逢甲大學

土木工程學系
水利工程學系

91~96 學年度
工程數學考古題

科目	工程數學	適用系所	土木工程學系 水利工程學系	時間	一〇〇分鐘

(1) Find a general solution of the third-order ordinary differential equation as follows:

$$y'''-3y''+3y'-y=30e^{x} \qquad (10\%)$$

(2) Solve the following second-order differential equation for y as a power series in powers of $(x-x_0)$

where $x_0=0$, $\quad y''-4xy'+(4x^2-2)y=0 \qquad$ (15%)

(3) Solve the following initial value problem by using Laplace transforms:

$$y''+y=2x,\quad y(\frac{1}{4}\pi)=\frac{1}{2}\pi,\quad y'(\frac{1}{4}\pi)=2-\sqrt{2} \qquad (15\%)$$

(4) Find the inverse matrix of the following matrix:

$$\begin{bmatrix} 19 & 2 & -9 \\ -4 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix} \qquad (10\%)$$

(5) (a) Find the Fourier series of $f(x)=x+\pi$, $-\pi<x<\pi$. (10 %)

(b) Use the result of (a) to show that $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$ (5 %)

(6) The vertical displacement $u(x,t)$ of an infinitely long string is determined from the initial – value

problem: $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ ， $-\infty<x<\infty$ ， $u(x,0)=f(x)$ ， $\frac{\partial u(x,0)}{\partial t}=g(x)$

(a) Find the d'Alembert's solution of $u(x,t)$. (10 %)

(b) If $f(x)=\sin(x)$ ， $g(x)=1$ ， find $u(x,t)$. (10 %)

(7) Evaluate the real integral $\int_0^{2\pi}\frac{\cos 2\theta}{5-4\sin\theta}d\theta$.

Hint: Using the Residue theorem, $z=e^{i\theta}, 0\le\theta\le 2\pi$. (15 %)

逢甲大學九十二學年度碩士班招生考試試題

科目	工程數學	適用系所	土木工程學系、水利工程學系	時間	一〇〇分鐘

1. Find the general solution of the given differential equation: (20%)

$$y'''+3y''+3y'+y=16e^{x}+x+3$$

2. Suppose f(t) is defined for $t \ge 0$, and its Laplace transform = $F(s)$. (10%)

   Find the inverse Laplace transform : $F(s)=\dfrac{2s-5}{s^2+16}$

3. Solve the following differential equation for y as a power series $y=\sum_{n=0}^{\infty} a_n x^n$ (20%)

$$y''+x^2 y=0$$

4. Let $f(x)=\begin{cases}-\pi/4, & -\pi<x<0\\ \pi/4, & 0\le x<\pi\end{cases}$, (a)Find the Fourier series of $f(x)$ on $[-\pi, \pi]$. (10%)

   (b) Using (a) find $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-+\ldots=?$ (5%)

5. Solve the Laplace's equation $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0,\ 0<x<\infty, 0<y<\pi$.

   subject to $u(0,y)=1, 0<y<\pi; u(x,0)=0, u(x,\pi)=0, 0<x<\infty; u(x,y)$ is bounded as $x\to\infty$. (15%)

6. Solve the PDE $x\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial t}=xt$, $x\ge 0,\ t\ge 0$, subject to $u(x,0)=0,\ u(0,t)=0$,

   by Laplace transform. (10%)

7. Evaluate the Cauchy principal value of $\int_{-\infty}^{+\infty}\dfrac{x^2}{(x^2+1)^2}\,dx$. (10%)

逢甲大學九十三學年度碩士班招生考試試題

科目	工程數學	適用系所	土木工程學系、水利工程學系	時間	一〇〇分鐘

1. Find the general solution of the given differential equation

(a) $\dfrac{dy}{dx}-\dfrac{2x}{1-x^2}y=1, y(0)=1$ (5%)

(b) $x^2y''-xy'+y=x\ell nx, y(1)=0, y'(1)=0$ (10%)

2. Solve the following differential equation, where $\delta(t-\pi)$ is the impulse function at t = π

$\ddot{y}+2\dot{y}+2y=\delta(t-\pi)$ (10%)

