朝陽科技大學

資訊工程系

90~97 學年度
工程數學考古題

朝陽科技大學九十學年度研究所碩士班招生考試試題

系別：資訊工程系
組別：一般生
科目：工程數學

1. Solve the following questions: (60%)

(a) Find the solution to the nonhomogeneous equation as

$$y''-2y'+1=8x+e^{x},\quad y(0)=1,\quad y'(0)=0$$

(b) Integrate a complex function $f(z)$ around C, where

$$f(z)=\frac{(1+z)\sin z}{(2z-1)^2},\quad C:|z-i|=2$$

(c) Find the determinant of matrix A, where

$$A=\begin{vmatrix}1 & 0 & 3 & 7\\ 4 & 2 & 0 & 1\\ 7 & 7 & 3 & 0\\ 5 & 0 & 6 & 8\end{vmatrix}$$

(d) Evaluate the double integral over the square region in Fig. 1

$$\iint_R (x^2+y^2)\,dxdy$$

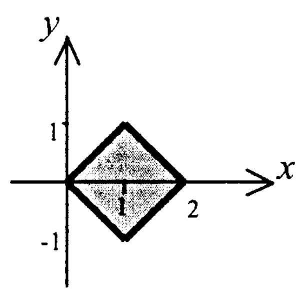

Fig. 1 Region in Qestion (d)

2. Let $f(t)$ be a given function that is defined for all $t\geq 0$. Define the transform as

$$F(s)=\int_0^{\infty} e^{-st}\cdot \ln(f(t))dt$$

if the resulting integral exists. Solve the following questions by this transform (20%)

(a) $f(t)=1$

(b) $f(t)=ae^{at}$

(c) $f(t)=\cos\omega t$

(d) Check whether this transform is linear or not.

朝陽科技大學九十學年度研究所碩士班招生考試試題

系別：資訊工程系
組別：一般生
科目：工程數學

3. Let $f(z)$ be analytic in the simply connected domain D that contains the circle C: $|z-z_0|=R$. Then, $f(z)$ has the Taylor series representation

$$f(z)=\sum_{n=0}^{\infty}\frac{f^{(n)}(z_0)}{n!}(z-z_0)^n \quad \text{valid for } |z-z_0|<R$$

Answer the following questions related to this Taylor series representation (20%)

(a) Show that $\exp(z)=\sum_{n=0}^{\infty}(1/n!)z^n$ is valid for all z;

(b) Use the first five terms in the series and obtain an approximation to $\exp((1+i)/8)$, where $i=\sqrt{-1}$.

1. (20%) Find a general solution of the following equations

(a) $y''+8y'+16y=0$

(b) $y''-2y'+10y=0$

2. (15%) Please prove that the Fourier series with the form

$$f(x)=a_0+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$$

can be derived to the complex form

$$f(x)=\sum_{n=-\infty}^{\infty}c_n e^{inx}$$

3. (20%) Laplace transform is defined as

$$L\{f(t)\}=F(s)=\int_0^{\infty}f(t)\cdot e^{-st}dt\ .$$

Convolution f(t) and g(t) denoted by ($f*g$) and defined by

$$f(t)*g(t)=\int_0^t f(\tau)\cdot g(t-\tau)d\tau\ .$$

(a) Please prove that

$$L\{f(t)*g(t)\}=F(s)\cdot G(s)$$

(b) Find $\sin t*\sin t$

4. (15%) Show that the given set is orthogonal on the given interval I and determine the corresponding orthonormal set.

