \end{bmatrix}.$$

Find a basis for $\ker(T)$ as a subspace of R^5.

4. Let $T\colon R^3 \to R^3$ be the linear transformation given by

$$T(x_1, x_2, x_3) = (x_1 - x_2 - x_3, x_1 + 3x_2 + x_3, -3x_1 + x_2 - x_3).$$

10%

If possible, find a basis B for R^3 such that the matrix for T relative to B is diagonal.

5. Find an orthogonal matrix P that diagonalizes

10%

$$A = \begin{bmatrix} 2 & 2 & -2 \\ 2 & -1 & 4 \\ -2 & 4 & -1 \end{bmatrix}.$$

國立高雄大學九十四學年度研究所碩士班招生考試試題

系所(組別)：電機工程學系碩士班（通訊組）　　科目：工程數學

考試時間：100 分鐘　　本科原始成績滿分 100 分

機率

1. Suppose X is uniformly distributed over $[-1, 3]$ and $Y = X^2$. Find the CDF $F_Y(y)$ and the PDF $f_Y(y)$.

10%

2. Observe someone dialing a telephone and record the duration of the call. In a simple model of the experiment, $1/3$ of the calls never begin either because no one answers or the line is busy. The duration of these calls is 0 minutes. Otherwise, with probability $2/3$, a call duration is uniformly distributed between 0 and 3 minutes. Let Y denote the call duration. Find the CDF $F_Y(y)$, the PDF $f_Y(y)$, and the expected value $E[Y]$.

10%

3. Let R be the uniform $(0, 1)$ random variable. Given $R = r$, X is the uniform $(0, r)$ random variable. Find the conditional PDF of R given X.

10%

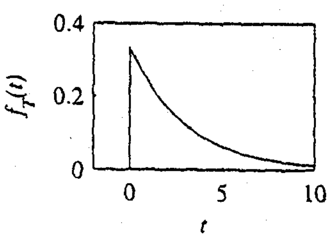
4. Find the PDF of $W = X + Y$ when X and Y have the joint PDF

10%

$$f_{X,Y}(x, y) = \begin{cases} 2 & 0 \le y \le 1, 0 \le x \le 1, x + y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

5. Suppose the duration T (in minutes) of a telephone call is an exponential $(1/3)$ random variable:

10%



$$f_T(t) = \begin{cases} (1/3)e^{-t/3} & t \ge 0, \\ 0 & \text{otherwise.} \end{cases}$$

For calls that last at least 2 minutes, what is the conditional PDF of the call duration?

國立高雄大學九十五學年度研究所碩士班招生考試試題

科目：工程數學(微分方程 50%、線性代數 50%)
系所：電機工程學系碩士班光電組
□可 ■否 使用計算機
考試時間：100 分鐘
本科原始成績：滿分 100 分

微分方程〔每題十分〕

1. Solve

$$(x^3 + y^3)\,dx + 3xy^2\,dy = 0$$

2. Solve

$$y'' + 4y' + 4y = (3 + x)e^{-2x}, \quad y(0) = 2, y'(0) = 5$$

3. Find power series solution about x=0:

$$2xy'' - y' + 2y = 0$$

4. Use the Laplace transform to solve

$$y'' - 3y' + 2y = e^{-4t}, y(0) = 1, y'(0) = 5$$

5. Solve

$$\frac{dx}{dt} = 2x - 7y$$
$$\frac{dy}{dt} = 5x + 10y + 4z$$
$$\frac{dz}{dt} = 5y + 2z$$

國立高雄大學九十五學年度研究所碩士班招生考試試題

科目：工程數學(微分方程 50%、線性代數 50%)
考試時間：100 分鐘

系所：電機工程學系碩士班光電組
本科原始成績：滿分 100 分

□可 ■否 使用計算機

線性代數〔每題十分〕

1. State (with a brief explanation) whether the following statements are true or false.
   (a) The vectors $(1, 2)$, $(-1, 3)$, $(5, 2)$ are linearly dependent in $\mathbf{R}^2$.
   (b) The vectors $(1, 0, 0)$, $(0, 2, 0)$, $(1, 2, 0)$ span $\mathbf{R}^3$.
   (c) $\{(1, 0, 2), (0, 1, -3)\}$ is a basis for the subspace of $\mathbf{R}^3$ consisting of vectors of the form $(a, b, 2a - 3b)$.
   (d) Any set of two vectors can be used to generate a two-dimensional subspace of $\mathbf{R}^3$.

2. Find the eigenvalues and corresponding eigenspaces of the matrix

$$A = \begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

3. Consider the linear transformation $T: \mathbf{R}^3 \to \mathbf{R}^2$, defined by $T(x, y, z) = (x + y, 2z)$. Find the matrix of T with respect to the bases $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and $\{\mathbf{u}'_1, \mathbf{u}'_2\}$ of $\mathbf{R}^3$ and $\mathbf{R}^2$, where

$$\mathbf{u}_1 = (1, 1, 0),\ \mathbf{u}_2 = (0, 1, 4),\ \mathbf{u}_3 = (1, 2, 3) \text{ and } \mathbf{u}'_1 = (1, 0),\ \mathbf{u}'_2 = (0, 2)$$

   Use this matrix to find the image of the vector $\mathbf{u} = (2, 3, 5)$.