3. Use the Fr$\ddot{o}$ benius Method to solve the given equation by power series of $(x-x_0)$ where $x_0=0$

$x^2y''+(x^2+x)y'-y=0$ (10%)

4. (a)Use Half-Range Expansion to find the Fourier Cosine Series of $f(x)$, where $f(x)=x$ for $0<x<\ell$ (10%)

(b)Use the result of (a) to show that $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$ (5%)

5. Find the eignevalues of the following matrices:

(a) $\begin{bmatrix}3&0&0\\5&4&0\\3&6&1\end{bmatrix}$ (b) $\begin{bmatrix}3&4\\4&-3\end{bmatrix}$ (10%)

6. Find the surface integral $\iint_S \vec{F}\bullet\vec{n}\,dA$ of the vector function $\vec{F}$ over the surface S, where

$\vec{F}=x^2\vec{i}+e^y\vec{j}+\vec{k},\quad S: x+y+z=1,\quad x\geq 0, y\geq 0, z\geq 0,$

$\vec{n}$ is the unit normal vector of S. (15%)

7. (a) Find the gradient ∇f, where $f=e^{x^2-y^2}\sin(2xy)$ (5%)

(b) Find the directional derivative of f at point P in the direction of $\vec{a}$, where

$f=x^2-y^2,\quad P:(2,3),\quad \vec{a}=\vec{i}+\vec{j}$ (5%)

(c) Find the divergence of the vector function $\vec{F}=y^2e^z\vec{i}+x^2z^2\vec{k}$ (5%)

8. Find the Fourier series of the following function, which is assumed to have the period 2π.

$$f(x)=\begin{cases} x & if\ 0<x<\pi \\ \pi-x & if\ \pi<x<\ 2\pi \end{cases}$$ (10%)

逢甲大學九十四學年度碩士班招生考試試題

科目	工程數學	適用系所	土木工程學系 結構組、大地組	時間	一〇〇分鐘

※ 請務必在答案卷作答區內作答。

1. Find the general solution of the given equation：
   (a) $x^2y' + 2xy = x - 1$ ....(5%)
   (b) $y'' - 2y' + 2y = e^x \cos x$ ....(10%)

2. Find (a) $L\left[\dfrac{\sin t}{t}\right]$ (b) $L\left[\int_0^t \dfrac{\sin t}{t}\,dt\right]$ (c) $\int_0^\infty \dfrac{\sin t}{t}\,dt$ ....(15%)

3. Suppose $f(t)$ is a given Continuous function, $b \neq 0$。Use Laplace Transformation to solve the following system of equations。

$$\begin{cases} \dot{x} = f(t) - (a^2 + b^2)y \\ \dot{y} = x + 2ay \end{cases} \quad x(0) = y(0) = 0 \text{ ....(15\%)}$$

4. The Infinitive Series $\sum_{n=1}^{\infty} \dfrac{(2n)!}{(n!)^2}$ is Convergent or divergent ? (5%)

5. Determine the rank of the following matrices:

(a) $[A] = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$ (b) $[B] = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ -3 & 4 & 4 \\ 1 & 2 & 4 \end{bmatrix}$ .... (15%)

6. Find the eigenvalues of the following matrices:

(a) $[A] = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $[B] = \begin{bmatrix} 7.3 & 0.2 & -3.7 \\ -11.5 & 1.0 & 5.5 \\ 17.7 & 1.8 & -9.3 \end{bmatrix}$ .... (15%)

7. (a) Find the divergence of the vector function $\vec{F} = e^x(\cos y\vec{i} + \sin y\vec{j})$
   (b) Find the Laplacian $\nabla^2 f$, where $f = 4x^2 + 9y^2 + z^2$ .... (10%)

8. Find the Fourier series of the periodic function $f(x)$, of period p = 2, where
   $f(x) = -1 \quad for \quad -1 < x < 0; \quad f(x) = +1 \quad for \quad 0 < x < 1$ .... (10%)

