嘉義大學

土木與水資源工程學系碩士班

91~97 學年度
工程數學考古題

國立嘉義大學九十一學年度

土木與水資源工程學系碩士班招生考試試題

科目：工程數學 （※禁止使用計算機）

一、若 $\sigma = \begin{bmatrix} \mathbf{2} & \mathbf{0} & \mathbf{-2} \\ \mathbf{0} & \mathbf{4} & \mathbf{0} \\ \mathbf{-2} & \mathbf{0} & \mathbf{5} \end{bmatrix}$ 代表一應力張量。試應用特徵值(Eigenvalues)與特徵向量(Eigenvectors)之方法求出： (15 分)

1.三個主應力大小；

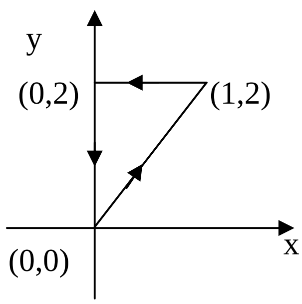
2.三個主應力方向之單位向量。

二、1.函數 $f(x) = \begin{cases} 1\ , & |x| < 1 \\ 0\ , & |x| > 1 \end{cases}$ 之 Fourier 積分式為何？ (15 分)

2.由 1.推求 $\int_0^\infty \frac{\sin\ x\ \cos\ x}{x} dx$ =？

三、寫出高斯(Gauss)積分定理，並舉例說明其物理意義。 (10 分)

四、限用 Green 定理求 $\oint_c \mathbf{(4x^2ydx + 2ydy)}$ 之值，其中積分路徑 C 為以（0 ,2），（0 ,0），（1,2）三點為頂點之三角形邊界（逆時針方向）如下圖。 (15 分)



五、一無限長承受均佈荷重之彈性基礎樑，撓度微分方程式可表示為：

$EI\dfrac{d^4y}{dx^4}+ky=q$ ，式中 EI、k、q 皆為常數時，請推求該樑撓度之全解

$y(x)$ =？(不需求出積分常數)　(15 分)

六、使用 Laplace 轉換方法計算下述系統之反應 $y(t)$ =？

$\dfrac{d^2y}{dt^2}+y=U(t^3-7t^2+14t-8)$ ，$y(0)=\dfrac{dy(0)}{dt}=0$ ，式中 U 為單位步階函數

(unit step function)。　(15 分)

七、請用複變積分計算下式之瑕積分值：(15 分)

$$\int_0^\infty \frac{\cos 2x}{x^2+9}\,dx$$

國立嘉義大學九十二學年度
土木與水資源工程學系碩士班招生考試試題

科目：工程數學

一、(1)解微分方程式 $y'' + 2y' + y = 2e^{-x}$。(10%)

(2)解 $Y'' = AY$ 其中 $A=\begin{bmatrix} -4 & 3 \\ 0 & 1 \end{bmatrix}$。(15%)

二、(1)求 $f(t) = e^{-2t}\sin 3t$ 之 Laplace Transform。(6%)

(2)利用 Convolution 定理求 $\dfrac{1}{S(S^2+4)}$ 之 Inverse Laplace Transform。(9%)

三、$\begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & -1 \\ -3 & 1 & 2 \end{bmatrix}$，試用 Cayley-Hamilton 定理證明 $A^3-3A^2+16A-9I=\mathbf{O}$，並求 A^{-1}，式中 I 為單位矩陣。(20%)

四、試用變數分離法解 PDE $y\dfrac{\partial u}{\partial x} - x\dfrac{\partial u}{\partial y} = 0$，$u = f(x, y)$。(15%)

五、求 f (x, y, z)= xy sin z 在 $\vec{a} = i + 2j + 2k$，點(1, 2, $\pi/2$)的方向導數。(10%)

六、試求 $f(x) = |\sin x|$，$-\pi < x < \pi$，$T = 2\pi$。(15%)

國立嘉義大學九十三學年度
土木與水資源工程學系碩士班招生考試試題

科目：工程數學

注意：1.本試題不可使用計算機

2.本試題如條件不足，請自行假設

1. Find the general solution for the following equations. (20%)

(a) $xy'' - 3y' + 4\dfrac{y}{x} = 0$

(b) $y' + y = xy^4$

2. Prove $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a orthogonal matrix and find it's Eigenvalues. (15%)

3. Use Convolution theorem of Laplace transform to solve the equation of

$$y(t) = e^{-2t} + \int_0^t y(\tau)e^{(t-\tau)}d\tau \qquad (15\%)$$

4. Use Laplace transform to solve the equation. (15%)

$$y'' - 3y' + 2y = 4e^{2t}, \quad (\ y(0) = -3, \quad y'(0) = 5)$$

5. Find the Fourier Series of function **f(t)**, the period **T**= $\dfrac{\pi}{2}$

$$\mathbf{f(t)} = \begin{cases} \mathbf{t} & \text{in } -\dfrac{\pi}{8} < \mathbf{t} < \dfrac{\pi}{8} \\ \left(\dfrac{\pi}{4} - \mathbf{t}\right) & \text{in } \dfrac{\pi}{8} < \mathbf{t} < \dfrac{3\pi}{8} \end{cases} \qquad (20\%)$$

