義守大學

電子工程學系碩士班
電機工程學系碩士班
材料科學與工程學系碩士班

91~96 學年度
工程數學考古題

義守大學 91 學年度碩士班研究所入學招生考試試題

系所	電子工程學系碩士班、電機工程學系碩士班、材料科學與工程學系碩士班	考試科目	工程數學	考試日期	4 月 14 日

※可使用計算機※

1. Solve the differential equation $y'' - 4y' + 5y = e^{2x} \csc x$. (16%)
2. Find the currents in Fig. 1 when R = 2.5 ohms, L = 1 henry, C = 0.04 farad, $E(t)$ = 169 $\sin t$ volts, and $I_1(0) = I_2(0) = 0$. (20%)
3. Find $h(t)$ by the convolution theorem from the given $H(s) = \mathcal{L}(h)$, where $H(s) = \dfrac{e^{-as}}{s(s-2)}$. (12%)
4. Using Green's theorem, evaluate the line integral $\oint_C \vec{F}(\vec{r}) \cdot d\vec{r}$ counterclockwise around the boundary C pf the region R, where $\vec{F} = [x\cosh 2y, 2x^2 \sinh 2y]$, $R: x^2 \le y \le x$. (16%)
5. Find the Fourier transform of $f(x)$, where $f(x) = \begin{cases} |x| & if \quad -1 < x < 1 \\ 0 & otherwise \end{cases}$. (16%)
6. Find the steady-state temperature in the plate in Fig. 2, where a = 24, with the upper and lower sides perfectly insulated, the left side kept at 0°C, and the right side at $f(y)$ °C. (20%)

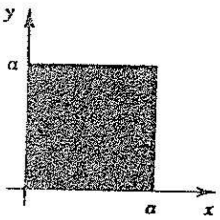


Fig. 1

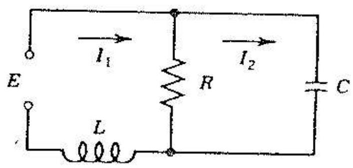


Fig. 2

備註：試題隨答案卷繳交

義守大學九十二學年度博碩士班研究所考試試題

系所別	電機工程學系、電子工程學系	考試日期	4 月 12 日
考試科目	工程數學		

本科可使用計算機

1. Evaluate $\oint_C zdx + xdy + ydz$, where C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $y + z = 2$. Orient C counterclockwise as viewed from above. (18%)
2. (a) The Laplace transform of a piecewise continuous function *f(t)* with period p is
   $$\mathcal{L}(f) = \frac{1}{1-e^{-ps}} \int_0^p e^{-st} f(t)dt \text{ , s > 0.} \qquad (1)$$
   Prove this theorem. (10%)
   (b) Using Eq. (1), find the Laplace transform of the saw-tooth wave in Fig. 1. (8%)

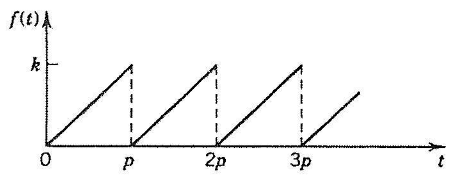
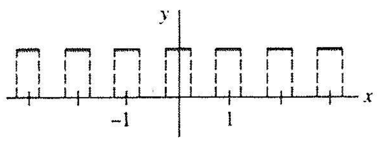


Fig. 1

3. Find out what type of conic section the following quadratic form represents and transform it to principal axes. (16%)
   $4x_1^2 + 12x_1x_2 + 13x_2^2 = 16$
4. Solve $xy'' - 4y' = x^4$. (16%)
5. Find the frequency spectrum of the periodic pulse shown in Fig. 2. The wave is the periodic extension of the function *f*:
   $$f(t) = \begin{cases} 0, & -1/2 < x < -1/4 \\ 1, & -1/4 < x < 1/4 \\ 0, & 1/4 < x < 1/2 \end{cases} .$$



(16%)

6. Integrate $\dfrac{\sin z}{4z^2 - 8iz}$ over the contour C which consists of the boundaries of the squares with vertices ± 3, $\pm 3i$ (counterclockwise) and ± 1, $\pm i$ (clockwise). (16%)

備註：試題隨卷繳交

義守大學九十二學年度博碩士班研究所招生考試試題

系所別	機械與自動化工程學系、土木與防災所、材料科學與工程學系	考試日期	4月12日
考試科目	工程數學		

本科可使用計算機

1. (20%) Solve the initial-value problem $\ddot{x} - 4\dot{x} - 5x = 6e^{-t}$, $x(0) = \dot{x}(0) = 0$.