逢甲大學95學年度碩士班招生考試試題

科目	工程數學	適用系所	土木工程學系結構組、大地組	時間	100 分鐘

※請務必在答案卷作答區內作答。

1. Find the general solution of the given equation:
   (a) $(x^2+2y)dx - xdy = 0$ (7%)
   (b) $x^2y' + 2xy = x - 1$ (8%)

2. Find the general solution of the given equation:
   (a) $y'' + y' - 2y = 1 + x + x^2$ (10%)
   (b) $x^2y'' + xy' + 4y = \ell nx(x > 0)$ (10%)

3. Given $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. Find the value of $\Gamma\left(-\frac{1}{2}\right)$ (5%)

4. Suppose $L[f(t)] = \dfrac{3s^2 - 2s - 1}{(s-3)(s^2+1)}$. Find f (t) (10%)

5. Let f(x)=1 for $0 \le x \le 5$. Compute the Fourier sine series of f(x) on [0,5]. (15%)

6. Determine the inverse of matrix A: (10%)

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 1 & 4 \\ 1 & 5 & -2 \end{bmatrix}$$

7. Find the rank of matrix A: (10%)

$$A = \begin{bmatrix} 2 & 1 & -3 & 4 \\ 2 & 4 & -2 & 5 \\ 0 & 3 & 1 & 3 \\ 2 & 1 & -3 & -2 \end{bmatrix}$$

8. Let $f(x,y,z) = xy - yz$, $\vec{v} = [2y, 2z, 4x+z]$, $\vec{w} = [3z^2, 2x^2 - y^2, y^2]$. (15%)
   (a) compute gradient of $f(x,y,z)$: $grad\ f$ at (2,0,7)
   (b) compute divergence of $\vec{v}$: $div\ \vec{v}$
   (c) compute div (grad f)
   (d) compute $grad(div\ \vec{w})$.

科目	工程數學	適用系所	土木工程學系結構組、大地組	時間	100 分鐘

※請務必在答案卷作答區內作答。

1. Find the general solution of the given equation：

(a) $\dfrac{dy}{dx} = -2xy$ ……(5%) (b) $x\dfrac{dy}{dx} + 2y = x^2$ ……(5%)

2. Find the general solution of the given equation：

(a) $x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = \left(\dfrac{dy}{dx}\right)^2$ …(10%) (b) $x^2\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} + y = x^3$ $(x > 0)$…(10%)

3. Find (a)$L(\sin t)$ (b)$L(t\sin t)$ (c)$L(\dfrac{\sin t}{t})$ (d)$\int_0^\infty \dfrac{\sin t}{t}dt$ (10%)

4. Find f(t) of the following Integral equation：(10%)

$$f(t) = e^{-t} - 2\int_0^t \cos(t-u)f(u)du$$

5. Let $f(x) = \begin{cases} 0 \cdots\cdots -\pi < x < 0 \\ x \cdots\cdots \ 0 < x < \pi \end{cases} \cdots f(x+2\pi) = f(x)$

(a)Draw the figure of f(x) ……(5%)

(b)Find the Fourier Series of f(x) ……(10%)

(c)Show that：$\sum_{m=1}^{\infty} \dfrac{1}{(2m-1)^2} = \dfrac{\pi^2}{8}$ ……(5%)

6. Let $f(x) = \begin{cases} 1 \cdots\cdots |x| < 1 \\ 0 \cdots\cdots |x| > 1 \end{cases}$ Find the Fourier Integral of f(x) and show that

$$\int_0^\infty \frac{\sin\lambda}{\lambda}d\lambda = \frac{\pi}{2}$$ (10%)

7. Solve the following boundary value problem：(15%)

P.D.E：$\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} \cdots\cdots 0 < x < \ell$, $t > 0$

B.C.：$u(0,t)=0$ $u(\ell,0)=0$ … $t>0$

I.C.：$u(x,0)=f(x)$ …… $0<x<\ell$

8. Find the eigen values of matrix A (5%)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

逢甲大學

機械工程研究所

91~96學年度
工程數學考古題

逢甲大學九十一學年度碩士班（碩士在職專班）招生考試試題

科目	工程數學	適用系所	機械工程研究所	時間	一OO分鐘

1. Solve the boundary value problem by the method of separation of variables.
$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\ 0<x<2,\ t>0,$$
$$u(0,t)=u(2,t)=0,\ t>0,\qquad u(x,0)=e^x,\ 0<x<2.$$
(20%)