$1, \cos x, \cos 2x, \cos 3x, \cdots,\qquad I: 0\le x\le 2\pi$

朝陽科技大學九十一學年度研究所碩士班招生考試試題

系所別：資訊工程系
組　別：一般生乙組
科　目：工程數學

5. (20%) Integrate $f(z) = \mathrm{Re}\{z\} = x$ from 0 to 1+2i

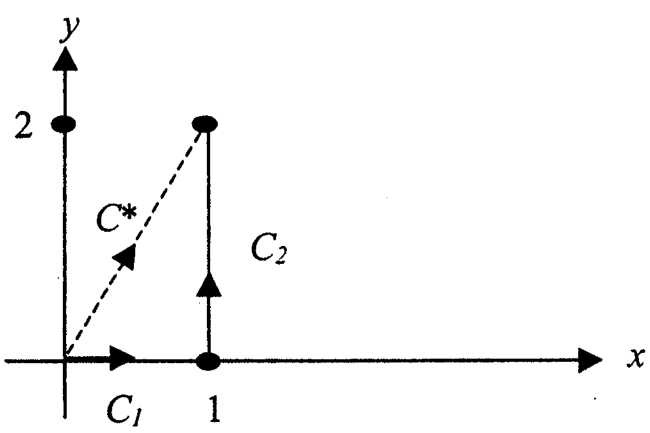

Fig. 1

(a) along C^* in Fig. 1.

(b) along C consisting of C_1 and C_2.

6. (10%) Find the inverse $\mathbf{A}^{-1}$ of

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

朝陽科技大學九十二學年度研究所碩士班招生考試試題

系所別：資訊工程系
組　別：一般生乙組
科　目：工程數學

1. Solve the initial value problems
(10%) (a) $y''+4y'+4y=0,\quad y(0)=1,\quad y'(0)=1.$
(10%) (b) $y'''-4y'=10\cos x+5\sin x,\quad y(0)=3,\quad y'(0)=-2,\quad y''(0)=-1.$
2. (15%) Find the inverse Laplace transform of the function $\ln\left(1+\frac{w^2}{s^2}\right)$.
3. (5%) Find all vectors $\mathbf{v}=[v_1\quad v_2\quad v_3]^T$ orthogonal to $\mathbf{a}=[1\quad 2\quad 0]^T$.
4. (10%) Using gradients, find a unit normal vector $\mathbf{n}$ of the cone of revolution $z^2=4(x^2+y^2)$ at the point P:(1,0,2).
5. (15%) Find the Fourier transform of e^{-ax^2}, where $a>0$.
6. (10%) Represent $(2.6+0.38i)^2$ in polar form, with the principal argument.
7. (10%) If $f(z)=\sum_{n=0}^{\infty}a_n z^n$ is even (i.e., $f(-z)=f(z)$), show that $a_n=0$ for odd n.
8. (15%) Find a basis of eigenvectors and diagonalize for the matrix $\begin{bmatrix}2 & 1\\ 2 & 1\end{bmatrix}$.

朝陽科技大學九十三學年度碩士班招生考試試題

系所別：資訊工程系
組　別：一般生乙組
科　目：工程數學

共八題

1. (10%) Use Gram-Schmidt process to find three orthonomal vectors from

$$s_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix},\ s_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -1 \end{bmatrix},\ s_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

2. (10%) Let $A = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$, use the Laplace transform to compute $\mathbf{e}^{\mathbf{A}}$.

3. (10%) Find the general solutions of the given differential equation

$$x^2y'' - 2xy' + 2y = 2\ln(x) + 1$$

4. (10%) Let **T** be the matrix transformation from $\mathbf{R}^2$ to $\mathbf{R}^3$ whose associated matrix is :

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ 2x_1 + x2 \\ -7x_1 + x_2 \end{bmatrix}.$$

Let $S = \left\{\begin{bmatrix} 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$ and $V = \left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}\right\}$

be ordered bases for $\mathbf{R}^2$ and $\mathbf{R}^3$, respectively. Determine the matrix representation of **T** relative to **S** and **V**.

5. (30%) Solve the initial value problem

(a) $y''-4y'+4y = 0,\quad y(0) = 3,\quad y'(0) = 1.$

(b) $y''+0.2y'+4.01y = 0,\quad y(0) = 0,\quad y'(0) = 2.$

6. (10%) Find the Fourier series of the function

$f(x) = x + \pi \quad if \quad -\pi < x < \pi, \quad and \quad f(x+2n\pi) = f(x),$ where n is an integer.