2. (10%) Using the convolution theorem $(f * g)(t) \equiv \int_0^t f(\tau)g(t-\tau)d\tau$, find the inverse of $F(s) = \dfrac{1}{s^2 + 3s - 10}$.

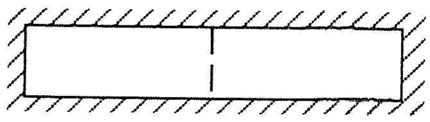
3. (20%) Find the eigenvalues and their corresponding eigenvectors of the following matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 2 & 0 \\ \sqrt{2} & 0 & 0 \end{bmatrix}.$$

4. (35%) Consider a cyclindrical compressed-gas container of length L, divided in half by a baffle (see the following sketch). To the left of the baffle is a gas of species A, and to the right of it is a different gas of species B. Suppose they are at the same pressure, so that when the baffle is removed at time $t = 0$ the two gases proceed to mix by diffusion alone. Considering species A, say, its concentration $c_A(x,t)$ moles/cm^3 is governed by the problem

$$D\frac{\partial^2 c_A}{\partial x^2} = \frac{\partial c_A}{\partial t}, \qquad (0 < x < L,\ 0 < t < \infty)$$
$$\frac{\partial c_A}{\partial x}(0,t) = \frac{\partial c_A}{\partial x}(L,t) = 0,\ (0 < t < \infty)$$
$$c_A(x,0) = \begin{cases} c_0 & 0 < x < L/2 \\ 0 & L/2 < x < L \end{cases}$$

where D is the diffusion coefficient and D and c_0 are constants. Solve for $c_A(x,t)$, and determine the steady state solution $c_{As}(x) = \lim_{t\to\infty} c_A(x,t)$.



$x = 0$ $\qquad$ $x = L/2$ $\qquad$ $x = L$

5. (15%) Locate and name all the singularities of $f(z) = \dfrac{\ln z}{(z^2+1)^3(z+3)^2}$.

APPENDIX: Some functions $f(t)$ and their Laplace transforms $F(s)$

	$f(t)$	$F(s) = \int_0^\infty e^{-st} f(t)\,dt$
1.	1	$1/s$ $(s > 0)$
2.	t	$1/s^2$ $(s > 0)$
3.	t^2	$2/s^3$ $(s > 0)$
4.	t^n $\quad$ ($n =$ positive integer)	$n!/s^{n+1}$ $(s > 0)$
5.	e^{at}	$1/(s-a)$ $(s > a)$
6.	$t^n e^{at}$ $\quad$ ($n =$ positive integer)	$n!/(s-a)^{n+1}$ $(s > a)$
7.	$\sin at$	$a/(s^2+a^2)$ $(s > 0)$
8.	$\cos at$	$s/(s^2+a^2)$ $(s > 0)$
9.	$\sinh at$	$a/(s^2-a^2)$ $(s > \lvert a\rvert)$
10.	$\cosh at$	$s/(s^2-a^2)$ $(s > \lvert a\rvert)$

備註：試題隨卷繳交

義守大學 93 學年度研究所碩士班入學考試

機動系、土木系、材料系『工程數學』參考試題

＊不可使用計算機

***共有七題，總分為 100 分。作答需標明題號，只寫最後答案而無計算步驟者不予計分。**

(1) 解微分方程式 $\dfrac{d^2 f(x)}{dx^2} - 2\dfrac{df(x)}{dx} + 6f(x) = e^x \sin x$，$f(x)$ 為何? (16 分)

(2) 解微分方程式 $\dfrac{d^2 f(x)}{dx^2} + \dfrac{df(x)}{dx} + f(x) = x^2$，$f(x)$ 為何? (16 分)

(3) 計算 Inverse Laplace 轉換 ： $L^{-1}\left\{\dfrac{s^2}{(s^2+1)^2}\right\}$。 [提示: convolution

$f(t) * g(t) = \int_0^t f(x)g(t-x)dx$， $L\{f(t) * g(t)\} = F(s)G(s)$] (16 分)