6. Find the unit normal vector at point (1,1,2) on the surface of the equation

$\mathbf{z^2 = 2(x^2 + y^2)}$。(15%)

國立嘉義大學九十四學年度
土木與水資源工程學系碩士班招生考試試題

科目：工程數學

(如有條件不足之情形，請自行假設。)

1. Find the general solution for the following equation: (20%)

$$(2x+4)^2 y'' - 4(2x+4)y' + 8y = 4\ln(2x+4)$$

2. (a) Derive the Laplace Transformation from Fourier Transformation. (10%)
   (b) Describe the difference between Laplace Transformation and Fourier Transformation from physical view. (10%)

3. $\vec{\mathrm{F}} = e^x \cos y\vec{i} - e^x \sin y\vec{j} + z^2\vec{k}$. Determine whether $\vec{F}$ is conservative in the entire space? (10%) If it is, find a potential function and evaluate $\int_C \vec{F} \cdot d\vec{r}$. Where C is the path from (0, 0, 2) to (1, π/4, 1) in the space, $\vec{r}$ is the position vector for any point on C. (10%)

4. (a) $f(t) = \begin{cases} \sin t, & 0 < t < 2\pi \\ 0, & t > 2\pi \end{cases}$ find Laplace Transform of *f(t)*. (10%)

   (b) $\mathrm{F}(s) = \ln(\dfrac{s^2+1}{s^2+9})$, find the Inverse Laplace Transform. (10%)

5. Use the residue theorem to evaluate $\oint_\Gamma \dfrac{z}{1+z^2} dz$, where Γ the circle $|z| = 4$ (oriented positively). (20%)

國立嘉義大學九十六學年度
土木與水資源工程學系碩士班招生考試試題

科目：工程數學

（如條件不足，請自行假設。）

1. The wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$, $u(0,t)=0$, $u(l,t)=0$, $u(x,0)=\sin\dfrac{\pi x}{l}$, $u_t(x,0)=0$.
   Solve the PDE by the method of separating variables. (20%)

2. Evaluate the following integral by residue integration method. (20%)

$$\int_0^\infty \frac{\cos 2x}{x^2+4}\,dx = ?$$

3. Matrix A=$\begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$, determine its Eigen values and Eigen vectors. (20%)

4. Derive Tylor series at a. (20%)

$$\boldsymbol{f(x) = f(a) + \frac{f'(a)}{1!}(x-a)^1 + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots\ldots\ldots\ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \ldots\ldots}$$

5. $\vec{F} = x\vec{i} + y\vec{j} + z^2\vec{k}$, $\iint_s (\vec{F}\cdot\vec{n})\,ds = ?$ (20%)
   Where s is the surface of the cylinder $x^2+y^2=4$, $0 \le z \le 3$.
   Find the values by direct surface integration.

國立嘉義大學九十六學年度
土木與水資源工程學系碩士班招生考試試題

科目：工程數學

（如條件不足，請自行假設。）

1. The wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, $u(0,t)=0$, $u(l,t)=0$, $u(x,0)=\sin\frac{\pi x}{l}$, $u_t(x,0)=0$.
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   Where s is the surface of the cylinder $x^2+y^2=4$, $0 \le z \le 3$.
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國立嘉義大學九十七學年度
土木與水資源工程學系碩士班招生考試試題

科目：工程數學

說明：如條件不足，請自行假設。

1. Solve the following ordinary differential equation: (20%)

$$y'' + y' - 2y = 6e^{x}$$

2. Solve the following ordinary differential equations using Laplace Transformation: (20%)

① $y(t) = 1 + \int_0^t y(\tau)\, d\tau$

② $y(t) = 1 + y'(t) \quad , y(0) = 1$

3. Solve the following ordinary differential equation: (20%)

$$y^{(4)} + 3y = e^{2x}$$

4. Define $\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$ as an operator, a function $F(x, y, z) = x^2 + y^2 + z^2$, and a vector $\vec{v} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ are known. Calculate results of the following questions: (20%)

① gradient $\nabla F = ?$ ② divergence $\nabla \cdot \vec{v} = ?$ ③ curl $\nabla \times \vec{v} = ?$

④ $\nabla\nabla F = ?$ ⑤ $\nabla \cdot \nabla F = ?$ ⑥ $\nabla \times \nabla F = ?$

⑦ $\nabla\nabla\vec{v} = ?$ ⑧ $\nabla \cdot \nabla \cdot \vec{v} = ?$ ⑨ $\nabla \times \nabla \times \vec{v} = ?$ ⑩ $\nabla \cdot \nabla \times \vec{v} = ?$

5. For the following matrix, (a) find the eigenvalues and eigenvectors;

(b) diagonalize the matrix **A** by using eigenvectors. (20%)

$$\mathbf{A} = \begin{bmatrix} -3 & 2 \\ -10 & 6 \end{bmatrix}$$

國立嘉義大學九十七學年度
土木與水資源工程學系碩士班招生考試試題

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