7. (10%) Find the Fourier transform of the function $f(x) = \begin{cases} k & if \quad 0 < x < a \\ 0 & otherwise \end{cases}$.

8. (10%) Find the all roots of $\sqrt[3]{1+i}$.

朝陽科技大學九十四學年度碩士班招生考試試題
系所別：網通所、資工系(所)乙組
組　別：一般生
科　目：工程數學

共八題，總分100分。

1. Solve the initial value problem
   (a)(10%) $y''+y=0$, $y(0)=3$, $y'(\pi)=-3$.
   (b)(10%) $y'''+3y''+3y'+y=30e^{-x}$, $y(0)=3$, $y'(0)=-3$, $y''(0)=-47$.

2. (10%) For the complex $z=x+iy$, show that $\cos(z)=\cos(x)\cosh(y)-i\sin(x)\sinh(y)$.

3. (10%) Find the Laplace transform of the function $f(t)=\cos at$.

4. (10%) Using Laplace transforms, solve the integral equation
   $$y(t)=t+\int_0^t y(\tau)\sin(t-\tau)\,d\tau$$

5. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$, $C=\begin{bmatrix}1 & 4\\ -2 & 3\end{bmatrix}$, find that:
   (a) (5%) $A(BC)$
   (b) (5%) AB^T
   (c) (5%) $B^{-1}C$

6. (15%) Find the complex Fourier series of $f(x)$ and plot some points of its frequency spectrum:
   $$f(x)=\begin{cases}0, & 0\le x<1\\ 1, & 1\le x<4\\ f(x+4), & for\ all\ x\end{cases}$$

7. (10%) Show that the given functions are orthogonal on the indicated interval:
   (a) (5%) $f_1(x)=x$, $f_2(x)=\cos x$, $[-\pi/2, \pi/2]$
   (b) (5%) $f_1(x)=\cos x$, $f_2(x)=\sin x$, $[0, \pi]$

8. (10%) Use Cramer's rule to solve the system of equations:
   $$x+2y+z=8$$
   $$2x-2y+2z=7$$
   $$x-4y+3z=1$$

朝陽科技大學 95 學年度碩士班招生考試試題

系所別：資訊工程系(所)
組　別：一般生乙組
科　目：工程數學　　　　　　　　　　　　　　　　第 1 頁 共 1 頁

共六題，總分100分。

1. (15%) Using Laplace transforms, solve the initial value problem

$$y'' - 3y' + 2y = 4t, \quad y(0) = 1, \quad y'(0) = -1.$$

2. The function $g(t)$ has the Fourier transform

$$G(w) = \frac{jw}{-w^2 + 7jw + 6}.$$

Find the Fourier transform of the following:

(a) (10%) $g(6t - 12)$

(b) (5%) $\dfrac{dg(t)}{dt}$

(c) (5%) $g(-t)$

(d) (5%) $\displaystyle\int_{-\infty}^{t} g(\tau)d\tau$

3. (10%) Find the Maclaurin series of $f(z)$ and find the radius of convergence.

$$f(z) = \frac{1}{1+z^2}.$$

4. According to the matrix $\mathbf{A}$, please answer the following questions.

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 2 \\ 4 & -1 & -1 \\ 0 & 0 & 3 \end{bmatrix}$$

(a) (5%) What are the eigenvalues and the corresponding eigenvectors of matrix $\mathbf{A}$?

(b) (5%) Can the matrix $\mathbf{A}$ be diagonalized? If yes, complete the diagonalization of matrix $\mathbf{A}$, or find the Jordan form of matrix $\mathbf{A}$ at least.

5. Solve the following differential equations:

(a) (10%) $y'' + 6y' + 9y = e^{-3x}$.

(b) (10%) $y' = \dfrac{y^2 + xy}{x^2}$.

(c) (10%) $y' - y \tan x = e^{\sin x}, \quad y(0) = 3.$

6. (10%) A surface S is defined as: $3x + 2y + z = 6,\ x \geq 0,\ y \geq 0,\ z \geq 0$. Please find out the surface integral value of $\displaystyle\iint_S (x + y + z)\, ds$.