(4) 假設一維的熱傳方程式可寫成 $\dfrac{\partial^2 u(x,t)}{\partial x^2} = \dfrac{\partial u(x,t)}{\partial t}$，而起始條件為 u(x, 0)=1 (0 < x < L)，邊界條件為 u(0, t) = u(L, t) = 0，使用分離變數法(separation of variables)解此偏微分方程式。 (16 分)

(5) 有一向量場為 $\vec{F} = x^2\hat{x} + y^2\hat{y} + z^2\hat{z}$，試利用散度定理(divergence theorem)計算面積分 $\oiint_S \vec{F} \bullet d\vec{S}$，S 為正方形 $(0 \le x \le 1,\ \ 0 \le y \le 1,\ \ 0 \le z \le 1)$ 的表面。

[提示 ： $\iiint_V (\nabla \bullet \vec{F})dV = \oiint_S \vec{F} \bullet d\vec{S}$] (12 分)

(6) 計算周期為 2L 的函數 f(x)，其傅立葉級數(Fourier series)為何?

$$f(x) = \begin{cases} -1 & (\ if \quad -L < x < 0\ ) \\ 1 & (\ if \quad 0 < x < L\ ) \end{cases}$$

[提示 ： $a_o = \dfrac{1}{2L}\int_{-L}^{L} f(x)dx$,

$a_n = \dfrac{1}{L}\int_{-L}^{L} f(x)\cos\dfrac{n\pi x}{L}dx$ ， $b_n = \dfrac{1}{L}\int_{-L}^{L} f(x)\sin\dfrac{n\pi x}{L}dx$] (12 分)

(7) 若 $\tilde{A}$、$\tilde{B}$、$\tilde{C}$ 為三個矩陣，且 $\tilde{A}\tilde{B} = \tilde{C}$，$\tilde{A} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$，$\tilde{C} = \begin{pmatrix} 3 & 2 \\ 1 & 3 \\ 2 & 1 \end{pmatrix}$，

試利用反矩陣計算，求出矩陣 $\tilde{B}$。 (12 分)

義守大學 93 學年度研究所碩士班入學考試

電機系、電子系、生醫系『工程數學』參考試題

＊可使用計算機

1. Solve $\left(3xe^{y} + 2y\right)dx + \left(x^{2}e^{y} + x\right)dy = 0$. 20%

2. Solve $y'' - 4y' + 4y = e^{2x}/x$. 20%

3. Solve $\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} -6 \\ 2 \end{bmatrix}e^{-2t}$. 20%

4. Evaluate $\int_{(0,0,1)}^{(1,\pi/4,2)} \left[2xyz^{2}dx + \left(x^{2}z^{2} + z\cos yz\right)dy + \left(2x^{2}yz + y\cos yz\right)dz\right]$. 20%

5. Solve $y(t) = t + \int_{0}^{t} y(\tau)\sin(t-\tau)d\tau$. 10%

6. Find the Fourier series of the function $f(x) = x + \pi$ if $-\pi < x < \pi$ and $f(x+2\pi) = f(x)$. 10%

義守大學 94 學年度碩士班入學考試試題

系所別	機動系、土木與生態系、材料系	考試日期	94/4/9
考試科目	工程數學	總頁數	1

＊ 此為試題卷，請將答案填寫於答案卷內，未寫於答案卷內者，不予計分。
＊ 不可使用計算機

1. (20%) Solve the initial-value problem $\ddot{x} - 2\dot{x} + x = 6e^t$, $x(0) = 2$, $\dot{x}(0) = 5$.

2. (10%) Using the convolution theorem $(f * g)(t) \equiv \int_0^t f(\tau)g(t-\tau)d\tau$, find the inverse of Laplace transform function $F(s) = \frac{1}{s^2+s}$.

3. (20%) From the eigenvectors of the following 3×3 matrix, construct an orthogonal basis for $\Re^3$.

$$\mathbf{A} = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

4. (10%) Sketch the extension function of half–range sine, and derive the corresponding expansion of the following function

$$f(x) = 5x, \text{ on } 0 < x < 4.$$

HINT: Fourier coefficients

$$\begin{aligned} a_0 &= \frac{1}{2l}\int_{-l}^{l} f(x)dx, \\ a_n &= \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi x}{l}dx, n = 1, 2, \cdots \\ b_n &= \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi x}{l}dx, n = 1, 2, \cdots \end{aligned}$$

5. (20%) Consider a 1-D diffusion problem with mixed boundary conditions and initial condition

$$D\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \qquad (0 < x < 2,\ 0 < t < \infty)$$

$$u(0,t) = 10,\ \frac{\partial u(2,t)}{\partial x} = -5,\ (0 < t < \infty)$$

$$u(x,0) = 10,\ (0 < x < 2)$$

where D is the diffusion coefficient. Solve for $u(x,t)$, and determine the steady state solution $u_s(x) = \lim_{t\to\infty} u(x,t)$.