2. Use Stokes' theorem to evaluate $\oint_C \vec{F}\cdot d\vec{r}$, where C is the boundary of S and oriented counterclockwise as viewed from above, $\vec{F}=z\vec{i}+x\vec{j}+y\vec{k}$, and S is that portion of the plane $2x+y+2z=6$ in the first octant. (20%)

3. Show that $\int_0^\infty \frac{\cos xw}{1+w^2}dw=\frac{\pi}{2}e^{-x}$, if $x>0$. (10%)

4. The 3D plot of the function $f(x,y)=xy$ and its contour lines are shown in Fig 1 and Fig 2, respevtively. Draw the gradient fields by a symbol of arrow， then list three your observations at least from the fiugres. (10 %)

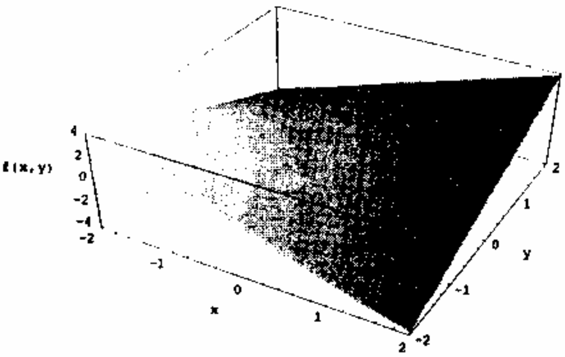


Fig 1

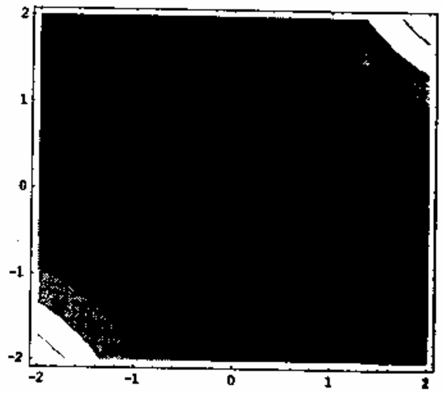


Fig 2

5. Solve the differential equation $EI\frac{d^4y}{dx^4}=w_0\delta(x-\frac{L}{2})$ of the beam subject to $y(0)=0$, $y'(0)=0$, $y''(L)=0$, and $y'''(L)=0$, as shown in Fig 3. (20%)

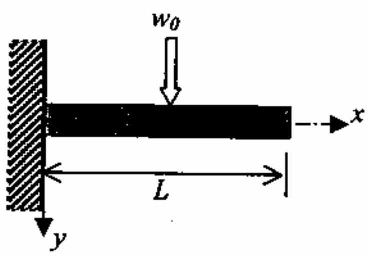


Fig 3

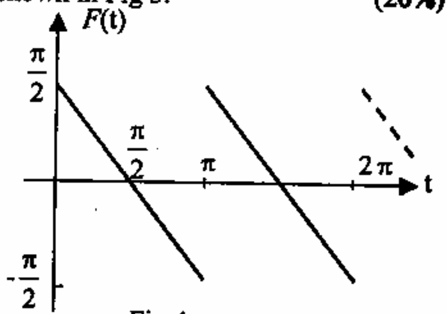


Fig 4

6. Find the displacement $x(t)$ of a mechanical oscillator governed by $mx''+cx'+kx=F(t)$, where m=1 kg, c=0.02 kg/sec, k=25 kg/sec^2,, and $F(t)$ is a periodic forcing function as depicted in Fig 4. (20%)

逢甲大學九十二學年度碩士班招生考試試題

科目	工程數學	適用系所	機械工程研究所	時間	一〇〇分鐘

1. A least squares line of best fit is used to analyze the following data:

   (-2, 0), (-1, 0), (0, 1), (1, 3), (2, 5)

   Determine which data point is farthest from the line of best fit. (15%)