朝陽科技大學 96 學年度碩士班招生考試試題	
系（所）別：資訊工程系 組　　　別：一般生乙組 科　　　目：工程數學	總分：100 分 第 1 頁共 1 頁

1. Let $f(x)=\begin{cases} 1, & 0<x<\pi \\ 0, & -\pi<x<0 \end{cases}$, where $f(x)$ is periodic with $T=2\pi$ ($f(x)=f(x+2\pi)$).

(a) (10%) Find the Fourier series of $f(x)$ = ?

(b) (10%) Based on the result of (a), compute $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots=?$

(c) (10%) Based on the result of (a), solve $y''(x)+16y(x)=f(x)$.

2. (10%) By Laplace technique, solve $y'''(t)+6y''(t)+11y'(t)+6y(t)=\delta^{(4)}(t)+3\delta'''(t)+2\delta''(t)+\delta'(t)$, $y(0)=y'(0)=y''(0)=y'''(0)=0$.

3 (10%) Given $f(t)=t\cdot\int_0^t e^{-2t}\sin 3t\cdot dt$, find its Laplace transform $F(s)=L(f(t))=?$

4. Given the matrixes A=$\begin{bmatrix} -1 & 1 & 1.5 \\ 1 & 0.5 & -3 \\ -0.5 & -1 & 0 \end{bmatrix}$, B=$\begin{bmatrix} -10 & 10 & 15 \\ 10 & 5 & -30 \\ -5 & -10 & 0 \end{bmatrix}$.

(a) (5%) Calculate $(AB)^T-B^TA^T$

(b) (5%) Find the eigenvalues and the corresponding eigenvecters of matrix A.

(c) (5%) Find the eigenvalues and the corresponding eigenvecters of matrix B.

5. A first-order linear differential equation can be written as $y'+p(x)y=r(x)$.

(a) (5%) Show that a Bernoulli equation ($y'+p(x)y=g(x)y^a$) can be reduced to linear differential equation.

(b) (5%) Solve $y'+xy=xy^{-1}$(show the details of your work).

6. Solve the following differential equations:

(a) (5%) $y''-y=0,\ y(0)=3,\ y'(0)=-3$.

(b) (10%) $y''-y=\sin 3x$ (show the details of your work).

7. (10%) Show that the line integral is independent of the path and evaluate the integral

for $\int_A^B e^{x-y+z^2}(dx-dy+2zdz)$, where A=(0,-1,1) and B=(2,4,0).

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朝陽科技大學 97 學年度碩士班招生考試試題

系（所）別：資訊工程系
組　　別：一般生乙組
科　　目：工程數學

總分：100 分

1. (10%) Please solve $\mathbf{Ax}=\mathbf{b}$, where $\mathbf{A}=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$ and $\mathbf{b}=\begin{bmatrix}1\\ 0\\ -1\end{bmatrix}$.

2. (15%) Use the Laplace transform to solve the following initial value problem.

$$y''(x)-y'(x)-2y(x)=0,$$
$$y(0)=1,\ \ y'(0)=0.$$

3. (15%) Please solve the following initial value problem.

$$\mathbf{x}'=\begin{bmatrix}-2 & 1\\ -5 & 4\end{bmatrix}\mathbf{x},\ \ \mathbf{x}(0)=\begin{bmatrix}1\\ 3\end{bmatrix}.$$

4. Let $f(x)=\begin{cases}0, & -2<x<-1\\ 1, & -1<x<1\\ 0, & 1<x<2\end{cases}$ and $f(x+4)=f(x)$ for all real x.

   (a) (10%) Find the Fourier series representation of $f(x)$.

   (b) (10%) Use (a) to find the sum of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}$.

5. (10%) Suppose $x(t)=\begin{cases}1, & |t|<\frac{1}{2}\\ 0, & otherwise\end{cases}$, and the Fourier transform of $x(t)$ is defined as $X(j\omega)$. Please find $X(j\pi)$.