HINT: you might start to solve this problem from the general solution by separation of variables

$$u(x,t) = A + Bx + (C\cos\kappa x + D\sin\kappa x)e^{-\kappa^2 Dt}.$$

6. (20%) By using residue theorem, evaluate

$$\oint_C \frac{z}{\sin z}dz$$

where C is the circle $|z| = 4$ in the positive sense.

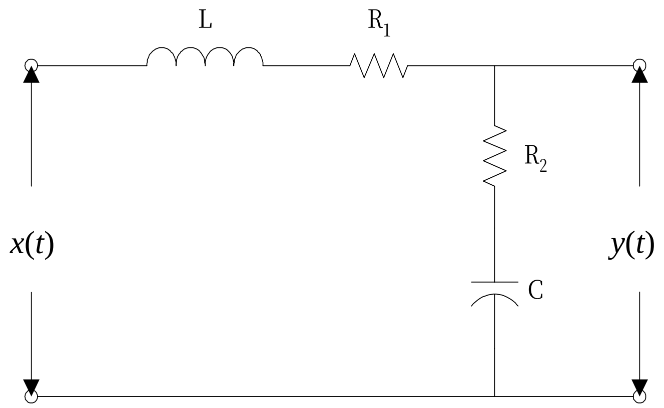
義守大學 94 學年度碩士班入學考試試題

系所別	電機系、電子系、生醫系	考試日期	94/4/9
考試科目	工程數學	總頁數	

＊此為試題卷，請將答案填寫於答案卷內，未寫於答案卷內者，不予計分。
＊可使用計算機

1. Solve the following differential equations by using the Laplace transforms method.
(a) $y'''(t)+3y''(t)+4y'(t)+2y(t)=2u(t)$, where $u(t)$ is the unit step function, (10%)
$y''(0)=y'(0)=y(0)=0$ and $y(t)$ for all $t<0$.
(b) $y''(t)+5y'(t)+4y(t)=2\delta(t)$, where $\delta(t)$ is the unit impulse function, $y'(0)=y(0)=0$ and $y(t)$ for all $t<0$. (10%)

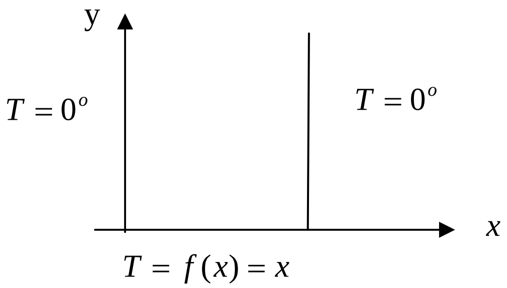
2. The electrical network shown in figure is characterized by the equation
$12y''(t)+7y'(t)+y(t)=2x'(t)+x(t)$ for $t>0$.
Find
(a) the frequency response $H(\omega)$, (10%)
(b) the magnitude and phase spectra of $y(t)$ assuming that $x(t)=e^{-t}u(t)$, (10%)
(c) the time-domain expression of $y(t)$. (10%)



3. A fair coin is tossed until a head appears. Let X denote the number of tosses required.
(a) Find the probability density function of X. (10%)
(b) Find the mean and variance of X. (10%)

4. Find the directional derivative of $\phi=x^2yz+4xz^2$ at point (1, -2, -1) in the direction $2\vec{i}-\vec{j}-2\vec{k}$. (10%)

5. Find the steady-state temperature distribution $\nabla^2\mathrm{T}=0$ for the semi-infinite plate, if the temperature of the bottom edge is T=f(x)=x degrees. The temperature of the other sides is 0 degree, and the width of the plate is 10 cm. (20%)



1025

義守大學九十五學年度碩士班入學考試試題

系所別	機械與自動化工程學系 土木與生態工程學系、材料科學與工程學系	考試日期	95.04.15
考試科目	工程數學	總頁數	1

＊此為試題卷，請將答案填寫於答案卷內，未寫於答案卷內者，不予計分。

＊ 本科不可使用計算機

1. (20%) Solve the initial-value problem $xy' - y = x^3$, $y(1) = 1$.