2. Find the work done in moving a particle in the force field $\vec{F} = 3x^2\,\vec{i} + (2xz - y)\,\vec{j} + z\,\vec{k}$ along the curve defined by $x^2 = 4y$, $3x^3 = 8z$, from $x = 0$ to $x = 4$. (15%)

3. Solve for the steady-state temperature distribution in a flat plate covering the region $0 \leqq x \leqq 1,\ 0 \leqq y \leqq 1$, if the temperature on the vertical sides and the bottom side are kept at zero and the temperature on the top side is kept at a constant T_0. (20%)

4. Solve $y'' - 4y' + 4y = x^{-1}e^{2x}$. (20%)

5. Solve the initial-value problem: $y'\cosh^2 x - \sin^2 y = 0,\ y(0) = \pi/2$ (15%)

6. Solve $y(t) = 1 - \sinh t + \int_0^t (1+\tau)\, y(t-\tau)\, d\tau$ (15%)

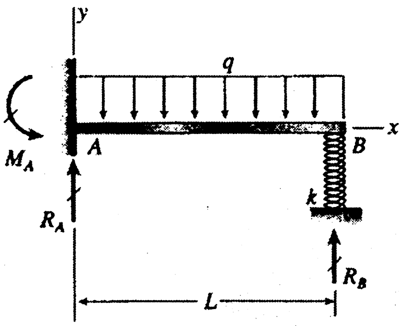
逢甲大學九十三學年度碩士班招生考試試題

科目	工程力學（含工程數學、應用力學、材料力學）	適用系所	機械工程研究所固力組	時間	一五〇分鐘

（共二頁）

＊ 以下共六題，每題 50 分

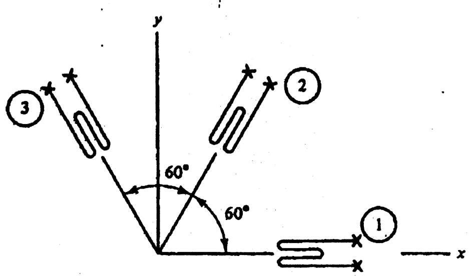
1. A cantilever beam AB of length $L = 5$ m has a fixed support at A and a spring support at B (see figure). The flexural rigidity of the beam is $EI = 1.2 \times 10^6$ N-m^2. The spring behaves in a linearly elastic manner with stiffness $k = 5$ MN/m. If a uniform load of intensity $q = 5$ kN/m acts on the beam, (a) what is the downward displacement δ_B of end B of the beam? (b) Find all reactions of the beam. (c) Draw the shear force and bending moment diagrams for the beam, labeling all critical ordinates.



2. Strain rosette readings are made at a critical point on the free surface in a structural steel member. The 60° rosette (see figure) contains three wire gages positioned at 0°, 60°, and 120°. The readings are

$$\varepsilon_1 = 190\,\mu,\ \ \varepsilon_2 = 200\,\mu,\ \ \varepsilon_3 = -300\,\mu$$

Determine (a) the in-plane principal strains and stresses and their directions, and (b) the maximum shearing strain and maximum shearing stress. The material properties are $E = 200$ GPa and $\nu = 0.3$.



3. Forced vibrations of the string of length L fixed at the ends $x = 0$ and $x = L$ under an external force acting normal to the string are governed by the equation

$$u_{tt} = c^2 u_{xx} + \sin \omega t$$

Find $u(x, t)$ of the string of length L, the initial velocity is zero, and the initial deflection is $0.1 \sin \pi x/L$