6. (15%) Please solve the following initial value problem.

$$x^3y'''-3x^2y''+6xy'-6y=0,$$
$$y(1)=2,\ \ y'(1)=1,\ \ y''(1)=-4.$$

7. (15%) Let $\mathbf{A}=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$, please compute $e^{\mathbf{A}t}$.

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朝陽科技大學

營建工程系

90~97 學年度
工程數學考古題

朝陽科技大學九十學年度研究所碩士班招生考試試題

系別：營建工程系
組別：甲組一般生
科目：工程數學

1. Find the solution of differential equation $x'' - 4x = 3\delta(t)$ using Laplace transform and discuss initial values at t=0 as limits from the right (i.e. $x(0^+), x'(0^+)$).(25%)

2. There are various differential equations which can be reduced to Bessel's equation. To illustrate this, use the indicated substitution and find the general solution of the following equation in terms of Bessel function.(25%)
$x^2y'' + xy' + (\lambda^2x^2 - \upsilon^2)y = 0 \qquad (\lambda x = z) \qquad \upsilon = \text{constant}$

3. Let $f(x) = 1 + x$ for $0 \le x \le 1$ (25%)
   (a) Expand f in a Fourier cosine series for a period of 2.
   (b) Expand f in a Fourier sine series for a period of 2.
   Hint： Half Range extension

4. A frictionless system of two masses coupled by two springs as shown in Fig. 1 is capable of oscillating as a whole in two different ways, called normal modes, with two different frequencies.(25%)
   (a) Obtain the governing differential equations of motion in matrix form.(Hint:：Identify degrees of freedom first.)
   (b) Find the normal modes(eigenvectors) and their associated natural frequencies.

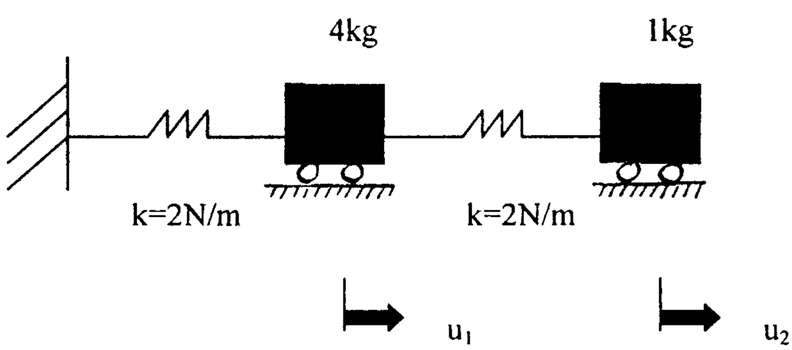

Fig.1

朝陽科技大學九十二學年度研究所碩士班招生考試試題

系所別：營建工程系
組　別：一般生甲組
科　目：工程數學

1. Find the general solution of the following system of simultaneous ordinary differential equations: (25%)

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 2e^{5t} \\ 3e^{2t} \end{bmatrix}$$

where $\mathbf{x} = [x_1 \quad x_2]^T$

2. Solve the differential equation $x'' - x = 2\sin t$, given $x(0) = 1$ and $x'(0) = -1$, by Laplace transform. (25%)

3. Find the complex Fourier series of the following periodic function: (25%)

$$f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ -1 & -1 \le t \le 0 \end{cases} \qquad f(t) = f(t+2)$$

4. Carry out the solution by separation of variables in the boundary-value problem for the wave equation as follows: (25%)

$$\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial t^2} = 0, \quad \text{for } 0 < x < \pi, \quad 0 < t < \infty;$$

$$w(0,t) = 0 \quad \text{and} \quad w(\pi,t) = 0, \quad \text{for } 0 \le t < \infty;$$

$$w(x,0) = \sin 3x \quad \text{and} \quad \frac{\partial w}{\partial t}(x,0) = 0, \quad \text{for } 0 \le x \le \pi$$