2. (10%) Using the convolution theorem $(f * g)(t) \equiv \int_0^t f(\tau)g(t-\tau)d\tau$, find the inverse of Laplace transform function $F(s) = \dfrac{1}{s^2 - a^2}$.

3. (10%) Find any two vectors in $\mathfrak{R}^3$ that span the plane $2x_1 + x_2 - 6x_3 = 2$.

4. (20%) From the eigenvectors of the following 4×4 matrix obtain an orthogonal basis for $\mathfrak{R}^4$.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}.$$

5. (20%) The temperature distribution $T(x,t)$ in a 2-m long brass rod is governed by the problem

$$\alpha^2 \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}, \qquad (0 < x < 2,\ 0 < t < \infty)$$

$$T(0,t) = T(2,t) = 0,\ (t > 0)$$

$$T(x,0) = \begin{cases} 50x, & (0 < x < 1) \\ 100 - 5x, & (1 < x < 2) \end{cases}$$

where α^2 is the diffusivity of the material. Solve for $T(x,t)$. What is the temperature distribution after a long time?

HINT: you might start to solve this problem from the general solution by separation of variables

$$T(x,t) = A + Bx + (C\cos\kappa x + D\sin\kappa x)e^{-\kappa^2\alpha^2 t}.$$

6. (20%) Find all possible series expansions, with center at $c = 1$, for

$$f(z) = \frac{1-z}{z-2}$$

in certain domains, and specify these domains.

義守大學九十五學年度碩士班入學考試試題

系所別	電機工程學系、電子工程學系	考試日期	95.04.15
考試科目	工程數學	總頁數	

＊此為試題卷，請將答案填寫於答案卷內，未寫於答案卷內者，不予計分。
＊本科可使用計算機

1. Solve the following differential equation: (10%)
   $x - xy - y' = 0$
2. Find a general solution of the following equation: (10%)

   $$y'' - 4y' + 4y = \frac{e^{2x}}{x}$$

3. Using Laplace transform to solve the given equation: (20%)

   $$y' + y = f(x),\ y(0) = 5,\ where\ f(x) = 3\cos t \cdot u(t - \pi)$$

4. Evaluate $\oint_{\Gamma} \frac{2z+1}{z^2 + 3iz} dz$, where Γ is the circle $|z + 3i| = 2$ (15%)

5. Using Stoke's Theorem to calculate $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = -5y\vec{i} + 4x\vec{j} + z\vec{k}$

   and C is the circle: $x^2 + y^2 = 4,\ z = 1$ (15%)

6. Find the Fourier coefficients of $f(x) = \begin{cases} -1, if\ -\pi < x \le 0 \\ 1, if\ 0 < x \le \pi \end{cases}$ and

   $f(x + 2\pi) = f(x)$ (10%)

7. a. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ (10%)

   b. Calculate A^{100} (10%)

系所別	電機系、電子系	考試日期	96/4/14
考試科目	工程數學	總頁數	1/1

＊此為試題卷，請將答案填寫於答案卷內，未寫於答案卷內者，不予計分。
＊可使用計算機

1. Solve the differential equation: (20%)

$$2y - 9x + (3x - 6\frac{x^2}{y})y' = 0$$

2. Solve the differential equation: (20%)

$$x^2 y'' - xy' + y = 2x$$

3. Use the indicated substitutions, find a general solution in terms of Bessel function.

$$x^2 y'' - 5xy' + 9(x^6 - 8)y = 0 \quad (y = x^3 u,\ x^3 = z)$$ (10%)

4. Solve the differential equation: (20%)

$$y'' + 4y' + 5y = [1 - u(t-10)]e^t - e^{10}\delta(t-10)$$
$$y(0) = 0,\ y'(0) = 1$$

5. If $\mathbf{F} = xy\vec{i} + y^2 z\,\vec{j} + z^3\vec{k}$, evaluate $\iint_S (\mathbf{F} \cdot \mathbf{n})dS$, where S is the unit cube defined by $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1$. (15%)

6. Expand the following function in Fourier integral. (15%)

$$f(x) = e^{-|x|}$$

1028

系所別	電機系、電子系	考試日期	96/4/14
考試科目	工程數學	總頁數	1/1

＊此為試題卷，請將答案填寫於答案卷內，未寫於答案卷內者，不予計分。

＊可使用計算機