4. Determine the kernel and the range of the transformation defined by the following matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 1 & 1 & 4 \end{bmatrix}$$

5. Find the least-squares linear approximation to $f(x) = e^x$ over the interval $[-1, 1]$.

國立高雄大學九十六學年度研究所碩士班招生考試試題

系所：電機工程學系光電組
電機工程學系微電子組-元件
電機工程學系微電子組-積體電路與系統

科目：工程數學
考試時間：100 分鐘
是否使用計算機：是
本科原始成績：100 分

微分方程（50%）

1. (10%) Solve $\dfrac{dy}{dx} = \dfrac{\cos(2y) + x}{2x\sin(2y)}$

2. (10%) Solve the given initial-value problem

$y'' + 4y' + 5y = 0$， $y(0) = 1$， $y'(0) = 1$

3. (10%) Solve the problem with given initial value.

$x^2y'' - 3xy' + 3y = 0$， $y(1) = 3$， $y'(1) = 5$

4. (10%) Solve the given system of differential equations

$$\frac{dx}{dt} = x - y + e^t$$

$$\frac{dy}{dt} = 2x - y$$

$x(0)= 2$, $y(0)= -2$

5. (10%) $d(t - t_0)$ is a Dirac delta function, Use Laplace transform of the differential function to solve

$y'' + 2y' = 4 + 2d(t - 1)$， $y(0)=0$， $y'(0) = 2$

國立高雄大學九十六學年度研究所碩士班招生考試試題

系所：電機工程學系光電組

科目：工程數學　　電機工程學系微電子組-元件　　是否使用計算機：是

考試時間：100 分鐘　　電機工程學系微電子組-積體電路與系統

本科原始成績：100 分

線性代數〔占 50%,每題十分〕

1. Let $L: P_2 \rightarrow P_2$ be the linear transformation defined by

$$L(y) = x^2 y'' - y' + y.$$

Compute the matrix M that represents the linear transformation L using the ordered basis $B = \{1, \ (x-1), \ (x-1)^2\}$ for the domain and $B' = \{1, \ (x-2), \ (x-2)^2\}$ for the target space.

2. Find an orthogonal basis for the solution set to

$$2x + y + 3z - w = 0.$$

3. Find a formula for A^k, where

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}.$$

4. Let W be the subspace of R^4 spanned by $A_1 = [1 \quad 2 \quad 1 \quad 1]^t$ and $A_2 = [1 \quad 0 \quad 1 \quad 0]^t$. Compute the projection of B onto W $\mathrm{Proj}_W(B)$ for $B = [1 \quad 2 \quad 3 \quad 4]^t$.

5. Find the distance of the point $X = [4 \quad 1 \quad 7]^t$ of R^3 from the subspace W consisting of all vectors of the form $[a \quad b \quad b]^t$.

科目：工程數學
考試時間：100 分鐘

系所：
電機工程學系碩士班光電組　是否使用計算機：是
本科原始成績：100 分

請按次序作答

1.(10%) $A=\begin{pmatrix}-3 & 3 & 3\\ 3 & -2 & -2\\ 3 & 1 & 0\end{pmatrix}$, find A^{-1}

2.(20%) $A=\begin{pmatrix}-1 & 1 & 0\\ 1 & 2 & 1\\ 0 & 3 & 1\end{pmatrix}$, find the eigenvalues and the corresponded eigenvectors.

3. (20%) $X'=\begin{pmatrix}1 & 0 & 0\\ 2 & 2 & -1\\ 0 & 1 & 0\end{pmatrix}X$, solve X

4. (10%) solve $\dfrac{dy}{dx}=-\dfrac{2y^2+3x}{2xy}$

5. (10%) $y''-2y'+5y=e^x\cos(2x)$, solve $y(x)$

6. (10%) $x^2y''-xy'+y=\ln x$, solve $y(x)$

7. (20%) $f(x)=|x|-x, -1<x<1$, expand f in a Fourier series.

國立高雄大學九十七學年度研究所碩士班招生考試試題

科目：工程數學
考試時間：100 分鐘

系所：
電機工程學系碩士班微電子組　　是否使用計算機：是
本科原始成績：100 分

微分方程（50%）

1. Determine the differential equations (a)~(e) are linear or nonlinear (10%)

(a) $(\frac{dy}{dx})^2 + \cos x = 0$

(b) $\frac{d^2y}{dx^2} + (\cos x)\frac{dy}{dx} = e^x$

(c) $\frac{dy}{dx} + \sin y = 0$

(d) $y\frac{dy}{dx} + 2x = 0$

(e) $\frac{dy}{dx} = x^2 y$

Solve y(x) for the given initial value problems from (2) to (4)

2. $\frac{d^2y}{dx^2} - 4y = 8x,\ \ y(0) = 4,\ \ y'(0) = 2$ (10%)