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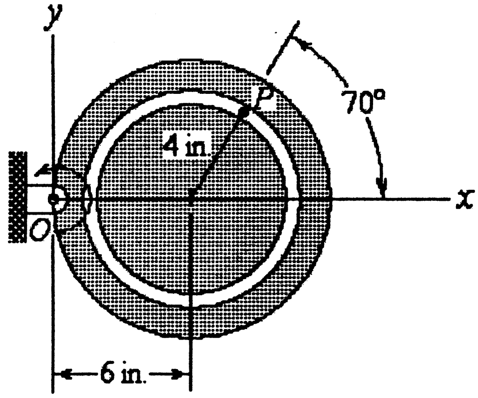
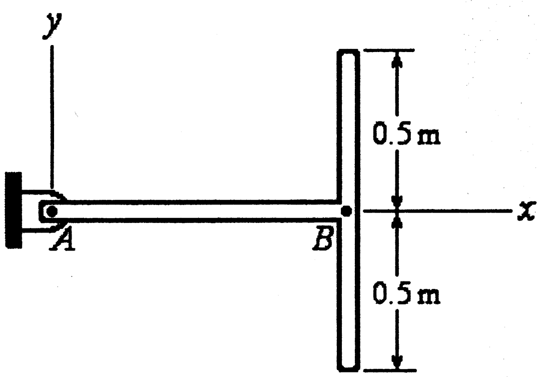
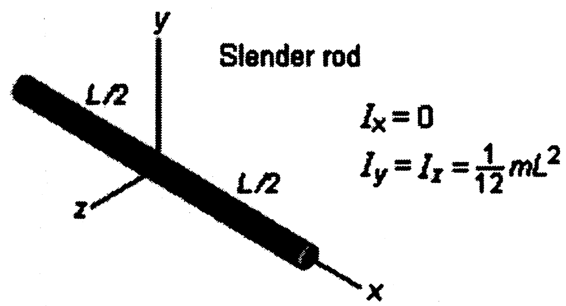
4. A pin P slides in the 4 in. radius circular slot cut in a 6 in. radius plate at a speed of 16 in./s and is decreasing at the rate of 5 in./s^2. At the instant of interest the plate is rotating with an angular velocity of 10 rad/s counterclockwise and is increasing at a rate of 18 rad/s^2. Find the x and y components of acceleration (as in./s) of the pin P at this instant.



5. The T-shaped assembly is made up of two uniform slender rods, each of which is 1 m long and weighs 20 N. Locate the center of mass of the assembly. Find the moments of inertia I_{xx}, I_{yy}, and I_{zz} of the assembly about point A.
If the assembly is released from rest, determine the angular acceleration of the assembly and the reaction force at point A immediately after the assembly has been released.





6. By using Green's Theorem, show that
(a) $\frac{1}{2}\oint_C x^2 dy = -\oint_C xydx = \frac{1}{3}\oint_C (x^2dy - xydx) = A\bar{x}$,
(b) $\frac{1}{3}\oint_C x^3 dy = -\oint_C x^2ydx = \frac{1}{4}\oint_C (x^3dy - x^2ydx) = I_y$,
where A is the area in the x-y plane bounded by a simple closed curve C, $(\bar{x}, \bar{y})$ is its center of mass, and I_y is its moment of inertia about the y axis.

逢甲大學九十四學年度碩士班招生考試試題

科目	工程數學	適用系所	機械工程研究所 固力組、熱流組、製造組、控制組	時間	一〇〇分鐘

※請務必在答案卷作答區內作答。

1. Diagonalize the matrix **A** and find an orthogonal matrix **P** such that $\mathbf{P}^{-1}\mathbf{AP}$ is a diagonal matrix **D**. (20%)

$$\mathbf{A} = \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & -2 \\ 2 & -2 & 0 \end{bmatrix}$$

2. Solve $\ddot{x} = \dot{x}(x-1)$ subject to $\dot{x}(0) = 1$ and $x(0) = 0$. Find the explicit form $x = x(t)$. (15%)
3. Solve the differential equation with variable coefficients . (15%)

$$ty'' - ty' - y = 0, y(0) = 0, y'(0) = 4$$

4. Given a vector function $\vec{F}(x,y,z) = p(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$, show that

$$div\left(curl\vec{F}\right) = 0$$ (15%)

5. Find the work done by the force $\vec{F} = -y\vec{i} + x\vec{j}$ acting along the cardioid $r = 1 + \cos\theta, 0 \le \theta \le 2\pi$.