朝陽科技大學九十三學年度碩士班招生考試試題

系所別：營建工程系
組　別：一般生甲組
科　目：工程數學

答案卷請依照順序作答，總分為 100 分

1. 試求下列一階微分方程式之通解：(25%)
   $y\cos x dx + 2\sin x dy = 0$
2. 解下列二階微分方程式：(25%)
   $y'' - 2y' + y = e^x$
3. 若 $f(x)$ 為如下列式子所示之週期函數，試求其傅立葉級數（Fourier series）展開。(25%)
   $$f(x) = \begin{cases} -k & ,-\pi < x < 0 \\ k & ,0 < x < \pi \end{cases}$$

4.試求 $A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ 之特徵向量(eigenvectors)，並證明其正交性。(25%)

朝陽科技大學 95 學年度碩士班招生考試試題

系所別：營建工程系(所)
組　別：一般生丁組
科　目：工程數學

滿分 100 分

計算過程必須標明題號，若需假設請輔以詳細說明。

1. solve $y''+y'=e^x\sin x$. <15%>
2. solve $x^2y''+2xy'-6y=0$ <20%>
3. 試求 $A=\begin{bmatrix}2 & 1 & -6\\ 0 & 1 & -3\\ 1 & -1 & -6\end{bmatrix}$ 之特徵值與特徵向量，並說明何謂正交性? <20%>
4. 考慮初值問題，$y''+4y'+3y=e^t$； $y(0)=0$，$y'(0)=2$，請以 Laplace 轉換法求解 $y(t)=?$<20%>
5. 長 10 公尺之繩，單位長度重ρ，原本靜止平衡懸掛於一根光滑之水平細桿上(即左右各 5 公尺掛於該桿上)，假設重力場強度 g=10m/s^2，並給予繩子初始擾動，使其開始向右側滑動，(1)請寫出繩運動的控制方程式(任選一個運動學物理量，如速度或某點之位移)，(2)請問繩脫離桿瞬間之速度為何? <25%>

   (註:第(2)小題未以(1)所列之微分方程解答者滿分 20 分，換言之，未利用解微分方程的方式而以其他物理學原理解答者仍可得分)

朝陽科技大學 96 學年度碩士班招生考試試題

系（所）別：營建工程系
組　　別：一般生甲組、丁組
科　　目：工程數學

總分：100 分
第 1 頁共 1 頁

答案卷請依照順序作答，總分為 100 分

1. 試以分離變數法解微分方程式 $y' + 3x^2 y = 0$。(20%)

2. 解微分方程式 $y'' - y' - 2y = 5\sin x$。(20%)

3. (1) 函數 $f(t) = 3e^t$ 當 $t > 0$，求該函數之拉普拉斯轉換(Laplace transform)。(15%)
   (2) 請以拉普拉斯轉換(Laplace transform)解微分方程式 $y'' - 5y' + 6y = f(t)$，$y(0) = y'(0) = 0$，其中 $f(t) = 3e^t, t > 0$。(15%)

4. (1)求矩陣 A=$\begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$ 之特徵值（eigenvalues）及特徵向量（eigenvectors）。(20%)
   (2) 解聯立微分方程式 $\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}$ 之通解，其中 $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$。(10%)

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朝陽科技大學 97 學年度碩士班招生考試試題

系（所）別：營建工程系	**總分：100 分**
組　　別：一般生甲、丁組	
科　　目：工程數學	第 1 頁共 1 頁

答案卷請依照順序作答，總分為 100 分

1. 試以分離變數法解微分方程式 $y'+2xy=0$。(25%)

2. 解微分方程式 $y''-2y'+y=e^{2x}+x$。(25%)

3. 若 $f(x)=x,\quad 0<x<L$，以該函數進行半幅展開(half-range expansion)，求其對應之傅立葉餘弦級數（Fourier cosine series）及傅立葉正弦級數（Fourier sine series）。(30%)

4. 求矩陣 $A=\begin{bmatrix} 5 & 0 & -15 \\ -3 & -4 & 9 \\ 5 & 0 & -15 \end{bmatrix}$ 之特徵值（eigenvalues）及特徵向量（eigenvectors）。(20%)

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