3. $4x^2 y'' + 4xy' - y = 0,\ \ y(1) = 6,\ \ y'(1) = 1$ (10%)

4. $\frac{dy}{dx} = \frac{y+1}{x-1},\ \ y(2)=0$ (10%)

5. Find the Fourier series of $f(t)$ on the given interval (10%)

$$f(t) = \begin{cases} 0, & -\pi < x < 0 \\ 1, & 0 \le x < \pi \end{cases}$$

國立高雄大學九十七學年度研究所碩士班招生考試試題

科目：工程數學
考試時間：100 分鐘

系所：
電機工程學系碩士班微電子組　　是否使用計算機：是
本科原始成績：100 分

線性代數（50%）

1. Find inverse matrix A^{-1}, given $A=\begin{bmatrix}0 & 4 & -1\\ 1 & 2 & 1\\ 1 & -1 & 3\end{bmatrix}$ (10%)

2. $A=\begin{bmatrix}0 & 4 & -1\\ 1 & 2 & 1\\ 1 & -1 & 3\end{bmatrix}$, use Cayley-Hamilton to find $A^5-5A^4+4A^3+6A^2$ (10%)

3 λ_1, λ_2 and λ_3 are the eigenvalues of the matrix A (10%)

$$A=\begin{bmatrix}0 & 4 & -1\\ 1 & 2 & 1\\ 1 & -1 & 3\end{bmatrix}, \qquad \lambda_1+\lambda_2+\lambda_3=?$$

4. Solve X, given $\mathrm{X}'=\begin{bmatrix}4 & 1\\ 3 & 6\end{bmatrix}\mathrm{X}+\begin{bmatrix}-3\\ 10\end{bmatrix}e^t$ (10%)

5. (a) Diagonalize $A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$, such that $B=C^{-1}AC$, (10%)

where C is orthonormal basis.

(b) Calculate $\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}^{10}$

背面尚有試題

國立高雄大學九十七學年度研究所碩士班招生考試試題

科目：工程數學
考試時間：100 分鐘

系所：
電機工程學系碩士班通訊組　　是否使用計算機：是
本科原始成績：100 分

注意：答案卷請先回答線性代數考題，再回答機率考題。

下列線性代數考試題目共三題。每題均需作答。請依序作答。

1. There exists a linear transformation $T: R^2 \to R^3$ such that $T(5,13) = (31,-53,-2)$ and $T(11,7) = (25,13,-26)$. Find $T(2,-1)$? (10%)

2. Given $X = (2,2,4,1)^T$ and $Y = (-2,1,2,0)^T$. Let θ be the angle between X and Y,
   a. find the square value of $\tan\theta$? (10%)
   b. find the vector projection of X onto Y? (5%)
   c. find the vector projection of Y onto X? (5%)

3. Given $A = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$.
   a. find e^A? (10%)
   b. find $\sin A$? (10%)

國立高雄大學九十七學年度研究所碩士班招生考試試題

科目：工程數學
考試時間：100 分鐘

系所：
電機工程學系碩士班通訊組　　是否使用計算機：是
本科原始成績：100 分

注意：答案卷請先回答線性代數考題，再回答機率考題。

下列機率考試題目共五題。每題均需作答。請依序作答。

1. A simple binary communication channel, regarded as one stage, carries messages by using only two signals, namely, 0 and 1. We assume that, for a given binary channel, 45% of the time a 1 is transmitted. The probability that a transmitted 0 is correctly received is 0.88, and the probability that a transmitted 1 is correctly received is 0.95. What is the probability that a 1 is received at the output? (10%)

2. By cascading two identical stages altogether in the previous question, given a 1 is received at the output of the second stage, what is the probability that 1 was transmitted? (10%)

3. The Q-function is defined by $Q(x)=\int_x^\infty \frac{1}{\sqrt{2\pi}}e^{\frac{-t^2}{2}}\,dt$. Due to the difficulty in computing the Q-function, one good approximation can be represented by $Q(x)\approx\left(\frac{1}{(\pi-1)x+\sqrt{x^2+2\pi}}\right)\cdot\sqrt{\frac{\pi}{2}}\cdot e^{\frac{-x^2}{2}},\quad x\geq 0.$ Suppose that the scores of an exam with twenty thousand attendants have normal distribution with the variance of 9, and half of the attendants have scores more than 75 points. With the information and the approximation of Q-function, how many attendants will have scores between 72 and 81 points? (10%)

4. One cumulative distribution function is represented by $F_X(x)=\begin{cases}0, & \text{for } x<0;\\ 1-\exp\left(\frac{-x}{2}\right), & \text{for } 0\leq x<2;\\ 1-0.3\cdot\exp\left(\frac{-x}{2}\right) & \text{for } x\geq 2.\end{cases}$

   Find the probability $P(1<x\leq 4)$. (10%)

5. It is observed that customers arrive at a store at an average rate of 36 persons per hour. Let T be the waiting time for the customer, what is the probability for the customer to wait for more than two minutes ? (10%)