(15%)

6. Solve the differential equation $y'' + 4y = U(t-2\pi)\sin t$ subject to $y(0) = 1$, $y'(0) = 0$. (20%)

逢甲大學95學年度碩士班招生考試試題

科目	工程數學	適用系所	機械工程研究所	時間	100 分鐘

※請務必在答案卷作答區內作答。

1. $I=\int_0^{2\pi}\frac{d\theta}{\cos\theta+2}$, I=? (20%)

2. (a) Find the inverse of $A=\begin{bmatrix}1&2&-1&2\\3&0&1&5\\1&-2&0&3\\-2&-4&1&6\end{bmatrix}$ (5%)

(b) Evaluate the determinant of A. $A=\begin{bmatrix}1&-2&0&3&6\\2&-4&2&-1&2\\3&3&-1&-3&-3\\2&1&5&0&1\\1&4&7&4&5\end{bmatrix}$ (5%)

(c) Determine the characteristic polynomials, eigenvalues of matrix A. $A=\begin{bmatrix}1&2&-1&2\\3&0&1&5\\1&-2&0&3\\-2&-4&1&6\end{bmatrix}$ (5%)

3. Use Laplace transform to solve the problems:
(a) $y''+y=\cos t$, $y(0)=0$, $y'(0)=0$ (7%)
(b) $y''+3y'+2y=e^{-t}$, $y(0)=0$, $y'(0)=0$ (8%)

4. Solve $x^2y''-2xy'+2y=x^3$ (15%)

5. Use Laplace transform to solve the wave equation $\frac{\partial^2u}{\partial x^2}=\frac{\partial^2u}{\partial t^2}$ subject to the given boundary and initial conditions: $u(-\tfrac{1}{2},t)=0, u(\tfrac{1}{2},t)=0,\ t>0$, $u(x,0)=0$, $u_t(x,0)=\cos\pi x$, $-\tfrac{1}{2}<x<\tfrac{1}{2}$. (20%)

6. Use Green's theorem to evaluate $\oint_C\frac{y^3}{3}dx+\left(xy+xy^2\right)dy$, where C is the boundary of the region in the first quadrant determined by the graphs of $y=0$, $x=y^2$ and $x=1-y^2$. (15%)

逢甲大學96學年度碩士班招生考試試題 編號：005

科目	工程數學	適用系所	機械工程研究所固力組、熱流組、製造組、控制組	時間	100 分鐘

※請務必在答案卷作答區內作答。 共 1 頁第 1 頁

1. Determine the characteristic polynomials, eigenvalues of matrix A.

$$A=\begin{bmatrix} 1 & 2 & -1 & 2 \\ 3 & 0 & 1 & 5 \\ 1 & -2 & 0 & 3 \\ -2 & -4 & 1 & 6 \end{bmatrix}$$ (14%)

2. I. Find all solutions of the equation shown. (a) $e^z+1=0$, (b) $z^3+(2+3i)=0$. (4%)

   II. Identify the branch points. (a) $\sin\sqrt{z}$ (b) $z^{\pi i}$ (c) $\sqrt{\dfrac{z-1}{z-2}}$. (6%)

   III. $I=\int_0^{2\pi}\dfrac{d\theta}{\cos\theta+2}=?$ (10%)

3. I. Compute the Laplace transform of the following functions. (a) $t^{-1/2}$, (b) $\sin^2 t$, (c) $t\sin\omega t$. (3%)

   II. Compute the inverse Laplace transform of the following functions. (a) $\dfrac{1}{s^2+as}$, (b) $\dfrac{1}{s}\left(\dfrac{s-a}{s+a}\right)$, (c) $\dfrac{s}{(s+3)^2+1}$. (3%)

   III. Solve the problem: $y_1''+y_2=-5\cos 2t$, $y_2''+y_1=5\cos 2t$, $y_1(0)=1$, $y_1'(0)=1$, $y_2(0)=-1$, $y_2'(0)=1$. (10%)

4. Solve $x^2y''-4xy'+6y=\ln x^2$. (20%)

5. Find a parametric equation for the normal line of the surface $x^2+2y^2+z^2=4$ at the point $(1,-1,1)$. (15%)

6. Use Green's theorem to evaluate $\oint_C \frac{1}{3}y^3dx+\left(xy+xy^2\right)dy$, where C is the boundary of the region in the first quadrant determined by the graphs of $y=0$，$x=y^2$，$x=1-y^2$. (15%)